# Hardware Development and Simulation Work for GRAMS REU Program at Columbia University - Nevis Labs

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1 Introduction

GRAMS (Gamma Ray and Anti-Matter Survey) is a next-generation balloon/satellite experiment whose purpose is two-fold: to discover/document MeV sources across the entire sky, as well as serve as an indirect dark matter search via measurement of antiprotons and antideuterons produced by dark matter annihilation or decay[2]. This REU project has two goals related to GRAMS. The project’s primary concern is to simulate how GRAMS will detect MeV gamma ray processes, with the end goal of quantifying the sensitivity of the detector (sensitivity being defined as the minimum photon flux needed to see a 5 sigma signal). There is also a secondary, experimental branch of the project where the gain of the photomultipliers (PM) to be used in MiniGRAMS (a smaller, proof of concept version of GRAMS) was tested.

2 Background and Motivation

2.1 Motivation

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{all_sky_images.png}
\caption{All sky images taken from a variety of collaborations that each scan different energy bands of the sky. The cut-out image denotes the MeV all-sky.}
\end{figure}

Compared to other energy bands of the sky, the MeV sky (which consists of gamma rays between 1 to 10 MeV) is very dark (See Fig. 1). COMPTEL (The Imaging COMPton TELeScope) was the first mission that catalogued this energy band in 1991; during its lifetime, only about 30 sources where identified [4]. Progress in imaging this regime of the sky has stalled since then. Having better knowledge of the sky in the MeV band would enable a number of scientific advancements. Examples include: direct observation of nuclear emission lines (which give direct experimental insight into nuclear synthesis processes), studies of the phase transition between thermal and non-thermal processes as observed in black holes, active-galactic nuclei, and neutron stars (said transitions can occur in the MeV spectrum), and an additional aid in multi-messenger astronomy [2].
2.2 Design of GRAMS

2.2.1 Operational Principles

GRAMS consists of a Liquid Argon (LAr) Time Projection Chamber (TPC) surrounded by two concentric rectangular arrays of plastic scintillators comprised of photomultipliers (See Fig. 2). On the top and bottom faces of the LArTPC are an anode and cathode plane respectively, between which an electric field is established. The liquid argon chamber and the readout electronics are held at $\approx 85^\circ K$, while the scintillators are at ambient temperature [2]. When gamma rays pass through the argon, they undergo a variety of processes; the relevant ones for detection are Compton scattering, photoabsorption, (where all the energy of the gamma ray is transmitted into an orbital electron of a Ar atom), and pair-production. Compton scattering and photoabsorption produce prompt scintillating light as well as ionized electrons. The scintillation light moves out and hit the PMs inside the tank. The electric field then drifts the ionized electrons towards the anode plane at the top. The timing difference between the scintillation light signal and the electron signal can be used to identify the $z$ position of the interaction via

$$z_{pos} = v_{drift} \times (t_{drift} - t_{scintillator})$$

where $v_{drift}$ is $0.1601 \frac{cm}{\mu s}$ at $E = 0.5 \frac{keV}{cm}$ ($v_{drift}$ varies with electric field strength) [1]. For gamma ray detection, the exterior scintillators are irrelevant. Spatial resolution in $x$ and $y$ can be determined via the location of the impact between the electron and the anode. The energy of the ionized electron is also determined on impact with the anode plane (See Fig. 3 for visualization). The anode plane can either be implemented as a wire mesh, as in MicroBooNE, or as a pixel-based charge readout, like in XENON. The wire mesh approach has the problem of being susceptible to vibrational noise (which can’t be suppressed during flight).

For pair production, a gamma ray decays and produces a positron and an electron. The two produced leptons propagate through the detector, ionizing the LAr electrons. These ionized tracks can be used to reconstruct the interaction point of the pair-production, as well as the momentum (and hence direction) of the gamma ray source. This report is primarily concerned with Compton scattering and photoabsorption, hence treats these products as noise in the simulation.
Figure 3: Geant4 renderings of GRAMS detector. Red denotes the LAr, cyan denotes the scintillators, and yellow denotes the separator sheets. The dimensions as specified in the original concept paper are: 140x140x20 cm for the active LAr size, 350x350x20 cm for the lengths of the exterior scintillators and 150x150x30 cm for the interior scintillators (These dimensions apply to Fig. 3a). The dimensions of the alternative Cubic Setup: 70x70x80 cm for the active LAr size, 350x350x20 cm for the lengths of the exterior scintillators and 80x80x90 cm for the interior scintillators (These dimensions apply to Fig. 3b ). The anode and cathode planes are directly above and below the LAr in both figures.

2.2.2 Design Motivations

TPCs are quite ubiquitous in high energy physics, seeing use in experiments like ALICE, XENON, and MicroBooNE to name a few[8][10][7]; all of the above use liquid noble gas TPCs as their detection method.

There also exists other TPC designs that utilize gaseous argon, semi-conductors and/or scintillation crystals as the active medium in the chamber [2]. The advantages that LAr has over other traditional TPC methods are [2]:

1. The density of LAr is much greater than gaseous noble gases. As such, the cross section for incident MeV gamma rays is markedly higher.

2. Conversely, LAr has the potential to have a larger fiducial volume compared to solid-based TPCs since the necessary amount of embedded readout electronics is lower (only the non-uniformity of the electric field really contributes to fiducial volume reduction)

3. Argon is plentiful and cheap to procure, and hence allows a relatively large detector as constrained by the weight requirements for balloon/satellite flights

An additional relevant design consideration are the separator sheets mentioned in Fig. 3. One experimental challenge is that a single Compton electron emitting scintillation light might be detected by two PMs. The two signals might then overlap enough to become indistinguishable from each other. To mitigate this problem, the separator sheets divides the LAr into cells, optically isolate each PM from each other. This reduces coincident measurements (although if two measurements occur within the same cell, the same problem occurs. See Sec 4.6 for further discussion).
3 Hardware Development

3.1 Experimental Setup

3.1.1 MicroBooNE PM Setup

A Hamamatsu R5912 photomultiplier tube (PMT) that was previously used in MicroBooNE was utilized. While this PMT is too large to be place in GRAMS, it provided a means to test the experimental setup until the lighter PM arrived. In addition, the MicroBooNE readout system could be used for the PMT; the hope is to re-purpose the readout system of MicroBooNE for MiniGRAMS.

![Figure 4: The PM Testing Stand. Fig 4a shows the inside of the dark box. Fig. 4b shows a schematic of the setup.](image)

In order to test the PMs, the set up in Fig. 4 was constructed. The high voltage source powers the PM. The signal generator that drives the LED is a GATE generator that has a FWHM (this is the Full Width at Half the Maximum) of 10 ns and a amplitude of 2 V. The light travels through to the PM and gets converted to a voltage signal which is then read by the oscilloscope. The oscilloscope then triggers on the GATE pulse so that the response curve of the the PM can be seen. The oscilloscope can then measure various parameters of the output (integral of PM, maximum voltage, rise time etc.)

3.1.2 Silicon Photomultiplier (SiPM)

![Figure 5: The SiPM Testing Stand. Fig 4a shows the inside of the dark box. Fig. 4b shows a schematic of the setup](image)

The SiPM setup (Fig. 5 was place in the same dark box as the PM in Fig. 4a. The readout board and programmable board run on 5V and draws .37 Amps. The bias of the SiPM is 33.2 V at .2 amps. The LED is driven by the same GATE generator[3].

3.2 Gain Calculation PMs

The primary test with the PMs that was performed was measuring the gain at various voltages (ie. how much did the photoelectron signals get increased by). When a large number of photoelectrons
interact with the PM in a given time interval, the PM pulse shape follows a Poisson distribution. This Poisson distribution can be approximated as a Gaussian for a large enough number of photoelectrons. The average number of PEs is then given by \( \langle PE \rangle = \mu^2 \), where \( \mu \) and \( \sigma \) are the area and standard deviation of the integral of the PM pulse respectively\[9\].

The gain is then given by \[9\]:

\[
\text{gain} = \frac{V \times t}{\langle PE \rangle \times C \times \Omega}
\]

where \( V \times t = \mu \), \( C \) is the charge of an electron, and \( \Omega \) is the impedance of the coaxial cable.

### 3.3 Hardware Results

#### 3.3.1 PM

When the measurement was performed with the PM, the following results were produced:

![Image of how the PMT output was captured. The oscilloscope triggered on the LED voltage going above a threshold, and then captured the output from both the LED and the PMT. The PMT output is smeared.](image)

**Figure 6:** Image of how the PMT output was captured. The oscilloscope triggered on the LED voltage going above a threshold, and then captured the output from both the LED and the PMT. The PMT output is smeared.

Upon performing the gain measurements, the following plots were made:

**Figure 7:** Comparison of gain measurements performed at Nevis to those done by the manufacturer.

The gain was over estimated at lower voltages and underestimated at higher voltages. While the upper boundary remains unexplained, the lower bound can be explained by the Fig. 8. There are actually two signals in the PMT output. This spread of values artificially drives up \( \sigma \) resulting in a higher gain. The origin of this bifurcation was not determined.
Figure 8: Close up image of the PMT Output at 600 V. There are two distinct curves in the output.

3.3.2 SiPM

(a) Oscilloscope output of the SiPM for single photoelectrons (filtered)   (b) Oscilloscope output of the SiPM for single photoelectrons (unfiltered)

Figure 9: Oscilloscope output of the SiPM, one with a filter between the LED and the SiPM, and one without the filter.

The SiPM was rated by the manufacturer to operate at 33.2 V. As such, analogous measurements could not be performed. The calculated gain came around to about $3.6 \times 10^6$, which is comparable to the gain measured at 600 V.

4 Simulation

4.1 Data Generation

GramsSim is a Geant-4 based simulation written by Dr. Seligman to study how gamma rays would react with the detector. The two relevant programs from this package used in this analysis were GramsSky and GramsG4. GramsSky simulates various kinds of particles generated from a sky. As it related to GRAMS, it generates gamma rays with a variety of positions on the northern hemisphere with varying energy distributions.

GramsG4 simulates particle transport through the detector via Geant4, which in turn is a library that simulates a variety of physics processes across many different materials and geometries. GramsG4 can either read in GramsSky output as input for the location of incident gamma rays, or generate the gamma rays itself.
4.2 Analysis Tools

The output of GramsG4 is written out in a format that is interpretable by ROOT. ROOT is a software package that allows large volumes of particle physics data to be stored, manipulated and analyzed in an efficient manner. As it relates to this project, ROOT enabled easy visualization of the output of GramsG4, as well as the rest of the analysis. The vast majority of processing and analysis of GramsG4’s output was done via Python scripting. Standard python packages such as numpy and matplotlib where utilized for the analysis. The details of particular usage can be found in Sec. 4.3.

4.3 Backpropagation Algorithm

![Compton telescope diagram](image)

**Figure 10:** Illustration of the working principle of a Compton telescope via "Compton cones". Ideally, event reconstruction would exclusively have events of the left-most and left-middle event type, where all the scatters in a series are detected. However, "escape events", where some interactions occur outside the detector, are useful as well [2]

Once a series of Compton scatters and photoabsorbtion events is recorded (an "event" consists of: the position of each interaction, the time at which they occurred at, and the deposition energy of the particle), possible candidate gamma-ray locations on the sky as well as their energies are extracted. The original method when Compton camera’s where first invented is called the backprojection algorithm, as illustrated in Fig. 10. While better Compton camera reconstruction algorithms exist, the backprojection algorithm is the most intuitive and serves as a good tool for a preliminary study. the energy of the incident gamma-ray can then be reconstructed via

$$E = \sum_{i=0}^{n-1} E_i$$

where $n$ denotes the total number of interactions and $E_i$ denotes the deposition energies of the i-th interaction.

Via relativistic kinematics, one can deduce the scattering angle of an incident gamma ray via the measured deposition energies. There are two cases: all in events and escape events. All in events denote scatters where all the interactions in a series are detected. The reconstruction for this case is given by:

$$\cos(\theta) = 1 - m_e c^2 \left( \frac{1}{E_{\text{initial}}} - \frac{1}{E_{\text{final}}} \right)$$

where $E_{\text{initial}}$ is the total energy as given by Eq. 3 and $E_{\text{final}} = E_{\text{initial}} - E_0$, where $E_0$ denotes the energy of the first deposition.
An escape event denotes cases where not all the scatters in a series where recorded by the TPC (for instance, the gamma ray exited the TPC). In this case, meaningful reconstruction can only be extracted if the number of detected scatters is 3 or more. If this is the case the energy and scattering angle can be reconstructed via:

\[
E = E_1 + E_2 + E_3
\]  

\[
\cos \theta = 1 - m_e c^2 \left( \frac{1}{E_2 + E_3} - \frac{1}{E_1 + E_2 + E_3} \right)
\]  

\[
E_3' = \frac{E_2}{2} + \sqrt{\frac{E_2^2}{4} + \frac{E_2 m_e c^2}{1 - \cos \theta'}}
\]

Where \(\theta'\) is the geometric angle as measured in Fig. 10.

A given scattering angle does not correspond to a unique location on the sky; rather, each angle is associated with an cone of possible incident directions \[5\]. these Compton Cones can be constructed via the scattering angle (as derived in Eq. 4), the position of the point of intersection of all the rays (as given by the position of the first interaction) and the axis of the cone (which is found by drawing the vector from the second interaction point to the first). The vectors on the surface of the cone are then projected to a very large sphere (representative of infinity).

Once enough Compton Cones have been gathered, one can map the projected vectors on the sphere to the x-y plane and examine regions where they overlap; said regions are the candidate source locations (See Fig 11 for example of reconstruction process). More technical details of the reconstruction can be found in Appendix A.

\textbf{Figure 11:} Example of the backpropagation algorithm. The source is taken to be a known MeV gamma ray source: blazar 3C279. The left most rendering shows the Compton circles being projected onto a sphere. The central image is a simple downward projection of all the points on the sphere (The red dot denotes the true location of the blazar. The rightmost image denotes the 2d histogram constructed from the central one. The small yellow dot denotes the most likely candidate for the source as determined by the algorithm; this aligns quite well with the truth.
## 4.4 Counting Study

### Table 1: Event counts for 1 million incident Gamma rays.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Configuration</th>
<th>No. of Events</th>
<th>% of Total</th>
<th>Mean</th>
<th>Std Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{evt}$</td>
<td>Flat</td>
<td>59951</td>
<td>5.9951</td>
<td>2.389</td>
<td>2.469</td>
</tr>
<tr>
<td>$N_{evt}$</td>
<td>Cube</td>
<td>58878</td>
<td>5.8878</td>
<td>2.974</td>
<td>3.043</td>
</tr>
<tr>
<td>$N_{Ar}^{evt}$</td>
<td>Flat</td>
<td>14374</td>
<td>1.4374</td>
<td>3.974</td>
<td>2.847</td>
</tr>
<tr>
<td>$N_{Ar}^{evt}$</td>
<td>Cube</td>
<td>20420</td>
<td>2.042</td>
<td>4.999</td>
<td>3.397</td>
</tr>
<tr>
<td>$N_{firstAr}^{evt}$</td>
<td>Flat</td>
<td>7897</td>
<td>0.7897</td>
<td>4.278</td>
<td>3.057</td>
</tr>
<tr>
<td>$N_{firstAr}^{evt}$</td>
<td>Cube</td>
<td>14348</td>
<td>1.4348</td>
<td>5.352</td>
<td>3.533</td>
</tr>
<tr>
<td>$N_{allAr}^{evt}$</td>
<td>Flat</td>
<td>4084</td>
<td>0.4084</td>
<td>4.277</td>
<td>3.031</td>
</tr>
<tr>
<td>$N_{allAr}^{evt}$</td>
<td>Cube</td>
<td>10109</td>
<td>1.0109</td>
<td>5.74</td>
<td>3.59</td>
</tr>
<tr>
<td>$N_{succ}^{evt}$</td>
<td>Flat</td>
<td>1524</td>
<td>0.1524</td>
<td>1.539</td>
<td>0.9337</td>
</tr>
<tr>
<td>$N_{succ}^{evt}$</td>
<td>Cube</td>
<td>2235</td>
<td>0.2235</td>
<td>1.893</td>
<td>1.354</td>
</tr>
<tr>
<td>$N_{unique}^{evt}$</td>
<td>Flat</td>
<td>1486</td>
<td>0.1486</td>
<td>1.462</td>
<td>0.7788</td>
</tr>
<tr>
<td>$N_{unique}^{evt}$</td>
<td>Cube</td>
<td>2181</td>
<td>0.2181</td>
<td>1.681</td>
<td>1.028</td>
</tr>
</tbody>
</table>

The original design of GRAMS is a flat geometry of 140x140x20 cm$^3$[2]. The motivation for this is because a large amount of events are to come from the zenith, so having a large cross-sectional area for this sky location would be beneficial. However, GRAMS is going to be an all sky detector. Therefore, it would be useful to see if having a more uniform cross section in all directions is better. The cubic geometry examined has the same volume 70x70x80 cm$^3$; this configuration also has this more uniform cross-sectional area property. A classification of events that occurred within the LArTPC of the two geometries was performed. The following metrics were examined:

- $N_{evt}$: Number of Compton scatters per event
- $N_{Ar}^{evt}$: Number of Compton scatters per event where scatters occurred in LAr
- $N_{firstAr}^{evt}$: Number of Compton scatters per event where the first scatter occurred in LAr
- $N_{allAr}^{evt}$: Number of Compton scatters per event where all scatter occurred in LAr
- $N_{succ}^{evt}$: Number of Compton scatters per event where all scatters occurred in successively-unique cells
- $N_{unique}^{evt}$: Number of Compton scatters per event where all scatters occurred in unique cells

"Unique cells" means that all the interactions occurred in different cells (See Sec. 2.2.2 for discussion). "Successively unique cells" means that adjacent interactions occurred in different cells. Taking the ratio of the number of events of each class, it is found that:

- In $N_{evt}$: Flat detected 1.018x the number of events than Cube
- In $N_{Ar}^{evt}$: Cube detects 1.421x the number of events than Flat
- In $N_{firstAr}^{evt}$: Cube detects 1.817x the number of events than Flat
- In $N_{allAr}^{evt}$: Cube detects 2.475x the number of events than Flat
• In $N_{\text{succ}}$: Cube detects 1.467x the number of events than Flat

• In $N_{\text{unique}}$: Cube detects 1.467x the number of events than Flat

The increased performance of the Cube geometry is in part due to the increased cross-sectional area that the Cube geometry has over the Flat geometry (this ratio is $\approx 1.8770$; see Appendix A for details on calculation). However, something that cannot be explained by the increased area is increase rate of $N_{\text{fullAr}}, N_{\text{succ}}, N_{\text{unique}}$. These events have all the interactions occur inside the LArTPC, and as such, give the most accurate energy and spatial reconstructions ($N_{\text{unique}}$ in particular are the ideal events to identify).

The increased rate of these event types can be attributed to the increased possible path length that the Cube geometry offers compared to the Flat geometry. While the ratio between the perceived depth that the source sees in the Cube versus in the Flat is 0.5 (See Appendix A.4), the actual path travelled by the gamma ray in the Cube geometry is consistently larger than the Flat geometry. This is probably due to the fact that the cube "fans out" more compared to the Flat, so when the gamma ray Compton scatters with a large transverse momentum, it has a higher chance to still be interact with the LAr. This is confirmed by the fact that for $N_{\text{fullAr}}, N_{\text{succ}}, N_{\text{unique}}$, the mean number of Compton scatters was higher in all categories in the Cube compared to the Flat (ie. the gamma ray travelled far enough in the TPC to undergo, on average, more Compton scatters in the Cube compared to the Flat).

### 4.5 Classification of Compton Scatters and Photoabsorbtions

![Classification of Compton Scatters and Photoabsorbtions](image)

**Figure 12:** Classification of Compton scatters and Photoabsorbtions into primary and secondary.

Compton scatters and photoabsorbtions can arrive not only from the original gamma ray source, but from other origins as well. Fig. 12 demonstrates these multiple possible origins. "Primary" denotes interactions that originate from the gamma-ray source. "Secondary" groups together interactions that do not originate from the gamma ray source (for instance, an electron from a pair-production could Compton scatter, a photon produced during bremsstrahlung could Compton scatter or get photoabsorbed etc.). The "secondary" interactions can lead to false positives if they are identified as part of a Compton scatter series.
4.6 Number of Scatters per Series

In order for escape event to be reconstructed at all, there must be at least 3 scatters in a series. As such, it would be beneficial to know how many scatters on average occur in the TPC. Fig. 13 accomplishes this by counting the number of scatters per incident gamma ray in a series where all the scatters occurred in the LArTPC. There is a heavy right skew of the data, where a large number of events have series lengths larger than 3, hence demonstrating that there are a sufficient number of events for escape reconstruction to be feasible.

This ties into a crucial experimental factor that GRAMS is dependent on. With a large number of scatters in a series, it is probable that some interactions might occupy the same optically isolated cells. In this case, the two signals would both arrive at the same PM. If the pulses overlap each other, then the PM cannot distinguish the two events. As such, the PM’s used by GRAMS need to have a fine enough temporal resolution to be able to distinguish the two signals.

There also exists a problem with the reconstruction that is related to the temporal resolution of GRAMS. The backpropagation reconstruction algorithm is only efficient if the scattering order is known. There is a method that are agnostic to the scattering order that amount to a brute-force search that tries all possible combinations and sees which orders are consistent with the kinematics and geometry of the series (see [5]). However, this method go as $O(n!)$, where $n$ is the number of scatters in the series. Therefore, having good timing information not only would yield more usable events, it would also make the analysis easier.

Figure 13: Counts the number of scatters in a series per incident gamma ray for a input of 1 million gamma rays
4.7 Energy and Spatial Reconstruction

4.7.1 All In Events versus Escape Events

Figure 14: Reconstruction of all in events that takes in 1 Million incident Gamma rays. The red dot denotes the true location of the source. Fig. 14b is a inset around the true source location. Axes of plots are location on down-projection of sky in cm

Figure 15: Reconstruction of escape events that takes in 1 Million incident Gamma rays.

Upon performing the reconstruction on all in events and escape events, one get Fig. 14 and Fig. 15 respectively. The two renderings give very similar results with regards to spatial reconstruction. This is in contrast to the energy reconstruction of the two classifications (See Fig. 3). Fig. 16a has near perfect energy reconstruction, while Fig. 3 has markedly poorer reconstruction ).
Figure 16: Energy reconstruction comparison between All In Events and Escape events. Both have a linear regression superimposed on top of the scatter plot. A slope of 1.00 and an intercept of 0.00 would denote a perfect energy reconstruction. Fig. 16a has a fit of $y = 1.037x - 0.071$ and Fig. 16b has a fit of $y = 0.619x - 0.005$, where $y$ is the reconstructed energy and $x$ is the incident gamma-ray energy.

In light of the above, it is of paramount importance to identify whether a scatter series is an all in event compared to an escape event; this is because this classification is essential to accurate energy reconstruction. However, GRAMS can be agnostic to the classification type when determining the direction of an MeV source.

4.7.2 No Photoabsorbtion

Figure 17: Reconstruction of series without the terminating photoabsorption.

Events where the series of Compton scatters do not terminate in a photoabsorption are also of interest. As seen in Fig. 17, the spatial reconstruction is essentially identical to Fig. 14 and Fig. 15, while the energy reconstruction (Fig. 17c) is markedly worse than even Fig. 16b. This reinforces the idea mentioned that spatial reconstruction is insensitive to the location of all the scattering events, while energy reconstruction relies heavily on knowing all the energies of the scatters in a series.
4.8 ARM

(a) Example of fitting a Lorentzian to an ARM distribution. The FWHM = |2\gamma|. \(\gamma\) is between absolute value bars because the non-normalized Lorentzian is unchanged under parity of gamma (See Eq. 9).

(b) A plot of the ARM as a function of energy. Incident from the zenith.

Figure 18: Calculation of ARM Distribution

One commonly used measure of a Compton camera’s resolution is called ARM (Angular Resolution Measure). It is defined as[6]:

\[
ARM = \theta_{geo} - \theta_{reconstruction} \tag{8}
\]

Where \(\theta_{geo}\) is the geometric angle between the axis of the cone and the true source location, and \(\theta_{reconstruction}\) is the reconstructed angle as calculated by Eq. 4 and Eq. 7.

Fig. 18a plots the ARM for the blazar source. In order to extract the angular resolution from this curve, some measure of the FWHM is needed. A Gaussian is an ill-suited fitting function since the tails of the ARM distribution are thicker. A literature review states that a Voigt function (the convolution of a Gaussian and a Lorentzian) is a proper fit function [6]. However, fitting said function yielded chi-squared values notably worse then just fitting a pure Lorentzian, defined as:

\[
A \frac{\gamma^2}{\gamma^2 + (x - x_0)^2} \tag{9}
\]

where \(x_0\) is the location parameter that specifies the peak of the distribution, and \(\gamma\) is the HWHM (half width half max). Therefore, pure Lorentzians were used as the fitting function all the ARM distributions. With this definition, one can define the angular resolution to be \(2\gamma\).

One would expect the ARM to be dependent on the energy of the incident gamma ray. The angular resolution for mono-energetic sources (between .2 and 10 MeV) was calculated, yielding Fig. 18b. There are two notable features of this distribution:

1. There is a sharp drop off in the angular resolution (ie. increased resolution) with regards to energy.

2. The curve stabilizes to an angular resolution of around .5 degrees for values above \(\approx 3\) MeV. This is consistent with the phenomena of Doppler broadening, which sets the theoretical limit of a Compton camera[6].
5 Conclusions and Looking Ahead

The goal of this project was to examine how the GRAMS detector will detect MeV gamma ray processes, as well as test the gain of PMs to be used in MiniGRAMS. With regards to the gain measurement, while the gain of the PMT did not align with those measured by the manufacturer, the SiPM performed reasonably well. Further progress could be made by performing the same gain test with the SiPM encased in a vat of LAr, at it would be on the actual MiniGRAMS flight.

With regards to the simulation, a cubic geometry was performed markedly better over the original flat GRAMS design. GRAMS has good spatial reconstruction, regardless of the event type. However, for acceptable energy reconstruction, being able to identify "All In" events from "Escape" events is essential. In addition, there are enough scatters on average to allow proper escape event reconstruction. Finally, the angular resolution of the detector is consistent with the theoretical limit of Compton cameras.

Further simulation work that could be done includes: adding smearing effects to account for experimental uncertainties, calculating the sensitivity as a function of incident gamma ray energy and sky position, seeing how the backprojection algorithm can reconstruct multiple point sources and diffuse sources, and finally, using a better reconstruction method other than the naive approach.

6 Acknowledgements

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7 Links

GramsSim can be found at https://github.com/wgseligman/GramsSim. Geant4 and ROOT can be found at https://geant4.web.cern.ch/ and https://root.cern/ respectively. Numpy and Matplotlib can be found at https://numpy.org/ and https://matplotlib.org/. Most of my code can be found at /nevis/milne/files/ms6556. Some of the larger files and condor jobs can be found split between /nevis/bleeker/data/ms6556 and /nevis/bleeker/share/ms6556.
A Algorithms

A.1 Quaternions

As mentioned in 4.3, a Compton Cone consists of an angle $\theta$, a axis vector, and a position vector for the tip of the cone. However, one still needs to generate the surface vectors of the cone from this information. One method is as follows:

1. Calculate any perpendicular vector ($\vec{P}$) to the axis vector ($\vec{A}$). As an example, define $\vec{A} = <a, b, c>$, where $a < b < c$. Then $\vec{P}$ can be constructed via the manipulation $(a, b, c) \Rightarrow (c, b, a) \Rightarrow (c, 0, -a)$, where by inspection $\vec{P} \cdot \vec{A} = 0$. Normalize both the $\vec{A}$ and $\vec{P}$.

2. Calculate the preliminary surface cone vector ($\vec{S}$) by taking the linear combination:

$$\vec{S} = \sin(\theta) \vec{P} + \cos(\theta) \vec{A} \quad (10)$$

3. Rotate the vector $\vec{S}$ around $\vec{A}$. This can be done by subdividing the interval $[0, 2\pi]$ in $N$ divisions, and then rotating the vector $\vec{S}$ by $\theta = 2\pi/N$ around $\vec{A}$ $N$ times in an iterative fashion. This generates the surface cones to be projected.

Step 3 was implemented via quaternions. Quaternions can be thought of a a 4-tuple $<w, x, y, z>$, which in turn represents $w + xi + yj + zk$. $\hat{i}, \hat{j}, \hat{k}$ are the standard Cartesian unit vectors. If one impose the algebraic constraints $\hat{i}^2 = \hat{j}^2 = \hat{k}^2 = \hat{i}\hat{j}\hat{k} = -1$ and the associative law, then these objects form the set of all quaternions. As a consequence of the above, one can prove $ij = -ji$ and other similar relations. In other words, quaternions are non-Abelian group under multiplication.

Quaternions of the form $q = <\cos(\theta/2), \sin(\theta/2) \ast u_x, \sin(\theta/2) \ast u_y, \sin(\theta/2) \ast u_z>$, (where $\vec{u} = <u_x, u_y, u_z>$ is a normalized unit vector) implement the exact same rotation as the classic matrix multiplication approach. Once can rotate a vector $\vec{U}$ via the equation

$$u_{\text{new}} = q \ast u \ast q^{-1} \quad (11)$$

where $q^{-1} = <\cos(\theta/2), -\sin(\theta/2) \ast u_x, -\sin(\theta/2) \ast u_y, -\sin(\theta/2) \ast u_z>$ is the inverse of $q$ and $u = <0, u_x, u_y, u_z$ Quaternions have the advantage in that numerical stability is less of an issue compared to matrices since it is trivially easy to normalize a quaternion (just treat the quaternion as a 4-vector and calculate the L2 norm). Re-orthogonalizing a rotation matrix is much more complex by comparison.

A.2 Spherical Projection

Once the surface vectors of the cones are know, each vector needs to be projected onto a large sphere. Let $\vec{P}$ denote the tip of the cone, and $\vec{D}$ be the unit vector in the direction of the surface vector. One can parameterize the line $\vec{L}$ that lies along $\vec{D}$ that passes through $\vec{P}$ as $\vec{L} = \vec{P} + t\vec{D}$, with $t \in \mathbb{R}$. When $\vec{L}$ intersects a sphere centered at the origin of radius $R$, $\|\vec{L}\| = R^2$. Expand out the dot product on the left and solve for $t$, one find that

$$t_{\pm} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (12)$$
Figure 19: GeoGebra rendering of maximum cross sectional area from various angles. The left denotes a source from the zenith and the right denotes a source with incidence from one of the corners of the cube.

where \( a = \| \vec{D} \|^2 \), \( b = 2 \times \vec{D} \cdot \vec{P} \), \( c = \| \vec{P} \|^2 - R^2 \). This will yield a sensible result if \( \vec{a}! = \vec{0} \) (which is trivial), and \( b^2 - 4ac \geq 0 \). Making the appropriate substitutions and rearranging yields the convergence condition:

\[
\frac{R}{\| \vec{P} \|} \geq \sin \theta
\]  

(13)

In other words, as long as the location of the interaction point in within the celestial sphere, two unique, real solutions are guaranteed (which is exactly the setup in GRAMS). One then take the \( t \) that yields a z-coordinate above the horizon.

A.3 Cross Section Calculation

In order to calculate the sensitivity of a source, one needs to know the effective cross-sectional area as seen by the source. As the source location moves around the detector, this cross sectional area changes (See Fig. 19). To calculate this area, a Monte Carlo simulation can be utilized. First, fix the LAr detector (assumed to be a parallelepiped) with its center at the origin and the normal vectors aligned with the x,y,z axes. Let the dimensions of the detector be \( < a, b, c > \), and WLOG let \( a < b < c \). Place a rectangle in the x-y plane centered at the origin and dimensions of \( < \sqrt{a^2 + b^2 + c^2}, c > \). Populate this rectangle with N points. Rotate the rectangle such that it’s normal is aligned with the source location. Let \( N_{in} \) denote the number of points that lie within the parallelepiped. The cross-sectional area can then be approximated via

\[
\text{Area} = \sqrt{a^2 + b^2 + c^2} \times \frac{N_{in}}{N}
\]  

(14)

This approximates gets better as \( N \) increases. It is know that this is the maximum cross sectional area since the plane that has the maximum cross-sectional area must run through the origin. For the area calculation of the Flat and Cube in Sec. 4.8, let \( N \) be 10 million points. The source location of 3C279 in the detector is along the direction \( < 282.79227, 95.44493, 30.31168 > \). Using the relevant parameters for each geometry as outlined in 3, one finds that the area of Flat is 3001.05 and the area of Cube is 5633.018. The ratio of Cube to Flat is \( \approx 1.8770 \).

A.4 Perceived Depth

In order for a scatter series to be classified as all in, it needs to terminate in a photoabsorbtion. Therefore, enough distance is needed for the gamma ray to propagate so that photoabsorbtion can occur. This number is between 5-30 cm for LAr at the relevant energy scales\[11\]. To calculate the perceived depth, define a plane to be \((\vec{p} - \vec{p}_o) \cdot \vec{n} = 0\), where \( \vec{p} \) is any point on the plane, \( \vec{p}_o \) is the
location of the plane’s origin, and \( \hat{n} \) is the normal of the plane. Define the source location in an analogous manner to Sec. A.2, where \( \vec{D} \) point towards the source location and \( \vec{P} \) be the coordinate origin (This is because the line that quantifies the max perceived depth passes through the origin). Plugging in the equation for the line into the plane equation and isolating \( t \) yields

\[
t = \frac{(\vec{p}_o) \cdot \hat{n}}{\vec{D} \cdot \hat{n}}
\]  

(15)
References


