

Study Guide
Physics C1402(001) – Final
Prof. H. Evans

Monday, May 7 – 1:00-4:00 pm
Room 309, Havemeyer

(ver.1) http://www.nevis.columbia.edu/~evans/c1402/pdf/f_guide.pdf

MATERIAL COVERED

- Concepts in Halliday, Resnick and Walker, **Chapters 22-37**
 - There will be 6 questions on the exam
 - The approximate breakdown of the questions will be:
 - Half covering Chapters 22-33 (midterms 1 & 2)
 - Half covering Chapters 34-37 (after midterm 2)
- Questions will be based on material covered in:
 - Lectures
 - Problems in the Homework
 - Sections of the Text not touched on in Lectures or Homework **will not be included** in the exam.
- Question Format:
 - Questions will be very similar to the homework

WHAT TO BRING

- The exam will be closed book, but
- You will be allowed to use one 8½ x 11 note sheet
 - You may use **both sides** of the note sheet
 - The notes must be in your handwriting – No photocopies
 - You may include anything you want on the note sheet (formulas, example problems, graphs, inspirational poetry...)
 - You will be asked to hand in your note sheet with your exam, so **put your name on it**
 - The note sheet will not be graded, but
 - Failure to hand in your note sheet will cost you points
- Remember to bring a **calculator**
- I will provide all necessary:
 - constants
 - unit conversions and prefixes (nano, pico, etc.)

GENERAL ADVICE

- **Studying for the Exam**
 - Do **all** the homework problems and understand the solutions.
 - Review your lecture notes.
 - Have a look at the “Questions” at the end of each chapter. If you find that you have problems with groups of them you should concentrate on understanding those areas.
- **Exam-Taking Strategies**
 - Before beginning the exam read over all the problems.
 - Before doing a problem read it carefully so you don't miss anything.
 - Start with the easiest problem.
 - If you get stuck – don't waste time. Go on to another problem.
 - Write legibly. If the grader can't read your solution he/she can't grade it.
 - Show your work. No work = no partial credit.
 - Draw detailed pictures - this can indicate to the grader that you understand the concept of the problem even if you don't do the math correctly.
 - Solve problems algebraically before plugging in numbers.
 - Check your answers for correct units and reasonable values.
 - **Don't worry if you can't do everything.** Grading will be on a curve.
- **Time Budgeting**
 - Do not allow yourself to miss out on points by wasting all your time on one problem.
 - **Make a time budget** at the beginning of the exam and stick carefully to it!
 - For a 3-hour exam with 6 questions a sample budget could be:
 - 10 min Read over all problems and decide the order to do them.
 - 25 min Spend this amount of time on each problem.
When a problem's time is up, stop, and go to the next one.
 - 20 min Check over your work and revisit any parts you couldn't do.
- **Partial Credit.** How to make sure you get it.
 - **Show your work.** You will not get credit if you simply write down the correct answer.
 - **Draw a diagram** of the problem with forces and a coordinate system.
 - A well drawn diagram will be worth points.
 - No diagram will make it difficult for the grader to give partial credit.
 - **Show clearly the steps** you have taken in attempting to solve the problem.
 - If the grader can't follow your reasoning you won't get much credit.
 - **Include units** in your answers.
 - Points will be taken off for answers without units.
 - **Use appropriate significant digits**
 - Points will be taken off for answers that have many (3-4) more significant digits than the information given.

Key Concepts

UNITS

- Always include units in answers!
- Know common units
 - $V = J/C$, $A = C/s$, etc.
- Be able to convert back and forth between different units

VECTORS

- Difference between Vectors and Scalars
 - Vector = magnitude and direction (at least two numbers)
 - Scalar = number with sign
- Vector Representation
 - Cartesian: $\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}$
 - Unit vectors
 - Polar: $\mathbf{v} = \text{magnitude}(v) \text{ and angle}(\theta)$
 - Calculating components of vectors along coordinate axes
- Adding and Subtracting Vectors
- Multiplying Vectors
 - Dot Product: $c = \mathbf{a} \cdot \mathbf{b} = ab \cos\theta$
 - Cross Product: $\mathbf{c} = \mathbf{a} \times \mathbf{b}$
 - $c = ab \sin\theta$
 - Direction of \mathbf{c} from Right-Hand-Rule

ELECTRIC FORCES & FIELDS

- Charge is quantized:
 - $e = 1.60 \times 10^{-19} \text{ C}$
- Electrostatic Force and Coulomb's Law:
 - $\vec{F} = \frac{kq_1q_2}{r^2} \hat{r}$
- Electric Fields: $\mathbf{E} = \mathbf{F}/q_o$
 - Be able to calculate Forces from Fields and vice versa
- Draw Field Lines for Simple Configurations of Charge
 - small numbers of discrete charges
 - simple continuous charge distributions: lines, planes, spheres, cylinders
- Simple applications of Coulomb's Law for E-Fields
 - no complicated integrals
- Electric Dipoles
 - field configuration
 - dipole moment: $\mathbf{p} = q\mathbf{d}$
 - torque in an external field: $\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$
 - energy in an external field: $U = \mathbf{p} \cdot \mathbf{E}$

- On exam – calculate Electric Fields using two methods:
 - 1) Superposition of E-Fields
 - apply this to find total fields for systems where you know the fields separately for each of the components
 - 2) Gauss’ Law
 - only appropriate when there is strong symmetry
- Electric Flux: $d\Phi = \mathbf{E} \cdot d\mathbf{A}$
 - understand meaning of flux (field lines per area)
 - direction of area element $d\mathbf{A}$
- Gauss’ Law: $\int_{\text{surf}} \mathbf{E} \cdot d\mathbf{A} = q_{\text{enc}}/\epsilon_0$
 - find E for simple shapes:
 - 1) points, spheres
 - 2) lines, cylinders
 - 3) planes
 - use superposition to find field for combinations
 - understand consequences of special cases:
 - $q_{\text{enc}} = 0$
 - $E = 0$ on surface
 - General Steps for Solving Gauss’ Law Problems
 - 1) Use symmetry to get:
 - > a) direction of E-Field vectors
 - > b) surfaces where E is constant
 - 2) Choosing your Gaussian surface:
 - > a) break up a simple surface so that E-Field is constant (or zero) on all elements of it
 - non-constant E-Fields make the integral very difficult
 - > b) it’s also helpful if the $d\mathbf{A}$ ’s of your surface are parallel to the field
 - non-constant angles \rightarrow difficult integration
 - > Note: in this class we’ve only considered three surfaces (spheres, cylinders and boxes). Your surface should be one of these!
 - 3) $\Phi = \sum E_{\text{surface}} A_{\text{surface}}$
 - 4) Evaluate Enclosed Charge

ELECTRIC POTENTIAL

- Definition of Electric Potential: $\Delta V = \Delta U / q$
 - Potential differences are path independent (Electric Force is conservative)
- Understand concept of “reference potential”
- Use conservation of Energy to describe how particles move in a potential
 - Kinetic vs. Potential energy
- Calculate Potentials for simple situations: $\Delta V = -\int_i^f \mathbf{E} \cdot d\mathbf{s}$
 - point charges
 - simple distributions: spheres, lines, cylinders, planes
- Superposition of Potentials
 - calculate potential of groups of objects

- Potential Energy required to build a charge distribution

PROPERTIES OF CONDUCTORS

- Note: applies only to electrostatics
- Always keep these in mind when confronted with a problem involving conductors!
- 1) All free charge on a conductor resides on the surface
 - Free Charge is:
 - a) charge that you put on the conductor
 - b) accumulations of negative or positive charge from the atoms in the conductor that are induced by external fields
- 2) $E = 0$ at all points inside a conductor
 - Gaussian surfaces which lie completely inside a conductor enclose zero net charge
- 3) At the surface of the conductor: \mathbf{E} points perpendicular to the surface at each point
- 4) All points on a conductor (or a set of connected conductors) are at the same potential.
- Be able to solve Gauss' Law problems involving conductors

CAPACITANCE

- Definition: $Q = CV$
- Calculate C for simple configurations
 - C from calculation of potential difference (V) between two conductors when charge (Q) is put on them
 - plates, cylinders, spheres
- Energy Stored in Capacitor: $U = \frac{1}{2} CV^2 = \frac{1}{2} Q^2/C$
- Energy Density: $u = \frac{1}{2} \epsilon_0 E^2$
- Equivalent Capacitances:
 - Series: $1/C = \sum 1/C_i$
 - Parallel: $C = \sum C_i$
 - Remember how these were derived
- Effect of Introducing a Dielectric: $C = \kappa C_{\text{vacuum}}$

CURRENT

- Definition: $I = dq/dt$
 - remember defined as motion of positive charges
- Getting current from drift velocity of electrons
 - $I = nev_d A$

RESISTANCE

- Definition: $R = \rho (L/A)$ (for most materials)
 - Resistivity depends on Temperature: $\rho = \rho(T)$
- Ohm's Law: $V = IR$

- understand concept of potential drop across resistor
- sign of potential drop
- Equivalent Resistance
 - Series: $R = \Sigma R_i$
 - Parallel: $1/R = \Sigma 1/R_i$
 - Remember how these were derived

DC CIRCUITS

- **Circuit Solving Techniques**
 1. Draw Equivalent Circuits when possible
 - Conductors in contact are at the same potential
 - Change shape of circuit to suit your taste, but:
 - Conserve number of Junctions
 - Only change *shape* of Loops
 - don't cut them
 - don't add extra loops
 - Replace groups of Resistors and Capacitors with Equivalent R or C using rules for combining in parallel and series
 2. Label the sides of each EMF to indicate high (+) or low (–) potential
 3. Find the Branches (Loops) in the circuit
 4. Choose a direction for the Current in each Element
 - *and draw an arrow indicating that direction*
 5. Apply the Junction Rule (currents) to the Currents going into and out of each Branch Point
 - $\Sigma I_{in} = \Sigma I_{out}$
 6. Apply the Loop Rule (voltages) to all the Loops in the circuit
 - $\Sigma \Delta V = 0$ around loop
 - for EMF's:
 - $\Delta V > 0$ - \rightarrow +
 - $\Delta V < 0$ + \rightarrow -
 - for Resistors:
 - $\Delta V > 0$ traversing element opposite to direction of I
 - $\Delta V < 0$ traversing element in same direction as I
 - for Capacitors:
 - $\Delta V > 0$ -plate \rightarrow +plate
 - $\Delta V < 0$ +plate \rightarrow -plate
 7. Solve the Simultaneous Equations
- Power
 - Supplied (or absorbed) by EMF: $P = IV$
 - Dissipated in Resistor: $P = I^2R = V^2/R$
 - Understand the how to tell whether power is supplied or absorbed
 - I through battery from – to + : power is supplied
 - I through batter from + to – : power is absorbed

MAGNETIC FIELDS AND FORCES

- Magnetic Force:
 - Charges: $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$ (point particle)
 - Currents: $\mathbf{F} = I\mathbf{L} \times \mathbf{B}$ (straight wire)
 - Use right-hand rule to find direction
 - Force between current carrying wires:
 - Two straight wires
 - Straight wire and Loop
 - Circular Path of Charged Particle in a Uniform Magnetic Field:
 - For \mathbf{v} perpendicular to \mathbf{B}
 - > $mv = qBr$
- Crossed Magnetic and Electric Fields
 - Understand examples of systems using both Electric and Magnetic Fields
 - Mass Spectrometer
 - Electron Discovery (cathode ray tube)
 - Generally:
 - E-Field used to accelerate particle
 - > $\Delta K = q \Delta V$
 - B-Field used to make particle move in a circle
 - > $mv = qBr$
 - Undeviated motion through region of crossed E and B
 - > $F_E = F_B$
- Magnetic Dipoles: Torque & Energy
 - $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$
 - $U = -\boldsymbol{\mu} \cdot \mathbf{B}$
 - $\boldsymbol{\mu}$ = Magnetic Dipole Moment
 - $\boldsymbol{\mu} = NIA$ (for current loop of N turns)
- Biot Savart Law (fundamental calculation of magnetic fields)
 - $d\vec{B} = \frac{\mu_o}{4\pi} i \frac{d\vec{s} \times \hat{r}}{r^2}$ currents
 - $\vec{B} = \frac{\mu_o}{4\pi} q \frac{\vec{v} \times \hat{r}}{r^2}$ charges
 - Simple Applications – no complicated integrals
 - Use with symmetry of problem to get direction of \mathbf{B}
- On exam – calculate Magnetic Fields using two methods:
 - 1) Superposition of B-Fields
 - apply this to find total fields for systems where you know the fields separately for each of the components
 - 2) Ampere's Law
 - only appropriate when there is strong symmetry
- Ampere's Law:
 - $\oint_{loop} \vec{B} \cdot d\vec{s} = \mu_o i_{enc}$

- Steps for using Ampere's Law (similar to Gauss' Law)
 - Determine direction of Magnetic Field using symmetry of problem
 - Choose Amperian Loop either parallel or perpendicular to Field
 - Do the integral
 - Calculate how much current is enclosed by loop
 - > sign of current from (yet another) right-hand rule
- Understand derivation of fields for:
 - 1) Straight Wires
 - 2) Planes of Current
 - 3) Solenoids
 - 4) Toroids
- Right-Hand Rules for Direction of Fields from Wires:
 - Straight Wires
 - Circular Loops
 - Solenoids

TIME VARYING MAGNETIC FIELDS

- Magnetic Flux: $\Phi = \int \mathbf{B} \cdot d\mathbf{A}$
 - Understand calculation for:
 - \mathbf{B} = constant (in space)
 - \mathbf{B} varies with position
 - > only very simple calculations here
- Faraday's Law / Lenz's Law
 - EMF Induced Around Loop = $-d\Phi_B / dt$
 - induced EMF opposes change in flux
 - Induced Current from Induced EMF
 - Direction of induced current
 - > sets up induced B-field to oppose change in flux from external B-field
 - Magnitude of induced current from:
 - > $I_{\text{ind}} = V_{\text{ind}} / R$
 - Understand Ways to get $d\Phi_B / dt$
 - Time Varying B
 - Area of Loop Changes
 - Loop Rotates in Constant Field
 - Loop moves through region of non-uniform field
 - Simple Calculations of induced EMFs and Currents

INDUCTANCE

- Mutual Inductance:
 - Know what this is and how it arises
- Self Inductance
 - Definition: $L = N \Phi / I$
 - Calculate for simple configurations (especially solenoids)
 - Voltage Drop across an inductor:

- $V_L = -L di/dt$
- Adding Inductances
 - Series: $L_{eq} = \Sigma L_i$
 - Parallel: $(1/L_{eq}) = \Sigma (1/L_i)$
- Energy in Inductor:
 - $U = \frac{1}{2} L i^2$
- Energy Density in Magnetic Field
 - $u_B = B^2 / 2\mu_0$

RC & RL CIRCUITS

- Know general forms of current and voltage in series RC and RL circuits when connecting or disconnecting from a battery of voltage V .

		Connect	Disconnect	
RC	$Q(t)$	$Q_f [1 - \exp(-t/\tau_C)]$	$Q_o \exp(-t/\tau_C)$	$\tau_C = RC$ $I_o = V / R$
	$I(t)$	$I_o \exp(-t/\tau_C)$	$I_o \exp(-t/\tau_C)$	
	$V_C(t)$	$V [1 - \exp(-t/\tau_C)]$	$V \exp(-t/\tau_C)$	
RL	$I(t)$	$I_f [1 - \exp(-t/\tau_L)]$	$I_o \exp(-t/\tau_L)$	$\tau_L = L/R$ $I_o = I_f = V / R$
	$V_L(t)$	$V \exp(-t/\tau_L)$	$V \exp(-t/\tau_L)$	

- Understand $t=0$ and $t=\infty$ values of current through parts of a circuit containing capacitors or inductors after connecting a voltage source
 - $t = 0$ Capacitor acts like a wire
Inductor acts like a block
 - $t = \infty$ Capacitor acts like a block
Inductor acts like a wire

ALTERNATING CURRENT CIRCUITS

- Form for Voltage Supplied and Current:
 - $v(t) = V_m \sin(\omega t)$
 - $i(t) = I_m \sin(\omega t - \phi)$
- Single Element Circuits
 - Phase Difference between v and i
 - Reactances: X_R, X_C, X_L
- RMS quantities
 - Definition: $A_{rms} = (\langle A^2 \rangle)^{1/2}$
 - $A = A_m \sin \omega t \rightarrow A_{rms} = A_m / \sqrt{2}$
- Series RLC Circuits
 - Total Impedance
 - $Z = [R^2 + (X_L - X_C)^2]^{1/2}$
 - Phase Angle & Current Amplitude
 - $\tan \phi = (X_L - X_C) / R$
 - $I_m = V_m / Z$
 - Resonance Condition: Driving Frequency = Natural Frequency

- $\omega_0 = 1 / \sqrt{LC}$
- understand how I_m , etc. changes as circuit moves off resonance
- RMS and Average Current and Voltages across elements
- Energy Stored in Capacitor and Inductor and how it changes
- Average Power and where it is dissipated
 - $\langle P \rangle = I_{\text{rms}}^2 R = I_{\text{rms}} V_{\text{rms}} \cos\phi$
 - understand how to calculate power usage in circuit
 - only resistors dissipate power
 - understand how power dissipation changes as the circuit moves off resonance
- Transformers
 - Relationships between V and I in Primary and Secondary Circuits
 - $V_s = V_p (N_s/N_p)$
 - $I_s = I_p (N_p/N_s)$
 - Equivalent Resistance
 - $R_{\text{eq}} = R (N_p/N_s)^2$
 - understand what is meant by impedance matching

MAXWELL'S EQUATIONS

- Know what these are

EM WAVES

- Form of EM Waves:
 - **E** perpendicular to **B**
 - **E, B** perpendicular to **Direction of Propagation**
 - $E/B = E_m/B_m = c = w/k = f\lambda = 1/(\mu_0\epsilon_0)^{1/2}$
 - For Plane Waves (the only kind we studied):
 - $\mathbf{E}(x,t) = \mathbf{E}_m \sin(kx - wt)$ $\mathbf{B}(x,t) = \mathbf{B}_m \sin(kx - wt)$
- Energy Flow – Poynting Vector
 - $\mathbf{S} = (\mathbf{E} \times \mathbf{B}) / \mu_0$
 - Direction of **S** = Direction of Propagation
 - $S = E^2 / c\mu_0$
- Intensity = Power / Area
 - Understand Variation of Intensity with distance from source
 - calculate for isotropic source of known power: $I(r) = I_0 / r^2$
 - Average Intensity:
 - $I = \langle S \rangle = E_{\text{rms}}^2 / c\mu_0$
- Radiation Pressure and Momentum
 - total reflection: $P_{\text{rad}} = 2I/c$
 - total absorption $P_{\text{rad}} = I/c$
 - Calculate Force from Radiation Pressure
- Polarization
 - Understand the difference between polarized and unpolarized light
 - Polarization Direction = Direction of oscillation of **E**-field

- Transmission of Light through Polarizing Filters
 - Unpolarized Light: $I_{\text{trans}} = \frac{1}{2} I_o$
 - Polarized Light: $I_{\text{trans}} = I_o \cos^2 \theta$
 - Be able to solve problems with multiple polarizing filters
- Reflection and Refraction:
 - Law of Reflection: $\theta_r = \theta_i$
 - Snell's Law: $n_1 \sin \theta_1 = n_2 \sin \theta_2$
 - θ measured with respect to the normal to the surface
 - Definition of Index of Refraction:
 - > $n = c/v$ ($v = \text{speed of light in medium} < c$)
 - Special Cases:
 - Total Internal Reflection
 - > $\sin \theta_{\text{crit}} = n_2 / n_1$
 - Polarization by Reflection
 - > Reflected Light is Polarized parallel to interface between media
 - > Total Polarization at Brewster's Angle: $\tan \theta_B = n_2 / n_1$

GEOMETRICAL OPTICS

- Optical Devices Studied:
 - Plane Mirrors
 - Spherical Mirrors
 - Spherical Refracting Surfaces
 - Thin Lenses
- Understand Conventions (see Optics Handout available on the Web)
 - What is convex and concave
 - Signs for radii of curvature and focii
 - Image Positions with respect to device and object
 - Magnification
 - Definitions of Virtual/Real Upright/Inverted Images
- Ray Tracing to understand general characteristics of Images
- Solve Problems involving:
 - Single optical devices
 - Two optical devices:
 - the image of the 1st forms the object of the 2nd

INTERFERENCE

- Understand relationships between frequency, wavelength, speed
 - $w = 2\pi/T = 2\pi f$ $k = 2\pi/\lambda$
 - In Vacuum: $w/k = f\lambda = c$
 - In Material (index of refraction n):
 - $v = c/n$
 - $f_n = f$ $w_n = w$
 - $\lambda_n = \lambda/n$ $k_n = nk$
- Amplitude and Intensity of Waves:

- Amplitude: $E(x,t) = E_m \sin \phi(x,t)$
- Intensity: $I = \langle E(x,t)^2 \rangle / c\mu_0$ (\propto Amplitude²)
- Adding waves together:
 - Add Amplitudes then square to get intensity
- Understand concept of phase of wave: $\phi(x,t)$
 - For Plane wave in a particular medium: $\phi(x,t) = k_r x - \omega t$
 - Calculate change in phase between two points on a single wave
 - Count number of oscillations between points A and B
 - > $\phi = 2\pi N$ $N = \Delta x / \lambda$
 - Be able to do this for:
 - > waves travelling in one medium between A and B
 - > waves moving through several media between A and B
- Understand concept of Coherence between waves:
 - Waves must oscillate at same frequency
 - Waves must have same phase at some point in space
- Sources of Phase Difference between initially coherent waves
 - 1) Travel over different distances
 - 2) Travel through different media
 - 3) Reflect off different surfaces
 - 4) All of the above
- Know how to calculate phase differences in terms of:
 - wavelengths: $\Delta N(x) = N_A - N_B$
 - radians: $\Delta\phi(x) = 2\pi \Delta N$
 - Be able to do this for all 3 possible ways initially coherent waves can get out of phase
- Conditions for Constructive and Destructive Interference
 - Constructive: $\Delta\phi = 2m\pi$ (even multiple of π)
 - Destructive: $\Delta\phi = (2m+1)\pi$ (odd multiple of π)
 - Apply these conditions to general situations involving two sources
- Some specific examples to understand:
 - Note: problems will generally be slight twists on these
 - 2-Slit (or 2 Source) Interference:
 - Conditions for Maxima and Minima of intensity
 - > Max: $d \sin\theta = m\lambda$
 - > Min: $d \sin\theta = (m+1/2)\lambda$
 - > know how these were obtained from constructive and destructive interference
 - Formula for Intensity vs. Angle
 - > $I = 4I_o \cos^2(\Delta\phi/2)$ $\Delta\phi = 2\pi d \sin\theta / \lambda$
 - Understand plot of Intensity vs. Angle
 - Interference of light passing through different thicknesses of transparent material
 - Thin Film Interference
 - Understand how to set up the conditions for interference in thin films for arbitrary indices of refraction

- > (don't just memorize the formula for thin films in air and apply it to everything)
- Sources of phase difference between rays reflected from top and bottom surfaces of thin film:
 - > 1) Phase shift on reflection:
 - low n to high n : $\Delta\phi = \pi$
 - high n to low n : $\Delta\phi = 0$
 - > 2) Difference in paths of two rays (including index of refraction)

DIFFRACTION

- Single-Slit Diffraction
 - Understand the concepts of how the equations are derived. No need to be able to reproduce the math.
 - questions here will ask you to apply the formulas we have derived
 - Position of Minima
 - Min: $a \sin\theta = m\lambda$
 - Maxima approximately halfway between minima:
 - Max: $a \sin\theta \sim (m+1/2)\lambda$
 - Intensity:
 - $I = I_o (\sin\alpha / \alpha)^2$ $\alpha = \pi a \sin\theta / \lambda$
 - Understand plot of Intensity
 - width of central maximum
- Diffraction by a Circular Aperture
 - Position of first minimum: $\sin\theta = 1.22 \lambda/d$
 - Width of diffracted beams
 - Resolvability
 - understand Rayleigh's criterion and how to apply it to find the minimum resolvable distance/angle between distant objects
- Double-Slit Diffraction
 - only need to know the general forms of the intensities from this
 - No detailed calculations