EXPERIMENT 2-6

e/m OF THE ELECTRON

GENERAL DISCUSSION

The "discovery" of the electron by J. J. Thomson in 1897 refers to the experiment in which it was shown that "cathode rays" behave as beams of particles, all of which have the same ratio of charge to mass, *e/m*. Since that time, a number of methods have been devised for using electric and magnetic fields to make a precise measurement of *e/m* for the electron. When combined with the value of the electron's charge, which is measured in the Millikan Oil Drop Experiment, the determination of *e/m* leads to an accurate value of the mass of the electron. In the present experiment, electrons are emitted at a very low velocity from a heated filament, then accelerated through an electrical potential V to a final velocity v, and finally bent in a circular path of radius r in a magnetic field B. The entire process takes place in a sealed glass tube in which the path of the electrons can be directly observed. During its manufacture, the tube was evacuated, and a small amount of mercury was introduced before the tube was sealed off. As a result, there is mercury vapor in the tube. When electrons in the beam have sufficiently high kinetic energy (10.4 eV or more), a small fraction of them will collide with and ionize mercury atoms in the vapor. Recombination of the mercury ions, accompanied by the emission of characteristic blue light, then occurs very near the point where the ionization took place. As a result, the path of the electron beam is visible to the naked eye.

The tube is set up so that the beam of electrons travels perpendicular to a uniform magnetic field B. B is proportional to the current I through a pair of large diameter coils (so-called "Helmholtz Coils") in which the coil separation is selected to produce optimum field uniformity near the center.

Experimental Apparatus

A. The Sealed Glass Tube

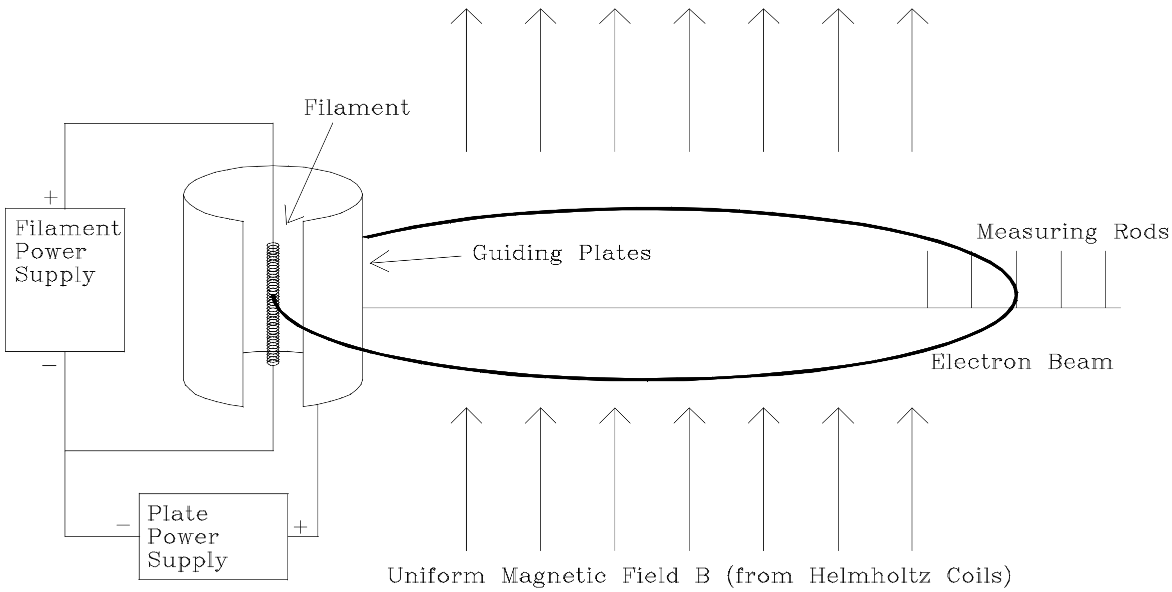

Figure 1. Diagram of the interior of the sealed glass tube.

Figure 1 shows the filament surrounded by a small cylindrical plate. The filament is heated by passing a current directly through it. A variable positive potential difference of up to 100 volts is applied between the plate and the filament in order to accelerate the electrons emitted from the filament. Some of the accelerated electrons come out as a narrow beam through a slit in the side of the cylinder. The entire tube is located inside a set of coils, which produce a uniform magnetic field B perpendicular to the electron beam. The magnitude of the field can be adjusted until the resultant circular path of the electron beam just reaches one of the measuring rods. These rods are located along a cross bar, which extends from the cylinder in a direction perpendicular to that in which the electron beam was emitted--i.e., along a diameter of the circular orbits.

B. The Helmholtz Coils and Uniform Magnetic Field

The magnetic field produced at the position of the electron beam by a current I flowing through the coils must be computed. For a single turn of wire of radius R, the field on the axis at a distance x from the plane of the loop is given by:

$$B' = \frac{m_0 R^2 I}{2\left(R^2 + x^2\right)^{3/2}} \ .$$

For the arrangement in Figure 2, there are two loops with N turns each, separated by a distance equal to the coil radius R. The coils contribute equally to the field at the center:

$$B_I = \frac{\mu_0 R^2 NI}{\left[R^2 + (R/2)^2\right]^{3/2}} = \frac{4\pi \times 10^{-7} NI}{R(1+\frac{1}{4})^{3/2}} = \text{constant} \times I \text{ Tesla}$$

where N = 72 is the number of turns of each coil and R = 33 cm is the radius of the coils used.

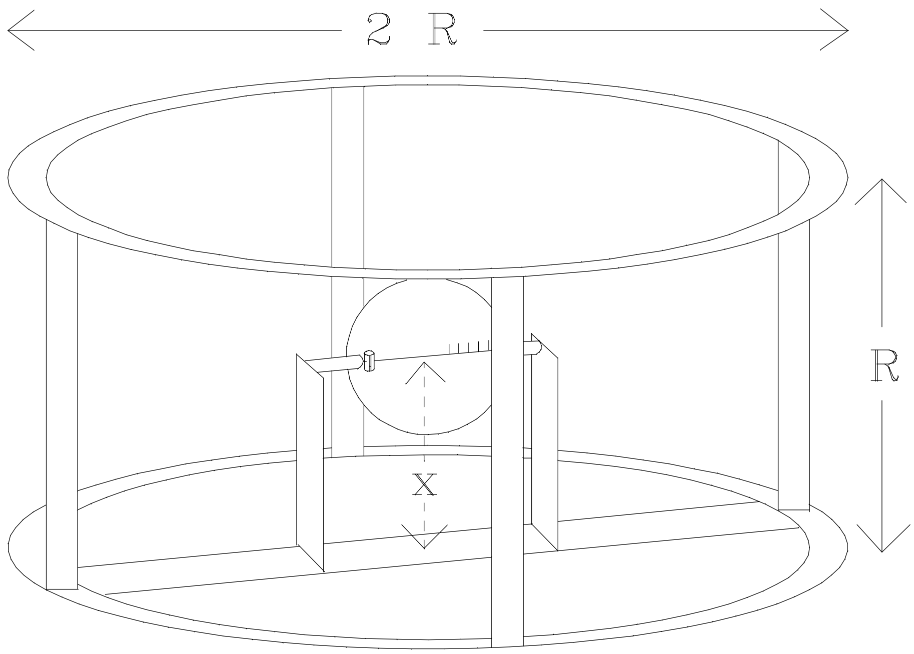

Figure 2. Helmholtz coils used to produce a uniform magnetic field.

This arrangement, called a pair of **Helmholtz coils,** yields a remarkably uniform field in the region at the center.

The net field B in which the electrons move is not B_I alone, but the resultant of the earth's field B_e and B_I. If the equipment is oriented so that the field of the Helmholtz coils is parallel to that of the earth, and if the current through the coils causes B_I to be directed opposite to B_e, then

$$B = B_I - B_e \qquad (1)$$

C. The Trajectories of the Electrons in the Glass Tube

If an electron of charge *e* and mass *m* starts nearly from rest and is accelerated through a potential difference *V* to a final velocity *v*, then

$$\tfrac{1}{2} mv^2 = eV \quad \text{or} \quad e/m = \frac{v^2}{2V} \ . \qquad (2)$$

If the electron then enters a uniform magnetic field *B* which is perpendicular to its velocity, it will move in a circular orbit of radius *r*, where

$$\frac{mv^2}{r} = evB \quad \text{or} \quad e/m = \frac{v}{Br} \ . \qquad (3)$$

If it were possible to measure the velocity directly, then *e/m* could be determined by measurements of either the electric or magnetic field alone. Since a direct measurement of *v* is not feasible in this experiment, *e/m* can be determined from the combination of electric and magnetic fields used. Specifically, by eliminating *v* from equations (2) and (3), *e/m* can be expressed directly in terms of *V*, *B*, and *r*.

Instead of determining *e/m* from a single measurement of *r* for given values of *V* and *B*, however, it is preferable to measure the variation of *r* with *B* (or *I*) at fixed values of *V*. In particular, the data can be presented in convenient form by plotting the **curvature** 1/*r* as a linear function of *I*.

$$\frac{1}{r} = \sqrt{\frac{e/m}{2V}} B_I + \sqrt{\frac{e/m}{2V}} B_e \qquad (4)$$

Derive (4) from (1), (2), and (3) for your report <u>before</u> coming to laboratory.

Note that equations (2), (3), and (4) apply strictly only to electrons with trajectories on the *outside edge* of the beam -- i.e., the most energetic electrons. There are two reasons why some electrons in the beam will have less energy:

1) There is a small potential difference across the filament caused by the heating current. Only electrons leaving the negative end of the filament are accelerated through the whole potential difference *V*.

2) Some of the electrons in the beam will lose energy through collisions with mercury atoms.

PROCEDURE

Orientation of the Apparatus

Orient the coil frame so that the axis of the coils is parallel to the earth's field. The direction of the field can be determined by using a compass and a dip needle. The compass needle will indicate the horizontal component of the field, and the dip needle, when its plane is aligned in this direction, will show the true field direction. Do not be surprised if the earth's magnetic field in a steel frame building is not in the direction you would expect.

Preliminary Adjustments

The supplies and controls for the Helmholtz coils and the filament are permanently wired on a board and are designed to minimize the possibility of damage to the tube or coils. Locate each control, and note the qualitative effects observed when the control is varied. In particular:

a) Figure 1 shows that the filament and its associated lead wires form a small loop. Since a 4 amp current is required to heat the filament, this loop creates a measurable field. The filament coil reversing switch permits you to study the effect of this field. The effect can be minimized in the experiment by rotating the tube slightly in its mounting so that the electrons come out parallel to the plane of the coils.

b) Note the direction of the coil current for each position of its reversing switch by using the dip needle to check the direction of the resultant field. Knowing the field direction, check the sign of the charge of the particles in the deflected beam. Also, determine whether the earth’s field adds to or subtracts from the coil field.

c) The beam will have a slight curvature in the earth's field when the coil current is zero. Make a preliminary measurement of the earth's field by adjusting the coil current to remove this curvature. The special Meter Switch and low current meter (200 mA) will enable you to measure the relatively small current needed, and the straight line trajectory can be checked by comparison with the light emitted from the filament.

Measurement of the Circular Orbits

With the accelerating voltage at an intermediate value, the current in the Helmholtz coils can be adjusted so that the outside edge of the beam strikes the outside edge of each bar in turn. Measure field current as a function of radius for the highest voltage V which allows you to adjust the beam with respect to all five bars.

For one measurement, test the reproducibility of the current setting as an aid to error analysis. The tube manufacturer supplies the following values for the *diameters* from the filament to the *outside* of each bar in succession:

6.48 cm, 7.75 cm, 9.02 cm, 10.30 cm, 11.54 cm

Calculations

Plot a graph of $1/r$ versus I, and draw the straight line that gives a best fit to the five measured points. Use equation (4) to calculate *e/m* from the slope of this line. Write your report up to this point and then proceed to the following:

Further Considerations

a) Calculate the percentage difference between your value of *e/m* and the accepted value which is $1.758\times 10^{11}\ \text{coulombs}/\text{kg}$. What do you think is your largest source of error? Can you account for this much error by a numerical estimate?

b) Calculate the actual maximum velocity of the electrons in your beam.

c) Make another run with a lower accelerating voltage. Do you find a consistent value of *e/m*?

d) Compare the intercept of your graph with the value of coil current you obtained by balancing the earth's field. If these numbers are not roughly the same, you may have made an error. Note that this current is not an important number, but it makes a good check on your technique. Physicists frequently check the consistency of their data by computing numbers which they do not "need."

e) Calculate the actual value of B_e.

f) Test the maximum error in your readings which could be caused by a change in the field of the nearest neighboring coil.