Noise Simulation for XENON experiment

Nevis Labs Summer 2017 REU Project

Olenka Jain
Harvard University

August 4, 2017
Abstract

The XENON experiment is a dark matter detection experiment housed underground at the Laboratori Nazionali de Gran Sasso in Italy (LNGS). The experiment uses a dual-phase Time Projection Chamber (TPC) to detect direct collisions between Xenon nuclei and weakly interacting massive particles (WIMPS). The most recent detection chamber, XENON1T, is the world’s most sensitive direct dark matter detector. To understand and analyze the data, a Monte Carlo simulation is used. My project this summer was to improve the noise simulation so that noise would be random and coherent.
Contents

1 Introduction

1.1 Evidence for Dark Matter

1.1.1 Λ-CDM Paradigm

1.1.2 Galaxy Clusters

1.1.3 Spiral Galaxies

1.1.4 The Bullet Cluster

1.1.5 Cosmic Microwave Background

1.1.6 Big Bang Nucleosynthesis

1.1.7 Microlensing

1.2 WIMPs

1.3 Detection with XENON1T

1.4 Monte Carlo Methods

2 My Project

2.1 Noise Data

2.2 Analyzing Noise

2.3 Old Noise Simulation

2.4 New Noise Simulation

2.5 Correlations in Noise

2.5.1 Between Frequencies

2.5.2 Between Channels

2.6 Summary

References
1 Introduction

1.1 Evidence for Dark Matter

1.1.1 Λ-CDM Paradigm

Evidence for dark matter is understood within the context of the Λ-CDM paradigm. This model proposes that about $10^{10}$ years ago the Universe experienced a Big Bang, which was a period of intense inflation. This inflation was followed by a period of cooling and expansion of the Universe, in which gravitational interactions follow the laws of General Relativity and the interactions of the known particles are determined by the standard model. This model predicts that the energy density of the Universe is split mainly between the vacuum energy density ($\Omega_\Lambda = .684$ currently) and the matter energy density ($\Omega_m = .316$). The total baryon density is only $\Omega_b h^2 = 0.0223$, while the dark matter component is $\Omega_c h^2 = 0.1199$. Therefore, this model predicts that most of the matter energy density of the Universe is made up of non-relativistic (cold) dark matter. (Goetzke, 2015)

1.1.2 Galaxy Clusters

The first evidence for the existence of dark matter came from Swiss Astronomer Fritz Zwicky from observing the Coma and Virgo clusters. A cluster is a group of galaxies that move together inside a gravitational field. To determine the mass of the system, Zwicky determined the galaxy velocities in the Coma Cluster by using the Doppler effect. He then used the Virial Theorum, to calculate the gravitational force acting upon each of the galaxies, and was able to calculate the mass of the system. He additionally measured the total light output of the cluster to calculate the light to mass ratio. When he compared this value to that of a nearby Kapteyn stellar system, he found that the light to mass ratio of the Coma Cluster was over 100 times lower than that of a single Kapteyn star. He hypothesized that the Coma Cluster must therefore contain non luminous matter, which he named “dark matter.”

1.1.3 Spiral Galaxies

Vera Rubin also discovered evidence of dark matter by studying spiral galaxies. Spiral galaxies, in particular, are useful systems to test the dark matter hypothesis because they are “rotationally sustained.” This means that it is possible to clearly define rotational motion for these galaxies. According to the Newtonian gravitational law, velocity as a function of radius is given by:

$$v(r) = \sqrt{\frac{GM(r)}{r}}$$

(1)
Because spiral galaxies can be defined by a central core, containing nearly all
the mass of the galaxy and whose motion can be described as a rigid body, \( M(r) \)
can be considered as constant. Therefore, the velocity outside the core of the
galaxy should fall as:

\[
v(r) \propto r^{-1/2}
\]

However, this does not occur. Instead the velocity distribution outside the disk
of the central core of the galaxy remains nearly constant. The velocity distribu-
tion is can be explained if there were additional non-luminous matter surround-
ing the galaxy, and adding to the gravitational pull of the galaxy. (Massoli,
2015) Rubin theorized that this non-luminous mass was a halo of dark matter
which surrounds the spiral galaxy. Her work with spiral galaxies gave credence
to Zwicky’s hypothesis (Ver)

1.1.4 The Bullet Cluster

The study of the Bullet Cluster has yielded evidence that supports the theory
of dark matter. The Bullet Cluster is made of two clusters which are passing
through each other. Each cluster has a stellar and gaseous component. During
a collision, the stellar component is acted upon by the gravitational field of the
other cluster while the gaseous components act as a collision particle fluid. The
interactions of the gaseous components cause an X-ray emission which can be
used to measure the baryonic matter distribution. Lensing, the phenomena in
which light is bent by gravity, can also be used to measure the baryonic matter
distribution. However, there is a discrepancy between these two calculated dis-
tributions. This discrepancy can be explained by the existence of dark matter.
While the stellar and gaseous components interact with each other, dark matter
particles can pass through each other without disturbing their path. Accounting
for dark matter, which is able to pass undisturbed through the collision to the
edges of the clusters, justifies the matter distribution. (Massoli, 2015)

1.1.5 Cosmic Microwave Background

The Cosmic Microwave Background (CMB) is electromagnetic radiation which
has been left over from an early stage of the Universe. Though generally
the CMB is incredibly isotropic, there exist small spatial anisotropies. These
anisotropies can be shown to correspond to cosmological parameters. Fitting a
power spectrum of the anisotropies gives precise limits on certain parameters of
the Lambda-CDM model which predict the total density of dark matter to be
\( \Omega_c h^2 = 0.1199 \pm 0.0022 \). (Goetzke, 2015)

1.1.6 Big Bang Nucleosynthesis

Studying Big Bang Nucleosynthesis (BBN) is another way to quantify upper
limits on the amount of baryonic matter in the universe. BBN tells us about
the formation rates of light elements such as H, D, $^4$He, $^3$He, and $^7$Li. The primordial quantities of these elements depend on the rate of expansion of the Universe in its early stages. Therefore, surveys of the abundance of these light elements can be used to constrain the total amount of baryonic matter present in the universe. D in particular, is useful for setting an upper limit on the amount of baryon energy density. This is because the abundances of light elements depend on stellar processes which can create or destroy light elements. D can be destroyed in stellar fusion, but there are no probable methods for D production. For this reason, the amount of D currently sets an upper limit on the total amount of D at any stage of the Universe. Measurements from Kirkman et al. (2003) give a baryon matter density of $\Omega_b h^2 = 0.0214 \pm 0.0020$. Because this agrees with the limits from the CMB, the measurement indirectly supports the theory that the majority of matter energy density is not baryonic. (Goetzke, 2015)

### 1.1.7 Microlensing

Studies on dark matter abundance and composition have used the *Microlensing* effect. This effect occurs whenever there is a source of a gravitational field strong enough to bend light between a distant source and a viewer. The light from the distant source is bent by the gravitational field resulting in multiple images or a distorted image of the source. Microlensing has been used to test the hypothesis that dark matter is made of astronomical bodies called MAAssive Compact Halo Objects (MACHOs). In microlensing, when the lens (source of gravitational field) is relatively small, a luminosity curve due to the relative motion between the distant source and the lens can be observed. The variation in luminosity happens on a time scale defined by the time needed by the source to cross the lens. The optical depth is defined as the probability of a background source being lensed. For the Milky way, the optical density is about $10^{-6}$. Approximately one in a million nearby galaxies will be lensed. By studying the micro-lenses in a certain direction, it is possible to characterize the lens population. From studying lenses in the Milky Way’s halo from sources in the Large and Small Magellanic Clouds, researches concluded that MACHOs could account for only about 20% of the halo mass. (Massoli, 2015)

### 1.2 WIMPs

Weakly interacting massive particles (WIMPs) are a general class of particles which have historically been considered likely candidates for dark matter. The observed energy density of dark matter could be explained by a particle with a mass anywhere 1 GeV - 100 TeV, which interacts with a strength on the weak scale. Because no new forces are needed to explain the interactions of WIMPs and because particles sharing properties of WIMPs (self-annihilation cross section, mass, and force through which they interact) are predicted by super symmetric extensions of the Standard Model, they are compelling candidates for dark matter. From the SUSY model, the sneutrino (superpartner
of the neutrino) and neutralino, the lightest stable supersymmetric partner, have been predicted as candidates of dark matter. (Goetzke, 2015) Another superpartner, the axino (superpartner of the axion) is a potential candidate. Kaluza-Klein dark matter is proposed in the Extra Dimensions model. If standard model particles propagate in Extra dimensions and there is KK parity, the lightest KK particle is stable and becomes another candidate for dark matter. Its mass ranges from hundreds of GeV to a few TeV and can be detected through elastic scattering or through annihilation products. Wimpzillas are super heavy dark matter with masses above $10^{10} \text{GeV}/c^2$. Wimpzillas could have been produced during the end of inflation of the Universe and have been proposed as an explanation of the ultra high energy cosmic rays. (Massoli, 2015)

1.3 Detection with XENON1T

The XENON experiments use direct detection to search for the scattering of WIMPs off of Xenon nuclei. Because the Milky Way is thought to be surrounded by a dark matter halo, there exists a local dark matter density of $0.3 \pm 0.1 \text{GeV/cm}^3$. The terrestrial XENON1T detector aims to detect particles from this local density. The rate of detection is expected to change annually as the interaction rate depends on the relative velocity of WIMPs and the target XE nuclei. This flux of WIMPs through the Earth is called “WIMP wind” and reaches a maximum in June when the velocities of the sun and earth are aligned. Understanding this “WIMP wind” has been proposed as a background for discriminating a true WIMP signal from background. (Goetzke, 2015)

Xe is a fast and efficient scintillating material which makes it useful for detecting WIMPs. It has a high density, atomic number, and electronic stopping power, which both increase the likelihood of particle interactions and shield against external radiation. Liquid Xenon atoms are transparent to their scintillation light and their ionization electrons can be drifted and collected efficiently in time projection chambers. Xe allows differentiation between two types of interactions: nuclear recoils (NR) and electron recoils (ER). In an ER interaction, a charged particle interacts mainly with the electron cloud of a Xe atom while in a NR interaction, an uncharged particle transfers significant momentum to the Xe nucleus by scattering off of the nucleus. In liquid Xe the ER have a reduced fraction of escaping electrons when in an electric field. This difference in fraction of electrons which recombine/escape allows for differentiation between the two interaction types. (Goetzke, 2015)

In the XENON1T detector, the energy deposited by a particle interaction in the liquid Xe is detected by two signals. The first is the scintillation light from recombination electrons (S1) which de-excite Xe molecules to the ground state $\text{Xe}^*$. S1 photons have a wavelength of 178 nm and are detected by photomultiplier tubes in the detector. An electric field is applied in the liquid Xe, causing the electrons which have escaped their atoms to drift upwards toward the liquid-gas interface. Once inside the gas, a much stronger electric field is applied which causes a second scintillation light (S2). The number of S2 photons is proportional to the type of interaction that caused S1. In this way the
relationship of S2 to S1 signal can be used to differentiate particle interactions. The pattern of hits from S1 and S2 signals on both the array of PMTs above the gas phase and the array below the liquid phase are used to reconstruct the position of the initial interaction. (Aprile et al., 2010)

1.4 Monte Carlo Methods

Monte Carlo methods were first invented by Stanislaw Ulam and Nicholas Metropolis, and have been since applied to many areas including high energy physics. It was named “Monte Carlo” by John Von Neumann who named it after a casino in Monaco where Ulam’s uncle would gamble. During World War II they were used to study radiation shielding and how far neutrons would tend to travel in a material. (N.Srimanobhas)

Monte Carlo simulations rely on multiple experiments using random number generation to understand and solve complex equations. A classic example of a probability theory problem is the birthday problem. Given that there are x people in a room, what is the probability that no two share a birthday? The classic approach is to consider the probability person by person: With only one person, there is a \(\frac{365}{365}\) chance of not sharing their own birthday with someone else. The next person has a \(\frac{364}{365}\) chance of not sharing a birthday. Now the third person cannot share the birthday of either of the two first individuals so they have a \(\frac{363}{365}\) and so on. Because these are all independent probabilities they can be multiplied together to get the probability that no two people share a birthday:

\[
\frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} \ldots
\]

The probability that two people share a birthday is simply 1 - this result. This problem can also be approximately solved by Monte Carlo methods. With a Monte Carlo simulation, for each trial x random numbers from (1,365) are generated for a group of x people. Then the program checks if any of the two numbers are the same. The program stores the result and continues to the next trial. As the number of trials increases, the result gets increasingly accurate. (Aldag)

In the XENON1T detector, Monte Carlo simulations allow predictions of events by simulating many possible particle interactions. Detector design and solving equations for every potential particle interaction in order to understand how data will look is a complex process. Monte Carlo simulations are able to analyze information from particle production generators and track the particles and their potential interactions through the detector. They are also able to simulate the response of the detector for the interaction. In this way, Monte Carlo simulations predict interactions and the way they will be seen by the detector.

In simulations for the XENON1T detector, randomness is useful for predictions in a variety of ways. Interactions happen with certain probabilities. For example, there is a probability of ionization when electrons are accelerated through the gaseous Xe. There is also a probability that the electrons will be caught by some impurity in the liquid Xe and will not drift to the gas phase to cause the S2 signal. All the various probabilities are taken into account by the Monte
Carlo simulation. Through multiple simulations, the Monte Carlo is able to converge on increasingly accurate results and predictions of what will happen in the detector and how it will appear to the detector. (Wlodek)

2 My Project

2.1 Noise Data

Noise is unwanted background signal that hinders the ability of a detector to clearly “see” an interaction. Noise is inherent in the XENON experiment because of electronic circuits which have intrinsic noise. The photo multiplier tubes (PMTs) which capture the S1 and S2 signal are routed through cables to a digitizer, where the signal from the PMTs is digitized. It is in the process of capturing scintillation light to converting it into a digital signal that noise can occur. Examples of noise include cross-talk, shot noise, and interference. Simulating noise and the response of the detector to noise is an important component of the Monte Carlo simulation. My task this summer was to implement a noise simulation that would be able to randomize noise. This new noise simulation can be used to understand how noise affects the ability of the detector to discriminate between S1 signals and unwanted noise.

The idea of the new simulation was to store the noise as Fourier coefficients, each of which could be randomized in some way to generate noise to add to an event. A Fourier transform is a way of breaking a function into its component frequencies. Each frequency is assigned an amplitude by the Fourier transform, which corresponds to its relative importance in the function. Fourier transforms are a way of taking noise from time space into frequency space. They are a good way to store noise because noise is made up of component frequencies and understanding the different components helps understand the noise and its causes. To add the noise back the the simulation, I would perform a reverse Fourier transform after applying randomness to the Fourier coefficients. This would allow me.

2.2 Analyzing Noise

I used a recent noise run to analyze the noise present in the detector. The noise run consisted of 65 noise events. I analyzed each event by taking the Fourier Transform and storing 50,000 coefficients for each channel. Each of the 260 channels corresponded to one of the 260 photo multiplier tubes in the detector. I looked at the power spectrum of the noise. The power spectrum shows the value of the Fourier coefficient squared for each frequency. Because my noise run came from 100 ms of noise data and included 100,000 data points, I looked at frequency increments per us. (Gao, 2017)
From Figure 1 we can see that there is a peak at 25 kHz. This peak is conserved across most PMTs. We can also see that the majority of the power spectrum comes from high frequency noise. Part of the reason high frequency noise seems so important may be because of how the noise run sampled noise. I looked at all 65 events from the noise run and found that throughout all events, the noise followed the same power spectrum. In order to have a power spectrum to compare my simulation with, I analyzed the distribution of a single Fourier coefficient within a single channel. I could see that the distribution was approximately Gaussian. I used this to model the power spectrum density of the 65 events by choosing a value for each Fourier coefficient for each channel by sampling from a Gaussian defined by the mean and standard deviation values for that channel and coefficient. In Figure 2, the distribution of the Fourier coefficient corresponding to 40,000 kHz is plotted using the 65 noise events. This distribution is approximately Gaussian and therefore I model it as such.

In Figure 3, I have plotted the power spectrum by modeling each coefficient from each channel as a Gaussian and choosing randomly from a Gaussian defined by that coefficient’s mean and standard deviation. I can see that this power spectrum preserves the features from the noise prevalent in each of the 65 events (such as the 25 kHz peak). This is the power spectrum with which I compare my noise model.

2.3 Old Noise Simulation

I compared the data to the current noise simulation. The current noise simulation uses noise from Xenon 1T noise run 5037. It takes randomly chosen noise
samples from a noise file, concatenates them, and then add them to the event for the entire pulse length. The problem is that noise can differ over time and concatenating the chosen noise samples may not be a realistic way to characterize noise. There is also the problem that because of the small size of the noise file, there is no randomness. Figure 4. looks at the power spectrum of the current simulation compared to the data from the latest noise run. The power spectra do not match. The 25 kHz peak, which is an important component of the noise is lost in the current simulation. Only the correct power spectrum for noise with frequencies above 10,000 kHz is preserved.
Figure 3: Top plot shows power spectrum for each channel for each of the 50,000 coefficients, by modeling each coefficient as Gaussian. The bottom plot shows the total power spectrum for each frequency (blue) and the cumulative (red).

Figure 4: The top plot shows the power spectrum from data. The middle plot shows the power spectrum from the current noise simulation. The bottom plot shows the total and cumulative power spectrum for each frequency (blue) and the cumulative (red).

2.4 New Noise Simulation

In the new noise simulation I stored the means and standard deviations of all 50,000 Fourier coefficients for each PMT channel. I was able to sample randomly from this normal distribution for each Fourier coefficient in each channel. I would then reverse transform the chosen Fourier coefficients back to time space.
I would add this to the event in the simulation. The new simulation is able to capture the power spectrum far more effectively than the old noise simulation (Figure 5). The slight discrepancies between the blue and green lines in Figure 5 are the result of adding randomness to the simulation. And though there is randomness, the shape and peaks of the power spectrum are preserved. The randomness is critical for the Monte Carlo simulation and this new simulation allows the detector to simulation randomized noise to understand how noise would affect signal.

Power Spectrum Comparison: New Simulation Vs. Data

![Figure 5: The power spectrum of data (blue) vs the power spectrum from new noise simulation (green) and old simulation (red)](image)

2.5 Correlations in Noise

2.5.1 Between Frequencies

To check that the noise model was accurate, I checked that there were no correlations within a channel or between channels. I first checked whether, for any Fourier coefficient, there was correlation within a channel. I was asking whether the value of any of the 50,000 Fourier coefficients in channel x was correlated with any other coefficient in that channel. In order to do this I used Pearson correlation coefficients. Pearson correlation coefficients measure the amount of linear correlation between two variables. The Pearson correlation coefficient between two variables is:

\[
    r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}
\]  

(3)

In order to plot the full distribution of Pearson correlation coefficients for a channel, I would have to plot the correlation coefficient for every pair of the 50,000 frequencies. This would be 50,000 choose 2 for each of the channels which is too computationally difficult. Instead, I focused on the high frequency
noise because that is what most affects the ability of the detector to see an "S1" interaction. I plotted the distribution of the high frequencies against the null distribution. The null hypothesis distribution is the distribution of Pearson correlation coefficients between uncorrelated parent populations and is defined as:

\[ p_r(r; v) = \frac{1}{\sqrt{\pi}} \frac{\gamma([v + 1]/2)}{\gamma[v/2]} (1 - r^2)^{(v-2)/2} \]  

where \( v = N - 2 \) is the number of degrees of freedom for an experimental sample of \( N \) data points.

The null hypothesis distribution is plotted Figure 6. with a smooth green line and the Pearson correlation coefficient distribution is plotted using a histogram in a blue line. The distributions match up and there is no significant correlation (Gao, 2013).

\[ \text{Pearson Correlation Coefficient Distribution in Data vs Null} \]

![Pearson Correlation Coefficient Distribution in Data vs Null](image)

Figure 6: The green line shows the expected distribution from uncorrelated parent populations. The blue line comes from data of the highest 1000 coefficients in channel 66

### 2.5.2 Between Channels

I analyzed the correlation between channels for a single coefficient/frequency. I plotted correlation matrices to show the value of the Pearson correlation coefficient for every pairs of channels for a single coefficient. I could see that there was correlations between the channels for many of the Fourier coefficients.

I analyzed the noise at 25 kHz to understand its source. In Figure 7. the correlation matrix for 25 kHz noise is plotted. The channels are grouped by digitizer module and then by connector. The inter and intra digitizer correlation is shown clearly in Figure 8. Analyzing the 25 kHz noise by plotting correlation coefficients in blocks of signal connector cables and digitizers shows that the cause of the 25 kHz is related to the digitizers. This understanding can help reduce the noise (as the 25 kHz peak is a relatively major contributor to the spectral power) because the source is known.
Correlation between Channels for 25 kHz Noise

Figure 7: There are clear blocks of red (positive correlation) and blue (negative correlation) that show correlations seem to be digitizer dependent.

Correlation between Two Digitizers for 25 kHz Noise

Figure 8: There is clear positive correlation within a digitizer and negative correlation between these two digitizers.

I also show the analysis of the highest frequency noise at 50 MHz in Figure 9. This correlation matrix is also grouped by digitizer and signal connector cable. This noise shows heavy positive and negative correlation, but there is no pattern based on digitizer or signal connector cable. The correlation between channels is very prevalent but there is no obvious source (de Perio, 2017).

Because there is no clear pattern to the noise correlation between channels as
Correlation between Channels for 50 MHz Noise

There is no clear grouping of correlation coefficients. The correlation changes for different frequencies, it becomes hard to model the noise. In order to account for the noise, I choose randomly from a Gaussian multivariate distribution that accounts for the covariance between channels. A Gaussian multivariate is simply a higher dimensional Gaussian distribution. Just as the Gaussian curve is characterized by a mean and standard deviation, a multivariate Gaussian is characterized by a mean vector \( \vec{u} \) and a variance-covariance matrix \( \Sigma(Shalizi) \).

To generate a random vector from a multivariate distribution, a Cholesky decomposition can be used. The Cholesky is a fast and efficient way to decompose a matrix so long as it is positive semi-definite. Since covariance matrices must be positive semi-definite, the Cholesky decomposition becomes very useful. In a Cholesky decomposition, a matrix is factored into a lower triangular matrix and its conjugate transpose:

\[
M = RR^T = \begin{bmatrix}
R_{1,1} & 0 \\
R_{2,n,1} & R_{2,n,2:n}
\end{bmatrix}
\begin{bmatrix}
R_{1,1} & R_{2,n,1} \\
0 & R_{2,n,2:n}
\end{bmatrix}
\] (5)

To generate \( n \) random samples (\( \vec{r} \)) from a multivariate distribution:

\[
\vec{r} = \vec{u} + R\vec{s}
\] (6)

where \( R \) is lower triangular from Cholesky of the covariance matrix, \( \vec{u} \) is a \( n \)-size vector of the means, and \( \vec{s} \) is a \( n \)-size vector of randomly generated numbers from \( N(0,1) \).
Applying the Cholesky to the covariance matrix for each coefficient allowed me to account for correlation between channels for the noise simulation. However, applying the Cholesky to a 260-260 matrix for each channel takes a long time. A Monte Carlo simulation requires many trials and so minimizing time per trial becomes important. In order to speed this process up, I used principal component analysis to reduce the matrix from 253x253 to 8x8. This reduction captured about 86% of the information in each covariance matrix. The n samples are then chosen in the new basis using the Cholesky method and transformed back afterwards. (Lin, 2017) This adds randomness and saves time. This method, however, has not been fully implemented in the simulation yet. Once this way of storing a reduced data file and sampling randomly from a multivariate distribution more efficiently has been implemented, the new noise simulation will be able to account for correlation between channels. (Rundel, 2012)

2.6 Summary

I was able to create a new noise simulation model that captured the power spectrum of actual noise data. My original new noise model was able to simulate noise efficiently, however it did not account for correlations between channels. When the new noise simulation is implemented (a current project of mine), it will have the ability to both preserve the power spectrum and account for correlation between channels. The noise model is also flexible. The mean and covariance of the Fourier coefficients can be recalculated with each new noise run. The difference in Fourier coefficients over time can be used as a diagnostic way to understand if sources of noise are growing/waning and from where they originate.

This new noise model is primarily useful for understanding the detectors ability to process an S1 signal. The size of the S1 signal will depend on the mass of the WIMP (which is unknown) and noise will make a difference in detection ability if the S1 signal is small enough. With this new noise simulation model, peak classification tests of S1 can be run. These peak classification tests show the percentage of S1's that the detector can classify. With the new noise simulation, the peak finder tests will be able to accurately and efficiently account for noise.
References


S. Aldag, MAT Exam Expository Papers.

T. Wlodek, “Monte carlo methods in hep = Available at http://ww-hep. uta.edu/~yu/teaching/summer02-5391/lectures/tomasz-061702.pdf (2017/08/03),”.


