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Nevis, REU 2001

Measurement of the top quark mass

Employing two thousand physicists from more than 150 universities and 34 countries, the ATLAS experiment is the largest collaboration ever attempted in the history of all physical sciences. Its general-purpose proton-proton detector will exploit the potential of the Large Hadron Collider (LHC), at CERN Laboratory in Switzerland. The main goal of ATLAS is to understand the origin of mass at the electroweak scale. The existence of the Higgs boson has been speculated in order to explain the origin of spontaneous symmetry-breaking mechanism in the electroweak sector of the Standard Model. Other important issues at hand are the discoveries of the heavy W^- and Z-like objects, supersymmetric particles, and compositeness of the fundamental fermions. During my stay at Nevis Labs, Columbia University, I was involved in the development of a simulation program for the analysis of proton-proton interactions in an environment similar to what ATLAS will experience. The effort of my research was focused mainly on the measurement of the top quark mass using several correction algorithms.

Review of Physics

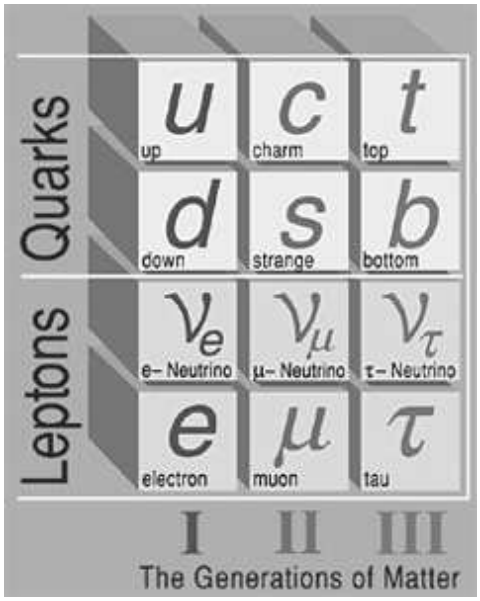
During the 1930's, our physical understanding of the world seemed almost complete. Decades before, in the famous gold foil scattering experiment, Sir Rutherford had shown that atoms have a relatively tiny but massive nuclei. Other phenomena such as atomic spectra and electron orbitals were understood using quantum theory. Furthermore, the discovery of the neutron explained the existence of nuclear isotopes. Thus, at that era, the fundamental building blocks of matter consisted simply of protons, neutrons, and electrons. However, a few puzzling questions remained unanswered. Such as, what binds protons and neutrons together in the nucleus? What are the processes of radioactive decay? To observe neutron-proton interactions, physicists needed a tool to probe within the tiny nucleus. As in earlier experiments, such a probe would be devised by scattering a particle from the nucleus. However, such a particle needed a very short wavelength and a very high momentum. De Broglie had proposed earlier that the wavelength, λ, of the associated wave of a particle is inversely proportional to its momentum, p, in the following way: $\lambda = h / p$, where h is Planck's constant. Hence, the more momentum a particle has, the shorter its wavelength. Thus, particle accelerators were invented.

Particle detectors grew more complex as the technology of particles accelerators excelled. They have been used extensively to study high-energy particle collisions. Unexpectedly, accelerator collisions lead to the discovery of vast amounts of new particles. Some of the new particles resembled protons and neutrons. They were named baryons. However, a completely new family of particles, known as mesons, was also discovered. By the 1960's a hundred or so types of particles had been discovered.

In 1964, Murray Gell-Mann and George Zweig independently recognized that all known particles are built from smaller entities known as quarks. From these quarks and their anti-partners alone, all the observed behaviors of baryons and mesons could be accounted for. We now know that quarks come in six flavors referred to as up, down, strange, charm, bottom, and top, in order of increasing mass. Several quark properties are worth mentioning. Unlike protons and electrons, quarks have fractional charge $2/3$ or $-1/3$, in units of the proton charge. However, they are never found alone, but in combinations yielding a net integer charge. Particles built from quarks are known as hadrons. Lastly, anti-quarks are the antimatter partners of quarks. They share the same mass as their corresponding quark but have opposite charge. When a quark and an anti-quark collide, they annihilate, releasing pure energy.

The Standard Model

Quarks have been extensively confirmed experimentally. They are now part of the Standard Model of Fundamental Particles and Interactions. There are also six other fundamental types of particles, including the electron, known as leptons. The Standard Model explains the strong, weak, and electromagnetic interactions of the quarks and leptons. Hence, nuclear bindings and decay are understood. Baryons and mesons were mentioned earlier. Baryons consist of any combination of three quarks while mesons consist of any combination of a quark and an anti-quark. Unlike quarks, all six leptons can be found alone in nature. The most famous lepton is the electron. Two other charged leptons are the muon and the tau, discovered in 1936 and 1975 respectively. They have the same charge as the electron, but are much heavier. The remaining three leptons are very elusive particles known as neutrinos. They have no electric charge and very little mass or none. There is one type of neutrino for each of the three types of electrically charged leptons. For all leptons, there is an anti-lepton with equal mass and opposite charge.

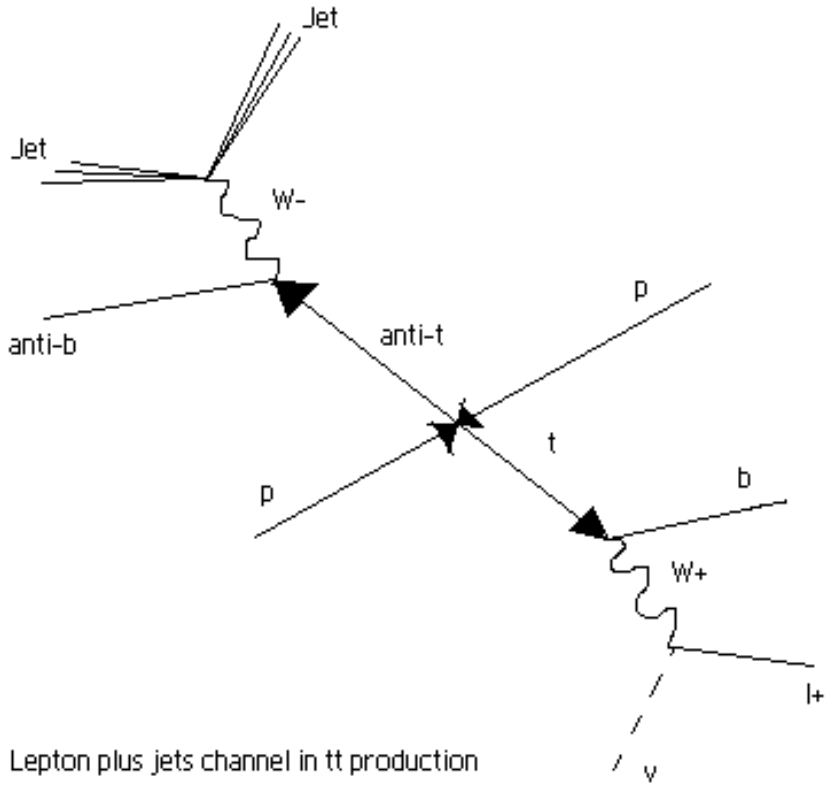


The Standard Model explains the structure and stability of matter quite well. However, several problems remain unsolved. For example, it incorporates all forces but gravity. Furthermore, why are there three families of quarks and leptons of each charge? Are there any more new fundamental particles to be discovered? Are the quarks and leptons fundamental or do they have substructure? What particles form the dark matter in the universe? ATLAS may finally unveil the truth behind these mysteries.

Events

The production of $t\bar{t}$ events (top, anti-top) was studied. Proton-proton events were selected in which at least 4 jets were generated, of which at least 2 were b-jets. Another requirement was the presence of at least one lepton (either a muon or an electron). Jets are streams of particles that deposit their energy within a small cone in the detector. They result from quark or gluon emission. PYTHIA generated the events by Monte Carlo. ATLFAST performed the detector simulation. The analysis program was written in Root and C++ (see attached code).

The collisions of the two protons yield a top quark and an anti-top quark. The top quark produced a b quark jet and a W^+ boson, which itself decayed to a neutrino and a positive lepton. On the other hand, the anti-top quark produced an anti-b quark jet and a W^- boson, which itself decayed to two jets.



Lepton plus jets channel in tt production

Preview of Mass Calculations

The invariant mass of the W boson, M_{jj}, was calculated from the two non-b jets after ΔR matching. ΔR matching is a statistical technique that is used to identify the two non-b jets produced from the W boson. It relies on the principal idea that a jet is more likely to have been produced from a particle that is lined up in its direction of motion than from one that is not. A more detailed explanation will be given later. The calculation was repeated twice with two different correction schemes. Then, M_{jj} was recalculated after matching on M_w, the real mass of the W. Finally, the last result of M_{jj} was combined with the second correction scheme. The invariant mass of the top quark, M_{bjj}, was calculated after ΔR matching. The calculation was repeated twice with the two previous correction schemes. Then, M_{bjj}, was recalculated after matching on M_{top}, the real mass of the top. Finally, the last result of M_{bjj} was combined with the second correction scheme.

Parton Analysis

The program began with a parton analysis. Partons are basically point-like quarks and gluons that make up the jets. The two non-b jets consist mainly of light quarks, which hadronize into other particles. Remember that quarks are not found alone in nature. After simulating the proton-proton interaction, the program identified all the remnants of the events and found its way back to the light quarks. Once these light quarks were identified, it remained to verify that they did indeed result from the decay of a W boson. As mentioned above, this test was performed using ΔR matching. This will be discussed later on. At this stage of the program, we also identified the b and anti-b quarks, muons, electrons, and neutrinos.

Calibration Corrections for b-Jets and Jets

All jets, b-jets and jets, were calibrated with the use of a third degree polynomial depending on the transverse jet momentum, Pt_{jet}. Pt_{jet} was calculated as $Pt_{jet} = \sqrt{Px_{jet}^{\,2} + Py_{jet}^{\,2}}$ and inserted into the Pt_{jet} polynomial expression, resulting in a correction factor. Lastly, the four-momentum of the jets were multiplied by this factor. The four-momentum is generally expressed as (E, Px, Py, Pz). The b-jets and jets were calibrated with their own set of coefficients for the Pt_{jet} polynomial. The two distinct sets of coefficients were obtained from previous study.

Cuts on Pt Lepton, Pt Jet, and Missing Et

The transverse momentum of the lepton, Pt_l, was calculated for the case of a muon or an electron. Only events for which Pt_l > 20 Gev/c were selected. In addition, only events where at least 4 jets had Pt_{jet} > 40 GeV/c were selected. The transverse missing energy was calculated as $Et_{miss} = \sqrt{Px_{miss}{}^2 + Py_{miss}{}^2}$ and only events where Et_{miss} > 20 Gev were selected.

Delta-R Matching

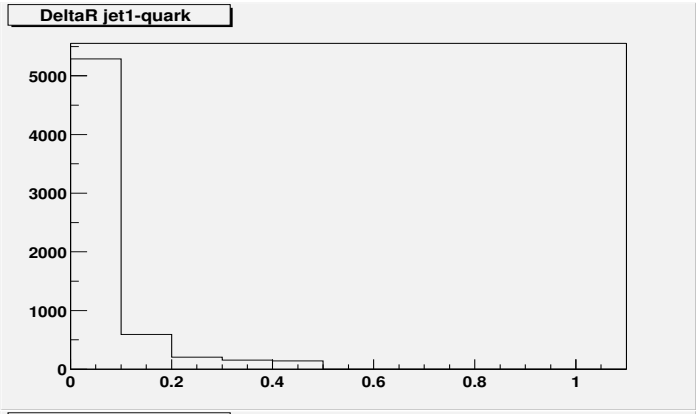
The objective of the ΔR matching is to match jets with their parent particle. It is a technique based on statistical means. A jet is more likely to have been produced from a particle that is lined up in its direction of motion than from one that is not. We define angles θ, ϕ, and η such that:

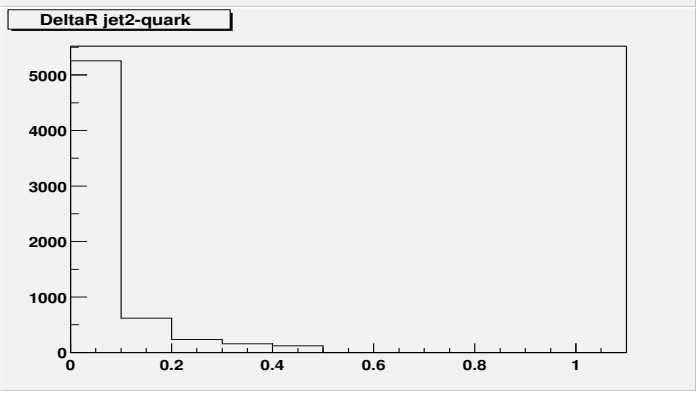
$$\theta = \cos^{-1}(Pz/P), \qquad \phi = \tan^{-1}(Py/Px), \qquad \eta = -\log(\tan(\theta/2))$$

for $P = \sqrt{Px^2 + Py^2 + Pz^2}$

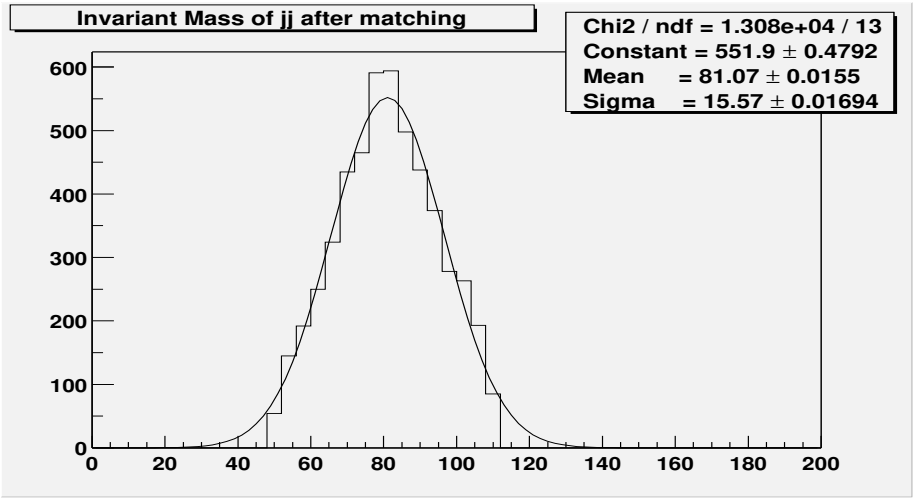
Let an axis be defined along the path of the two colliding protons. Then, variable θ is defined as the angle between a jet and the axis. Variable ϕ is defined as the angle swiping around the axis. The protons are accelerated at velocities very close to the speed of light. For those reasons, one must take into account relativistic effects such as Lorentz transforms. The angle θ between the jet being considered and the axis lies in the laboratory frame of reference. In the particle frame of reference, this angle has no meaning due to Lorentz tranformation. Instead, pseudorapidity, η, is used to represent θ in the particle's frame of reference. $\Delta\eta$ and $\Delta\phi$ are defined as $\Delta\eta = \eta_{jet_kin} - \eta_{jet_recon}$ and $\Delta\phi = \phi_{jet_kin} - \phi_{jet_recon}$. Where, η_{jet_kin} is calculated using the light quarks from W (generated) and η_{jet_recon} is calculated from the reconstructed jet (measured). A similar interpretation is applicable to $\Delta\phi$ variables. Lastly, ΔR is then defined as $\Delta R = \sqrt{\Delta\eta^2 + \Delta\phi^2}$ and is calculated twice because there are two jets coming from W.

Several cuts were made in the ΔR matching. Jets were accepted as being produced from the W boson only when ΔR <= 0.5 was satisfied. The resulting plots (see next page) indicated a sharp spike, reflecting that these were the events where the light quarks and the reconstructed jets were most probably in the same direction. Events with E_{jet} <= 20 GeV were ignored and only events where both jets were identified were considered.



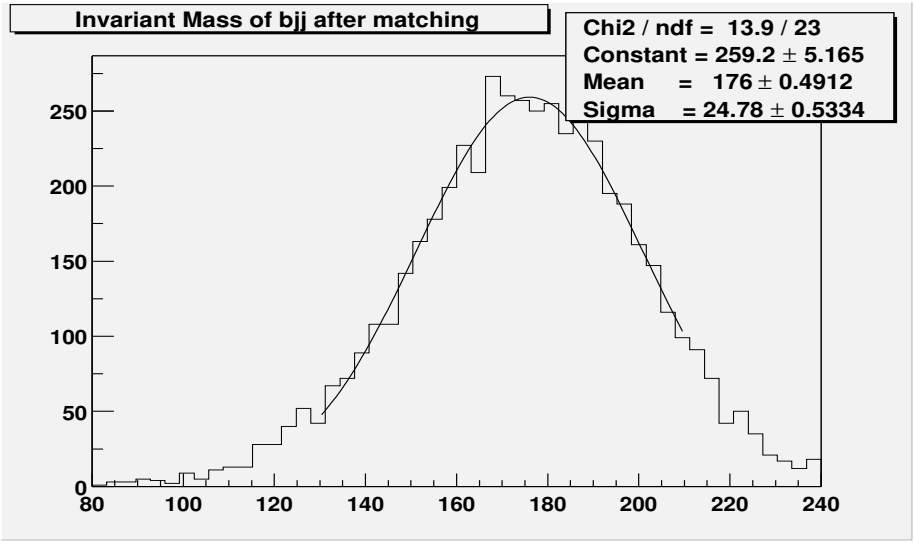


M_{jj} was plotted after the ΔR matching (see below plot). In doing so, all events with M_{jj} value outside of $50 < M_{jj} < 110$ were ignored. The resulting gaussian distribution was centered at $81.07 \pm 0.02\ GeV/c^2$ with a sigma value of $15.57 \pm 0.02\ GeV/c^2$. The accepted value of the mass of the W boson is $80.419 \pm 0.056\ GeV/c^2$ (as of 2000).



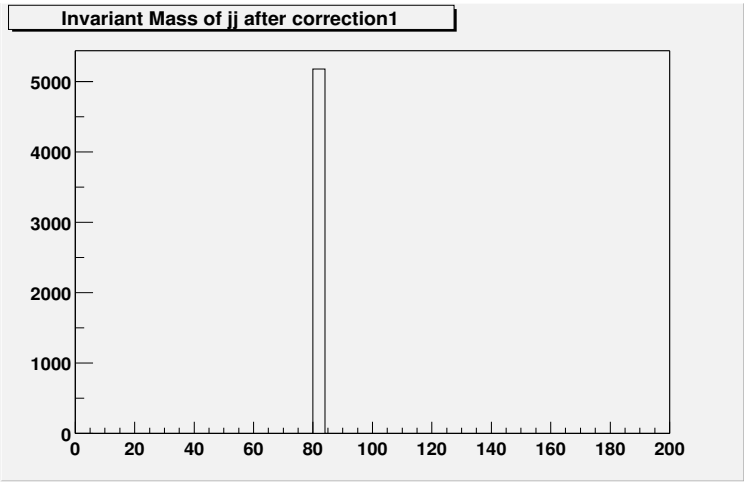
M_{bjj} was plotted (see plot below). The same ΔR restrictions were used. The resulting gaussian distribution was centered at $176.0 \pm 0.5\ GeV/c^2$ with a sigma value of 24.78

$\pm$ 0.53 GeV/c^2. The accepted value of the mass of the top quark is 174.3 $\pm$ 5.1 GeV/c^2 (as of 2000).

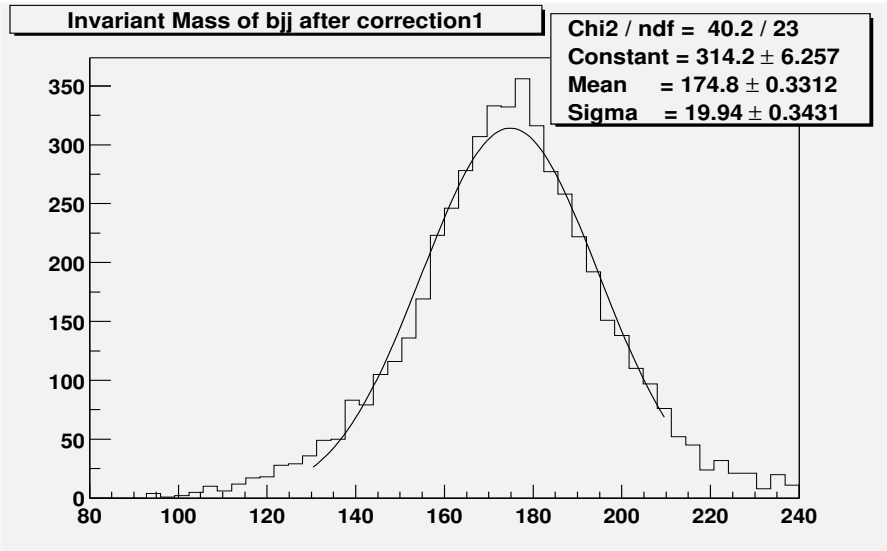


First Correction Scheme

The four-momentum of both non-b jets, (E, Px, Py, Pz), were multiplied by a scaling factor f defined as $f = (M_w / M_{jj})$. Where, constant M_w is the real mass of the W boson and M_{jj} is the mass after ΔR matching. Then, M_{jj} and M_{bjj} were plotted again. The purpose of the correction scheme is to force our M_{jj} measurement to be exactly the mass of W, in order to obtain a better measurement of the top mass, M_{bjj}. We can force this condition because the mass of the W boson is very well known (80.4 GeV/c^2). A better plot of M_{bjj} is obtained when sigma is minimized. As expected, the corrections lead to a delta function for M_{jj} (see plot below).



Most importantly, the newer plot of M_{bjj} yield a smaller sigma value of 19.94 ± 0.34 GeV/c^2 than previously (24.78 ± 0.53 GeV/c^2). The gaussian distribution for M_{bjj} was centered at 174.8 ± 0.3 GeV/c^2. The accepted value of the mass of the top quark is 174.3 ± 5.1 GeV/c^2 (as of 2000).



Second Correction Scheme

The second scheme was more complex than the first correction attempt. As previously, its purpose was to adjust the energies of the two non-b jets making up W, in order to obtain better measurement of the top quark. The scheme began with the following relation: $m_w{}^2 = 2E_1E_2(1-\cos\theta)$. This expression relates the mass of the W to the real energy of the two non-b jets, E_1 and E_2 respectively, and angle θ. Then, the following constant was defined: $\lambda = \dfrac{m_w{}^2}{2(1-\cos\theta)} = E_1E_2$

It followed, $E_2 = \dfrac{\lambda}{E_1}$.

Then, $E^1{}_m$ and $E^2{}_m$ were defined as the two non-b jet energy measurements. Variables σ_1 and σ_2 were defined as their respective errors.

κ^2 was then calculated as:

$$\kappa^2 = \left((E_1{}^m - E_1)/\sigma_1\right)^2 + \left((E_2{}^m - E_2)/\sigma_2\right)^2$$

Substituting E_2 with $E_2 = \dfrac{\lambda}{E_1}$, κ^2 reduced to

$$\kappa^2 = \left((E_1{}^m - E_1)/\sigma_1\right)^2 + \left((E^2{}_m - \lambda/E_1)/\sigma_2\right)^2.$$

Taking the first derivative to minimize κ^2,

$$\frac{\partial \kappa^2}{\partial E_1} = 0 = 2\left((E_1{}^m - E_1)/\sigma_1\right)\left(-1/\sigma_1\right) + 2\left((E_2{}^m - \lambda/E_1)/\sigma_2\right)\left(\lambda/E_1{}^2\sigma_2\right).$$

After further simplification,

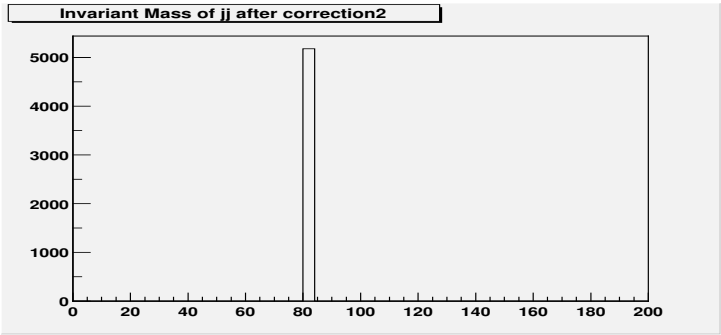
$$\left(E_1 - E_1{}^m\right)\sigma_2{}^2 E_1{}^2 + \left(E_2{}^m - \lambda/E_1\right)\lambda\sigma_1{}^2 = 0$$

Lastly, one obtains a quartic in E_1 expressed as:

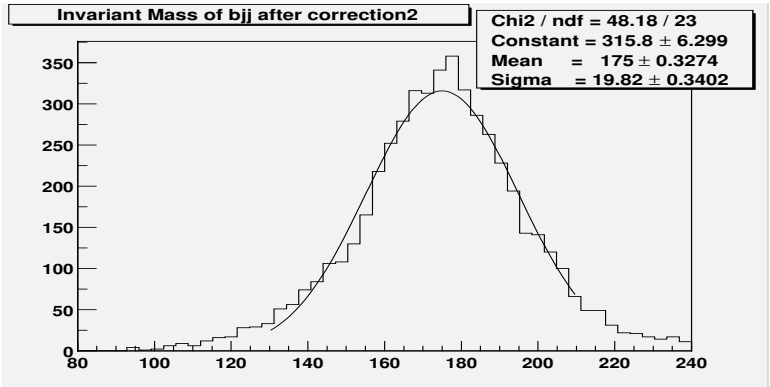
$$\sigma_2{}^2 E_1{}^4 - \sigma_2 E_1{}^m E_1{}^3 + \lambda\sigma_1{}^2 E_2{}^m E_1 - \lambda^2\sigma_1{}^2 = 0$$

Solving for the two real roots of this equation (ignore the complex solutions), $E_1{}^{rs}$ and $E_2{}^{rs}$, one defined correction factors, f_1 and f_2, as $f_1 = \dfrac{E_1{}^{rs}}{E_1{}^m}$ and $f_2 = \dfrac{E_2{}^{rs}}{E_2{}^m}$.

The four-momentum of the two non-b jets, j_1 and j_2, were multiplied by f_1 and f_2 respectively. Then, M_{jj} and M_{bjj} were recalculated and plotted. After the correction, M_{jj} yield another delta function as expected.

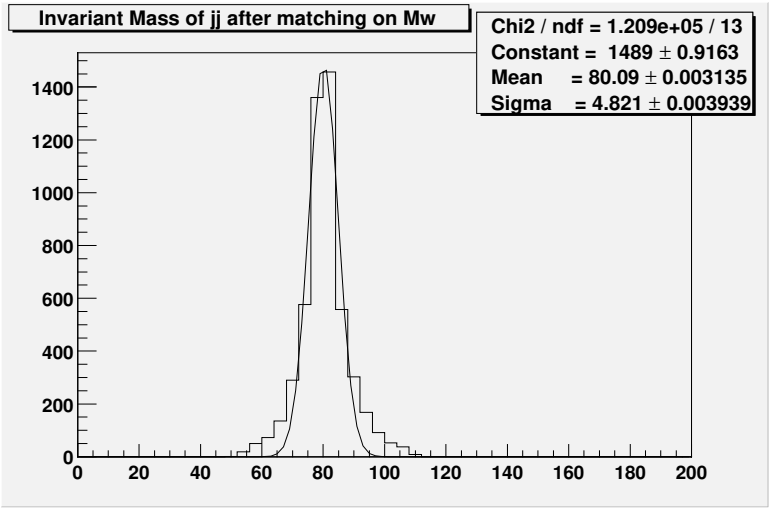


M_{bjj} yield a gaussian distribution centered at 175.0 ± 0.3 GeV/c^2 with a sigma value of 19.82 ± 0.34 GeV/c^2. Recall that the accepted value of the mass of the top quark is 174.3 ± 5.1 GeV/c^2 (as of 2000). Thus, the second correction was well justified.



Matching on M_w

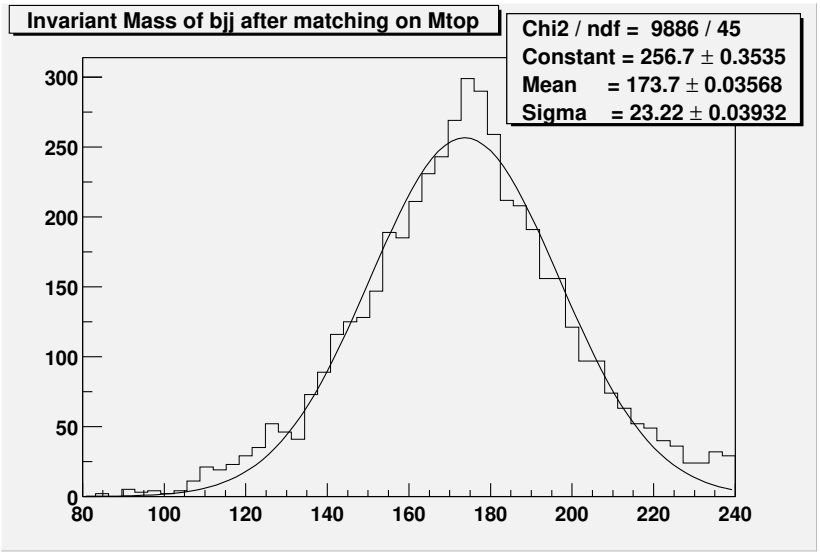
Next, the analysis was redone with another type of matching that also required our knowledge of M_{jj}. However, unlike the ΔR matching, this technique did not require knowing about the light quarks that are generated from the W boson. Only reconstructed information was necessary. The invariant mass M_{jj} of all the different combinations of the non-b jets was computed. The pair with the invariant mass closest to the W mass was selected. To remove further combinatorial background, the W candidate was accepted if and only if $\left|M_{jj} - 80.2\right| < 20 \ \ GeV/c^2$. The new plot of M_{jj} (below) yield a gaussian distribution centered at 80.090 ± 0.003 GeV/c^2 with a sigma value of 4.821 ± 0.004 GeV/c^2. The accepted value of the mass of the W boson is 80.419 ± 0.056 GeV/c^2 (as of 2000). Our latest result is better than the previous of 81.07 ± 0.02 GeV/c^2 (after ΔR matching).



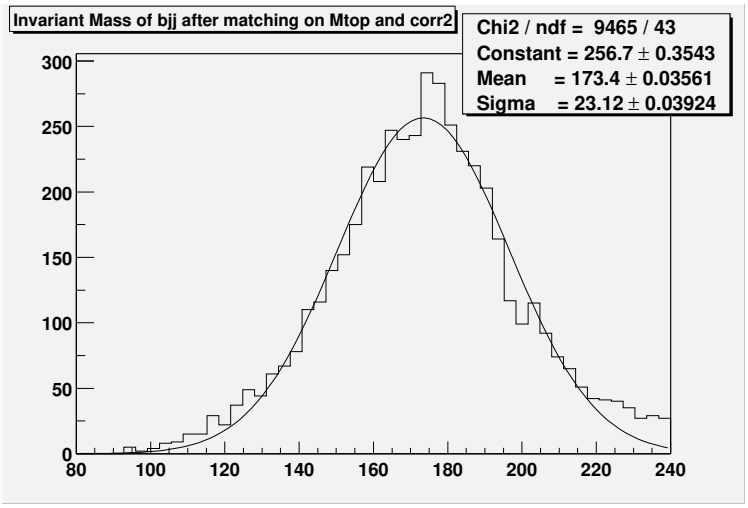
Then, the matching on M_w was combined with the second correction scheme. The new M_{jj} plot revealed a delta function as expected.

Matching on M_{top}

A similar matching as for M_w was performed, but for the top mass. The purpose of such a matching was to identify the b-jet that is related to the W boson that produces the two non-b jets. In doing so, M_{bjj} was calculated using the two possible b jets. The choice yielding a value closest to the known value of the top quark mass (174.3 ± 5.1 GeV/c^2) was accepted as having our b-jet. A newer M_{bjj} plot resulted in a gaussian distribution centered at 173.70 ± 0.04 GeV/c^2 with a sigma of 23.22 ± 0.04 GeV/c^2.



Lastly, the matching on M_{top} was combined with the second correction scheme. The new M_{bjj} plot was centered at 173.40 ± 0.04 GeV/c^2 with sigma of 23.12 ± 0.04 GeV/c^2.



Summary

The two non-b jets were identified and matched with the W boson using two different techniques. The first, known as ΔR matching, required knowing about the light quarks that are generated from the W. However, the second method, which was referred to as matching on M_w or M_{top}, depending on the application, relied only on reconstructed information. Thus, the second matching could actually be used with real data. Two schemes were employed to correct the four-momentum of the non-b jets. The first one relied on a simple scaling factor defined as the quotient of two quantities, the known mass of the W over the calculated invariant mass, M_{jj}, after matching. The second correction scheme was more complex, requiring the solution to a quartic energy equation.

Both correction schemes forced the invariant mass M_{jj} to be exactly its accepted value, improving our measurement of the top quark, M_{bjj}. The table below is a summary of all calculations. The second correction and matching schemes produced the best mass values (lowest sigma values).

The values in italic and bold were generated using ΔR and M_{top} matching respectively.

Mjj GeV/c^2	Sigma GeV/c^2	Mbjj GeV/c^2	Sigma GeV/c^2
81.07 +/- 0.02	*15.57 +/- 0.02*	*176.0 +/- 0.5*	*24.78 +/- 0.53*
80.090 +/- 0.003	4.821 +/- 0.004	*174.8 +/- 0.3* *1st correction*	*19.94 +/- 0.34*
		175.0 +/- 0.3 *2nd correction*	*19.82 +/- 0.34*
		173.70 +/- 0.04	23.22 +/- 0.04
		173.40 +/- 0.04 2nd correction	23.12 +/- 0.04

Accepted M_{jj}: **80.419 +/- 0.056** GeV/c^2 Accepted M_{top}: **174.3 +/- 5.1** GeV/c^2