Momentum determination through multiple coulomb scattering for the MicroBooNE experiment

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This paper describes a method of determining muon momentum for the MicroBooNE experiment. The technique uses information from multiple coulomb scattering to compute the muon’s momentum through the maximization of a likelihood algorithm. This method was applied to both simulation and data, with momentum resolutions for both measured to be roughly 20%. Given this, multiple coulomb scattering provides a promising route towards energy determination for muons escaping the detector, ultimately assisting with neutrino energy estimation in MicroBooNE.

I. INTRODUCTION

MicroBooNE (Micro Booster Neutrino Experiment) is an experiment based at the Fermi National Accelerator Laboratory (Fermilab) that uses a large Liquid Argon Time Projection Chamber (LArTPC) to investigate the excess of low energy events observed by the MiniBooNE experiment [1] and to study neutrino-argon cross-sections. MicroBooNE is part of the Short-Baseline Neutrino (SBN) physics program, along with two other LArTPCs: the Short Baseline Near Detector (SBND) and the Imaging Cosmic And Rare Underground Signal (ICARUS) detector.

The MicroBooNE detector is currently the largest LArTPC in the United States. It consists of a rectangular time projection chamber (TPC) with dimensions 2.3 m × 2.6 m × 10.4 m located 470 m away from the Booster Neutrino Beam (BNB) target. Time projection chambers, filled with a gas or liquid volume, are used to analyze particle interactions in three dimensions. The x-direction of the TPC corresponds to the drift coordinate, the y-direction is the vertical direction, and the z-direction is the direction along the beam. The amount of active liquid argon in the TPC is 86 tons, with the total cryostat containing 170 tons of liquid argon. Liquid argon is chosen to fill the volume for a variety of reasons: argon contains a high number of nucleons, which allows for a greater rate of interactions with particles within the medium; argon ionizes easily; it produces scintillation light which is not reabsorbed; it has a high electron lifetime; and it is inexpensive.

Photomultiplier tubes and three wire planes are located in the TPC to aid with event reconstruction (Fig. 1). In an event, a neutrino interacts with the argon and its charged interaction products move through the medium, losing energy and leaving an ionizing trail. The resulting electrons drift to the anode side of the TPC, containing the wire planes, away from the negatively charged cathode plate on the opposite side. The movement of electrons induces a current in the wires, which is used to reconstruct the event from these signals in three dimensions.

One of the aims of MicroBooNE is to explore neutrino oscillations. For the two neutrino case, the probability that a neutrino with an initial flavor $\alpha$ will be observed as a neutrino with a different flavor $\beta$ when measured is:

$$P_{\alpha \rightarrow \beta} = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

where $\theta$ is the mixing angle, $L$ is the distance from the neutrino source to the detector, $E$ is the energy of the neutrino, and $\Delta m^2$ is the neutrino squared mass difference. $\theta$ and $\Delta m^2$ are the two free parameters of interest in the above equation. The probability $P$ can be determined through a measurement of observed neutrino events compared to expected events, the distance $L$ is
readily determined and constant, and $E$ can be computed from the energies of the neutrino’s interaction products.

The BNB is predominantly composed of muon neutrinos, which can undergo charge-current interactions in the TPC and produce muons. For muon tracks that are completely contained in the TPC, it is easy to calculate the momentum through the length of the particle’s track. Around half of muon tracks are not fully contained in the TPC, however. Using length-based calculations for these uncontained tracks is not a possibility. There is a way to account for the amount of energy deposited per segment of track, however, and use this information to predict the initial energy of the muon, if one takes into consideration the effects of multiple coulomb scattering on the particle.

A. Multiple coulomb scattering

The phenomenon of multiple coulomb scattering (MCS) occurs when a charged particle enters a medium and undergoes electromagnetic scattering with the atoms. This scattering deviates the original trajectory of the particle within the material (Fig. 2). The collection of these deflections within the material form a Gaussian distribution with mean at 0 and standard deviation given by the Highland formula [4]:

$$\theta_0 = \frac{13.6 \text{ MeV}}{p\beta c} \sqrt{\frac{\ell}{X_0}} \left[ 1 + 0.0038 \ln \left( \frac{\ell}{X_0} \right) \right]$$

(2)

where $\beta$ is the ratio of the particle’s velocity to the speed of light, $\ell$ is the distance traveled inside the material, and $X_0$ is the radiation length of the target material. The particle’s momentum, $p$, can be found if the angular deflections of the particle inside the material—and thus the standard deviation $\theta_0$ of these angular deflections—are known.

B. MCS implementation using the maximum likelihood method

To determine the angular deflections of the particle, its track through the medium is first split into segments. The angles of deflection between adjacent segments are then taken. The maximum likelihood method is then used to find the momentum of the particle. Individual angle deflections are given as input to the likelihood product:

$$L = \prod_{(i,j)} f_{ij}$$

(3)

which is then maximized to find the momentum, $p$.

The probability to measure a certain deflection is given by:

$$f_{ij} = \frac{1}{\sqrt{2\pi(\Delta\theta)_{\text{std}}}} e^{-\frac{(\Delta\theta)^2}{2(\Delta\theta)_{\text{std}}^2}}$$

(4)

where $f = f((\Delta\theta)_{ij};(\Delta\theta)_{\text{std}})$ is a Gaussian function with mean 0 and standard deviation given by $(\Delta\theta)_{\text{std}} = \sqrt{\theta_0^2 + \delta\theta_0^2}$ (where the angular resolution $\delta\theta_0$ is given a fixed value of 0.5 mrad from Monte Carlo information [5]). The momentum and standard deviation are updated along the track using the following energy-range relation:

$$p_i \approx p - \frac{k_{\text{cal}} L_i}{c}$$

(5)

where $p_i$ is the momentum along the track for a certain segment $i$, $p$ is the initial momentum of the particle at the neutrino interaction vertex, $L_i$ is the distance of the $i$th segment from the starting point of the track, and $k_{\text{cal}}$ is the ionization constant for muons in liquid argon, given to be $2.1 \times 10^{-3}$ GeV/cm.

For this implementation, a minimum reconstructed track length of 100 cm was used. A relativistic limit approximation was also used, meaning $\beta \approx 1$ and $E \approx p$.

The idea and implementation of MCS using the maximum likelihood method for this project is credited to Leonidas Kalousis, a former member of the MicroBooNE collaboration. Further details regarding the technique can be found in his internal notes concerning both Monte Carlo simulated tracks [3] and reconstructed tracks [5].

II. ANALYSIS OF MONTE CARLO BOOSTER NEUTRINO BEAM DATA

The performance of MCS was first tested on simulated data. Monte Carlo (MC) BNB and CORSIKA [6] cosmic data files (with Pandora [7] producer pandoraNuPMA) were used. These data files contain 173,400 simulated neutrino interaction events. Each event contains hundreds of MC tracks, most of which originate from cosmic sources rather than the BNB.
To work with the simulated data, certain filters were required to be implemented. First, each MC track was ensured to match a detected neutrino interaction vertex located in a fiducial volume 20 cm within the walls of the detector in the x-direction, 26.5 cm in the y-direction, 20 cm in the z-direction from the front, and 36.8 cm in the z-direction from the back. These passing MC tracks were then filtered for two requirements: being the track of a muon, and originating from the BNB. For the first requirement, each MC track’s Particle Data Group (PDG) code was compared to that of a muon (PDG code = 13) and passed if it met the requirement. A similar filter was set in place for origin from the BNB (a value of 1 for origin from BNB, 2 for origin from cosmic sources). After filtering out tracks originating from cosmic sources, the majority of events contained only one MC track. In the cases where the event contained no passing MC tracks, or in the rare cases where there were 2 or more passing MC tracks, the event was discarded.

With each event containing only one MC track at this point, the MC track was matched with one reconstructed (reco) track, for which there were around 10 per event. The requirement for filtering one reco track from the rest to match the MC track is as follows: the direction of the chosen reco track had to be the most parallel to the MC track, and the distance between the reco track and the MC track had to be the smallest of the entire selection of reco tracks. To compare for direction, the absolute value of the dot product of two linear vectors approximating the directions of the reco and MC tracks was calculated, and the track with the biggest value per event was chosen to be the matching reco track. To compare for distance, the location of the start and end points of the reco track (to account for mistaken direction) was compared to the start point of the MC track. A required threshold of 3 cm distance between the points was required to pass (in which case, if there were no passing reco tracks, the event was discarded), and the track with the smallest distance was selected to be the matching reco track. With both of these requirements met, one reco track was chosen to match each single BNB muon MC track per event. This selection resulted in 20,789 reconstructed neutrino events able to be used for analysis. With the reconstructed track length minimum of 100 cm in place, this number of events dropped to 13,773.

Reconstructed muon momentum from MCS for this simulated data was then compared with MC truth values to determine the accuracy of the reconstruction (Fig. 3). A linear correlation with no spread means the MCS reconstruction matches truth precisely. In the plot, we see there is a positive and linear correlation, but with some spread. We quantify this spread by first defining a value ∆P:

\[
\Delta P = \frac{p_{mc} - p_{mcs}}{p_{mc}}
\]  

(6)

where the MCS momentum is \(p_{mcs}\) and the truth momentum is \(p_{mc}\). This value allows us to determination deviation from truth; with perfect MCS momentum reconstruction, \(\Delta P\) equals 0.

Taking the mean of \(\Delta P\) gives the average bias from truth. Fig. 4 shows a plot of mean \(\Delta P\) against truth energy for bins of 0.2 GeV. For muons contained in fiducial volume, the mean bias is under 5%. For escaping muons, the mean bias is worse, but still within ±10%.

![Fractional Mean Bias vs Truth Energy](image)

FIG. 4. A plot of fractional mean bias versus truth energy for simulated BNB data. Contained muons are shown in blue and uncontained in red.

Taking the standard deviation of \(\Delta P\), we obtain the fractional momentum resolution. The plot of this value against different truth energy ranges is shown in Fig. 5. For contained muons, the resolution is around 10%, and overall, does not surpass 20%. For uncontained muons, the resolution stays between 20-30%. We must note, however, that the higher energy ranges contain fewer statistics; there are around 100 events for the 1.8 and 2 GeV data points, compared to above 1,000 for those in lower energy ranges. Excluding the last couple of data points, the resolution for uncontained muons is roughly 23%.

![Fractional Resolution vs Truth Energy](image)

FIG. 3. A plot of truth momentum against the momentum reconstructed from MCS for all muons (both contained and uncontained in fiducial volume) using simulated BNB data with a minimum reconstructed track length of 100 cm.
A. Implementation of range-based energy

When MC truth information is not available—as with data—another way to determine a reliable momentum measurement for comparison is through range-based calculations, which take into account the length of a track to return an energy value. Energies are calculated from track length using a muon energy-loss table containing momentum and range values [8]. This method only works for contained tracks, however, since range-based energy calculations would always underestimate the starting energies of uncontained tracks. For this analysis, we implement range-based calculations to allow for comparison to MCS reconstructed momentum in preparation for working with data.

First, to check the accuracy of range-based momentum calculations compared to MC truth, we plot the two variables against each other to observe the correlation and spread (Fig. 6). The correlation is positive and linear with minimal spread, meaning we can replace MC truth momentum with range-based momentum when analyzing the performance of MCS.

Next, we analyze the performance of MCS against range-based momentum on simulated BNB data. This plot is shown in Fig. 7. We can see that there is a positive, linear correlation with some spread, meaning the performance of MCS is accurate compared to range-based values. Given this, we are ready to perform the same analysis on BNB data.

III. ANALYSIS OF BOOSTER NEUTRINO BEAM DATA

The files used for working with data were 5e19 POT BNB with cosmics.
FIG. 8. A plot of momentum from range versus MCS reconstructed momentum for data. Only contained tracks were used for this analysis.

range, where MCS is overestimating the momentum. The cause of this second population is still unknown, and is currently being investigated.

We quantify the spread of the data in Fig. 8 in a similar fashion to that of the plot of truth versus MCS momentum with simulated BNB data (Fig. 3). We first compute the mean of $\Delta P$, which is now given as:

$$\Delta P = \frac{p_{\text{range}} - p_{\text{mcs}}}{p_{\text{range}}}$$  \hspace{1cm} (7)

FIG. 9. Fractional mean bias versus range-based muon energy for data. Only contained tracks were analyzed.

Fig. 9 shows a plot of mean $\Delta P$ versus each range-based energy range in bins of 0.2 GeV. We observe that the overall mean bias here is within 15%. However, it is important to note we include the second population in this analysis. If we ignore the 0.4-0.6 GeV range, the mean bias hovers around 5%.

The standard deviation of $\Delta P$ is given in Fig. 10. We can see that the resolution is as high as 55% at lower energy ranges, but drops down to 10% at higher energies. Ignoring the 0.4-0.6 GeV energy ranges due to the second population, we observe a momentum resolution of no greater than 35% for contained muons using data.

FIG. 10. A plot of fractional momentum resolution against range-based muon energy for data. Only contained tracks were used for this analysis. Error bars assume a Gaussian distribution.

IV. CONCLUSION

A MCS algorithm based on the maximum likelihood method has been applied to both simulation and data. We have shown that MCS reconstructed momentum calculations are accurate using MC simulation, achieving a $\sim$10% resolution for contained tracks and $\sim$25% resolution for uncontained tracks. MCS was also used on data with contained tracks only to determine momentum values between 10% and 30% resolution. This accuracy is comparable to the upper limit of the MCS fractional momentum resolution achieved by the ICARUS collaboration [10]. Since MCS does not require that a track be contained in fiducial volume when calculating momentum values, we hope that this algorithm may also be applied on uncontained tracks in data.

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