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MEASURING DILUTION AND ITS SYSTEMATICS USING TIME INTEGRATED MIXING

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1 INTRODUCTION

1.1 Time averaged mixing probabilities

$$\chi_q = \frac{N_q^{mix}}{N_q^{mix} + N_q^{unm}} \quad (1)$$

$$= \frac{x_q^2 + y_q^2}{2(x_q^2+1)} \quad (2)$$

where:

$$x_q \equiv \frac{\Delta m_q}{\Gamma_q} \quad ; \quad y_q \equiv \frac{\Delta\Gamma_q}{2\Gamma_q}$$
$$\Gamma_q \equiv \frac{\Gamma_{H,q}+\Gamma_{L,q}}{2} \quad ; \quad \Delta\Gamma_q \equiv \Gamma_{H,q} - \Gamma_{L,q} \quad (3)$$

and *H, L* are the heavy and light mass eigenstates of B_q^0, with $q = d,s$ [1, 2].
Relating the true χ_q to values measured in the experiment requires the introduction of efficiency and dilution.

1.2 Dilution

$$D = \frac{R-W}{R+W} = 1 - 2\eta \quad (4)$$

where R, W are the number of right and wrong tagged events and η is the mis-tagging probability.

1.3 Efficiency

$$\varepsilon = \prod_i \varepsilon_{\rm i} \quad (5)$$

where $\varepsilon_{\rm i}$ are the relative efficiencies of each step in the analysis, given in table 1.

Table 1: Steps in B_q mixing selection.

Selection Step	**Eff. wrt to Prev. Step**
B-hadrons produced	$N_B = 2\sigma_{b\bar{b}} \int \mathcal{L} dt$
B_s produced	f_{Bs}
$B_s \rightarrow$ final state	$\mathcal{B}(B_s \rightarrow f)$
Kinematic cuts	$\varepsilon_{\rm kin}$
Trigger cuts	$\varepsilon_{\rm trig}$
$B_s \rightarrow f$ selection	$\varepsilon_{\rm sel}$
Tagging	$\varepsilon_{\rm tag}$
Fit for Mixed/Unmixed (should be =1)	$\varepsilon_{\rm fit}$

2 MEASURING THE DILUTION

The value of the dilution can be measured by comparing the measured χ_q ($\chi_q^{\rm fit}$) to its true value ($\chi_q^{\rm true}$). First some definitions:

- $N_{\rm true}^{\rm tot,mix,unm}(q)$:
  the true number of B_q mesons (tot) and those that have mixed (mix) or not (unm).

- $N_{\rm fit}^{\rm tot,mix,unm}(q)$:
  the fitted number of B_q mesons (tot) and those that have mixed (mix) or not (unm). These numbers are determined from fits to mass spectra.

We can then define the true and fit time-averaged mixing probabilities:

$$\chi_q^{\rm true} = \frac{N_{\rm true}^{\rm mix}(q)}{N_{\rm true}^{\rm tot}(q)} \quad \text{and} \quad \chi_q^{\rm fit} = \frac{N_{\rm fit}^{\rm mix}(q)}{N_{\rm fit}^{\rm tot}(q)} \tag{6}$$

$N_{\rm fit}(q)$ can be related to $N_{\rm true}(q)$ using the efficiency and dilution:

$$N_{\rm fit}^{\rm tot}(q) = \sum_{i=u,d,s...} \varepsilon_{\rm i} N_{\rm true}^{\rm tot}(i) = \varepsilon_{\rm q} N_{\rm true}^{\rm tot}(q) \left[1 + \sum_{i \neq q} \frac{\varepsilon_{\rm i}}{\varepsilon_{\rm q}} \frac{N_{\rm true}^{\rm tot}(i)}{N_{\rm true}^{\rm tot}(q)} \right] \tag{7}$$

$$\begin{aligned} N_{\rm fit}^{\rm mix}(q) &= \varepsilon_{\rm d} \left[(1-\eta_d) N_{\rm true}^{\rm mix}(d) + \eta_d N_{\rm true}^{\rm unm}(d) \right] \\ &+ \varepsilon_{\rm s} \left[(1-\eta_s) N_{\rm true}^{\rm mix}(s) + \eta_s N_{\rm true}^{\rm unm}(s) \right] \\ &+ \varepsilon_{\rm u} \left[\eta_u N_{\rm true}^{\rm tot}(u) \right] \\ &= \varepsilon_{\rm d} D_d N_{\rm true}^{\rm mix}(d) + \varepsilon_{\rm d} \eta_d N_{\rm true}^{\rm tot}(d) + \varepsilon_{\rm s} D_s N_{\rm true}^{\rm mix}(s) + \varepsilon_{\rm s} \eta_s N_{\rm true}^{\rm tot}(s) + \varepsilon_{\rm u} \eta_u N_{\rm true}^{\rm tot}(u) \end{aligned} \tag{8}$$

where $\varepsilon_{\rm q}$ is the total efficiency for fitting B_q decays and we have assumed that small contributions from non-q B-mesons appear in the fits for B_q. If we assume that these backgrounds are negligible then:

$$\chi_q^{\rm fit} = D_q \chi_q^{\rm true} + \eta_q \quad \Rightarrow \quad D_q = \frac{\chi_q^{\rm fit} - 1/2}{\chi_q^{\rm true} - 1/2} \tag{9}$$

Since $\chi_s^{\rm true} = 1/2$, this method cannot be used to measure D using time-averaged B_s oscillations.

2.1 Effect of Backgrounds

Since the above estimate for D relies on a correct measurement of $\chi_q^{\rm fit}$, it is important to understand the effect of backgrounds on this measurement. Because only the number of fitted signal events is used, and not the properties of the events, the only backgrounds that matter are those that peak near the signal's mass peak. Combinatoric backgrounds from other B-decays that have been mis-identified, for example, will not contribute to $N_{\rm fit}(q)$, but will be contained in the $N_{\rm fit}^{\rm bgrd}(q)$.

If peaking backgrounds do exist in the signal sample, however, their effect can be included in eqn. 9. The topologies of these peaking backgrounds will, necessarily, be similar to that of the signal allowing us to assume that all mistag probabilities are the same:

$$\eta_d = \eta_s = \eta_u \equiv \eta \tag{10}$$

for a given sample. Then, using B_d mixing as an example:

$$\chi_d^{\rm fit} = \frac{\varepsilon_{\rm d} D N_{\rm true}^{\rm mix}(d)\left[1+\frac{\varepsilon_{\rm s}}{\varepsilon_{\rm d}}\frac{N_{\rm true}^{\rm mix}(s)}{N_{\rm true}^{\rm mix}(d)}\right] + \varepsilon_{\rm d}\eta N_{\rm true}^{\rm tot}(d)\left[1+\frac{\varepsilon_{\rm u}}{\varepsilon_{\rm d}}\frac{N_{\rm true}^{\rm tot}(u)}{N_{\rm true}^{\rm tot}(d)}+\frac{\varepsilon_{\rm s}}{\varepsilon_{\rm d}}\frac{N_{\rm true}^{\rm tot}(s)}{N_{\rm true}^{\rm tot}(d)}\right]}{\varepsilon_{\rm d} N_{\rm true}^{\rm tot}(d)\left[1+\frac{\varepsilon_{\rm u}}{\varepsilon_{\rm d}}\frac{N_{\rm true}^{\rm tot}(u)}{N_{\rm true}^{\rm tot}(d)}+\frac{\varepsilon_{\rm s}}{\varepsilon_{\rm d}}\frac{N_{\rm true}^{\rm tot}(s)}{N_{\rm true}^{\rm tot}(d)}\right]} \tag{11}$$

But:

$$\frac{N_{\rm true}^{\rm tot}(i)}{N_{\rm true}^{\rm tot}(d)} = \frac{f_{Bi}}{f_{Bd}} \quad \text{and} \quad \frac{N_{\rm true}^{\rm mix}(s)}{N_{\rm true}^{\rm mix}(d)} = \frac{\chi_s^{\rm true}}{\chi_d^{\rm true}}\frac{f_{Bs}}{f_{Bd}} \tag{12}$$

Then, using $f_{Bu} = f_{Bd}$:

$$\chi_d^{\rm fit} = D\chi_d^{\rm true}\frac{\left[1+\frac{\chi_s^{\rm true}}{\chi_d^{\rm true}}\frac{\varepsilon_{\rm s}}{\varepsilon_{\rm d}}\frac{f_{Bs}}{f_{Bd}}\right]}{\left[1+\frac{\varepsilon_{\rm u}}{\varepsilon_{\rm d}}+\frac{\varepsilon_{\rm s}}{\varepsilon_{\rm d}}\frac{f_{Bs}}{f_{Bd}}\right]} + \eta \equiv D\chi_d^{\rm corr} + \eta \tag{13}$$

where

$$\chi_d^{\rm corr} \equiv \chi_d^{\rm true}\frac{\left[1+\frac{\chi_s^{\rm true}}{\chi_d^{\rm true}}\frac{\varepsilon_{\rm s}}{\varepsilon_{\rm d}}\frac{f_{Bs}}{f_{Bd}}\right]}{\left[1+\frac{\varepsilon_{\rm u}}{\varepsilon_{\rm d}}+\frac{\varepsilon_{\rm s}}{\varepsilon_{\rm d}}\frac{f_{Bs}}{f_{Bd}}\right]} \tag{14}$$

The efficiency ratio can be estimated from MC allowing a measurement of the dilution using:

$$D = \frac{\chi_d^{\rm fit} - 1/2}{\chi_d^{\rm corr} - 1/2} \tag{15}$$

2.2 Other Methods of Measuring the Dilution

The dilution can also be measured as part of the fit to time-dependent B_d mixing [3]. Because this method performs a simultaneous fit to Δm_d and η, however, it should result in a slightly less statistically significant measurement of η than that obtained by the method outlined here, which takes advantage of the independently well-measured value of χ_d.

3 ESTIMATING DILUTION ASYMMETRIES USING χ_s

A check that we will want to make as part of the B_s-mixing analysis is whether our assumption that the dilution is the same for the mixed and unmixed samples is correct. The time-averaged B_s mixing probability provides a convenient way to make this test. As can be seen from equation 9, D cannot be extracted from a comparison of $\chi_s^{\rm fit}$ and $\chi_s^{\rm true}$ since $\chi_s^{\rm true} = 1/2$. However, this comparison is sensitive to possible differences in D between mixed and unmixed events.

Note: in the following we will assume that $\chi_s^{\rm corr} = \chi_s^{\rm true}$ because peaking backgrounds are negligible under the $D_s \to \phi\pi$ peak.

Dilution differences could arise for a number of reasons, but fall basically into two classes.

1. Differences in mis-tag probability for mixed or unmixed events. These could arise, for example, because of unmodelled, peaking backgrounds that populate one category in preference to the other.
2. Differences in efficiency to select mixed or unmixed events. An example of such a difference could arise from charge-dependent tracking efficiencies.

If we define dilution and efficiency differences:

$$\begin{array}{llll} \delta\varepsilon \equiv \varepsilon_{\rm mix} - \varepsilon_{\rm unm} & \text{and} \quad \varepsilon \equiv \frac{\varepsilon_{\rm mix}+\varepsilon_{\rm unm}}{2} & \Rightarrow & \varepsilon_{\rm mix/unm} = \varepsilon -/+ \frac{\delta\varepsilon}{2} \\ \delta\eta \equiv \eta_{\rm unm} - \eta_{\rm mix} & \text{and} \quad \eta \equiv \frac{\eta_{\rm mix}+\eta_{\rm unm}}{2} & \Rightarrow & \eta_{\rm mix/unm} = \eta +/- \frac{\delta\eta}{2} \end{array} \tag{16}$$

then we can compare fitted to true time-averaged mixing probabilities in B_s events to constrain $\delta\varepsilon$ and $\delta\eta$. The B_s system is used for this check because measurements of χ_s are insensitive to the average dilution (see eqn. 9).

Assuming that both $\delta\varepsilon$ and $\delta\eta$ are non-zero, the total number of observed signal events, from fits to the B_s mass spectrum can be written as:

$$\begin{aligned} N_{\rm fit}^{\rm tot}(s) &= \varepsilon_{\rm mix} N_{\rm true}^{\rm mix}(s) + \varepsilon_{\rm unm} N_{\rm true}^{\rm unm}(s) \\ &= \varepsilon N_{\rm true}^{\rm tot}(s) + \frac{\delta\varepsilon}{2}\left[N_{\rm true}^{\rm unm}(s) - N_{\rm true}^{\rm mix}(s)\right] \\ &= \varepsilon N_{\rm true}^{\rm tot}(s) \end{aligned} \tag{17}$$

where we have assumed that no backgrounds peak under the B_s signal so that $\varepsilon_{\rm d} = \varepsilon_{\rm u} = 0$ and have used the fact that $N_{\rm true}^{\rm mix}(s) = N_{\rm true}^{\rm unm}(s)$ for B_s.

The number of mixed B_s events returned from the fit is:

$$\begin{aligned} N_{\rm fit}^{\rm mix}(s) &= \varepsilon_{\rm mix} N_{\rm true}^{\rm mix}(s)(1-\eta_{\rm mix}) + \varepsilon_{\rm unm} N_{\rm true}^{\rm unm}(s)\eta_{\rm unm} \\ &= \varepsilon N_{\rm true}^{\rm mix}(s)\left[1 - \frac{\delta\varepsilon}{\varepsilon}\frac{D}{2} - \delta\eta\right] \end{aligned} \tag{18}$$

Thus

$$\frac{\chi_s^{\rm fit}}{\chi_s^{\rm true}} = \left[1 - \frac{\delta\varepsilon}{\varepsilon}\frac{D}{2} - \delta\eta\right] \tag{19}$$

Since there is only one observable in eqn 19, $\chi_s^{\rm fit}$, only the *effective mis-tag asymmetry* can be constrained:

$$\begin{aligned} \delta\eta_{eff} &\equiv \delta\eta + \frac{\delta\varepsilon}{\varepsilon}\frac{D}{2} \\ &= 1 - \frac{\chi_s^{\rm fit}}{\chi_s^{\rm true}} = 1 - 2\chi_s^{\rm fit} \end{aligned} \tag{20}$$

using $\chi_s^{\text{true}} = 1/2$.

If $\delta\eta_e ff$ is found to be significantly non-zero, further investigation will be necessary to pin down the cause.

3.1 Other Methods to Estimate Dilution Asymmetries: $\mathbf{B^\pm \to J/\psi K^\pm}$

Another method of measuring the dilution asymmetry would be to measure η separately in events containing $B^+ \to J/\psi K^+$ and $B^- \to J/\psi K^-$ decays. Using the charge of the kaon to verify the tagging in the event, which will only be valid for opposite side tags, the mistag probabilities for K^+ and K^- decays are given by:

$$\begin{aligned} \eta_+ &= \frac{N^+_{\text{fit}}(wrong)}{N^+_{\text{fit}}(tot)} \\ \eta_- &= \frac{N^-_{\text{fit}}(wrong)}{N^-_{\text{fit}}(tot)} \end{aligned} \quad (21)$$

where $N^{+/-}_{\text{fit}}(wrong)$ are the number of fitted $B^\pm \to J/\psi K^\pm$ events where the tag disagrees with the sign of the kaon.

The mis-tag asymmetry is then:

$$\delta\eta = \eta_+ - \eta_- \quad (22)$$

Although this method measures $\delta\eta$ directly, it suffers from several problems if it were to be applied to B_s mixing analyses.

1. The $B^\pm \to J/\psi K^\pm$ topology is different than that found in semi-leptonic B_s decays. So it is not clear that $\delta\eta$ measured in $B^\pm \to J/\psi K^\pm$ events is directly comparable to that in B_s events.

2. Even if the comparison can be made, the $B^\pm \to J/\psi K^\pm$ sample will have lower statistics than the semi-leptonic B_s sample because of the low $B^\pm \to J/\psi K^\pm$ branching ratio.

3. $B^\pm \to J/\psi K^\pm$ events cannot be used to measure mis-tag asymmetries for same-side taggers.

References

[1] K. Anikeev, *et al.*, "B Physics at the Tevatron: Run II and Beyond", `hep-ph/0201071`.

[2] 2004 PDG Mixing Review

[3] G. Borissov, S. Burdin, A. Nomerotski, DØNote 4370.