SYNOPSIS

- Session 1: Introduction
- Session 2: History of Particle Physics
- Session 3: Special Topics I: Special Relativity
- Session 4: Special Topics II: Quantum Mechanics
- Session 5: Experimental Methods in Particle Physics
- Session 6: Standard Model: Overview
- Session 7: Standard Model: Limitations and challenges
- Session 8: Neutrinos Theory
- Session 9: Neutrino Experiment
- Session 10: LHC and Experiments
- Session 11: The Higgs Boson and Beyond
- Session 12: Particle Cosmology

Lecture materials are posted at:
http://home.fnal.gov/~georgiak/shp/
Last time’s agenda

- Historical background (will not cover this; see lecture 2)
- SM particle content
- SM particle dynamics
  - Quantum Electrodynamics (QED)
  - Quantum Chromodynamics (QCD)
  - Weak Interactions
  - Force unification
- Lagrangian/Field formulation
- Higgs mechanism (later lecture)
- Tests and predictions
Last time: SM particle content

**Fermions:**
- Quarks and Leptons
- Spin-1/2 particles

**Bosons:**
- Force mediators and Higgs field
- Integer-spin particles

(+ corresponding antiparticles...)

**Quarks**
- u
- c
- t
- d
- s
- b

**Leptons**
- e
- μ
- τ
- ν_e
- ν_μ
- ν_τ
In particle physics, we define fields like \( \varphi(x,t) \) at every point in spacetime.

These fields don’t just sit there; they fluctuate harmonically about some minimum energy state.

The oscillations (Fourier Modes) combine to form wavepackets.

The wavepackets move around in the field and interact with each other. We interpret them as elementary particles.

Terminology: the wavepackets are called the quanta of the field \( \varphi(x,t) \).
Particle/Field formulation

- How do we describe interactions and fields mathematically?

- Classically, the Lagrangian \( L = \text{kinetic energy} - \text{potential energy} \)

- Particle physics:
  - Same concept, plus use **Dirac equation** to describe free spin-1/2 particle:
    \[
    L = \overline{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi
    \]
    - \( \Psi = \text{wavefunction} \)
    - \( m = \text{mass} \)
    - \( \gamma^\mu = \mu^{\text{th}} \text{gamma matrix} \)
    - \( \partial_\mu = \text{partial derivative} \)
Lagrangian mechanics

- **Lagrangian**: Equation which allows us to infer the dynamics of a system.

- Developed by Euler, Lagrange, and others during the mid-1700’s.

- This is an energy-based theory that is equivalent to Newtonian mechanics (a “force-based” theory).
Standard Model Lagrangian

\[ \mathcal{L} = \mathcal{L}_0 + \mathcal{L}' \]

\[ \mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\psi} \gamma^\mu \partial_\mu \psi \]

\[ \mathcal{L}' = e \bar{\psi} \gamma^\mu A_\mu \psi \]

\[ L = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi \]

from previous slide, but with m=0 (massless fermions)
Standard Model Lagrangian

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from previous slide, but with \( m=0 \) (massless fermions)

Gauge Bosons

Fermions

Free Fields

Interaction

Fermion-Boson Coupling

strong interaction

e A_\mu = \frac{g_s}{2} \lambda_\nu G_{\mu \nu} + \frac{g}{2} \bar{\tau}_W W_\mu + \frac{g'}{2} Y B_\mu

F_{\mu \nu} F^{\mu \nu} = G_{\mu \nu} G^{\mu \nu} + W_{\mu \nu} W^{\mu \nu} + B_{\mu \nu} B^{\mu \nu}

gluons

combinations give \( W, Z, \gamma \ldots \)

electromagnetic and weak interactions
Standard Model Lagrangian

- Encompasses all theory:
  - From the Lagrangian to cross section predictions

\[ \sigma \sim \langle f | S | i \rangle^2 \]

[Def.: \( | t = +\infty \rangle \equiv | S | t = -\infty \rangle \)]

\[ \langle f | S | i \rangle \approx \delta_{fi} - i \int_{-\infty}^{\infty} dt' \langle f | H'(t') | i \rangle \]

\[ H'(t) = -\int L'(x, t) \, d^3x \]

Inelastic Cross Section

Feynman rules

Lagrangian of Interaction
Let’s go back a step...
Integral over “all possible paths”

- Where does the integral notion come from?
- QM picture of free particle motion: There is some \textit{amplitude} for a free electron to travel along \textit{any} path from the source to the some point \( p \).
  – not just the straight, classical trajectory!
  - Note that when we use the word “path”, we not only mean a path \( x(y) \) in space, but also the time at which it passes each point in space.
  - In 3-D, a path (sometimes called a worldline) is defined by three functions \( x(t), y(t), \) and \( z(t) \). An electron has an amplitude to take a given path \( x(t), y(t), z(t) \).
- The total amplitude for the electron to arrive at some final point is the sum of the amplitudes of all possible paths. Since there are an infinite number of paths, the sum turns into an integral (a path integral).
The quantum mechanical amplitude

- Feynman: Each path has a corresponding probability amplitude. The amplitude $\psi$ for a system to travel along a given path $x(t)$ is:

$$\psi[x(t)] = \text{const. } e^{iS[x(t)}/\hbar$$

where the object $S[x(t)]$ is called the **action** corresponding to $x(t)$.

- The total amplitude is the sum of contributions from each path:

$$\sum \psi[x(t)]$$

over all paths
The quantum mechanical amplitude

\[ \psi[x(t)] = \text{const} \cdot e^{iS[x(t)]/\hbar} \]

1) What is that thing \( e^{iS[x(t)]/\hbar} \) (called the phase of \( \psi \))? 
2) What is the action \( S[x(t)] \)?
Understanding the phase

- You may not have seen numbers like $e^{i\theta}$, so let’s review.
- Basically, $e^{i\theta}$ is just a fancy way of writing sinusoidal functions; from Euler’s famous formula:
  
  $$e^{i\theta} = \cos \theta + i\sin \theta$$

- Note: those of you familiar with complex numbers (of the form $z=x+iy$) know that $e^{i\theta}$ is the phase of the so-called polar form of $z$, in which $z=r\cdot e^{i\theta}$, with:

  $$r = \sqrt{x^2 + y^2}$$
  $$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$
Comments on the amplitude

- Now we can understand the probability amplitude $y[x(t)] \sim e^{iS[x(t)]/\hbar}$ a little better:

- The amplitude is a sinusoidal function—a wave—that oscillates along the worldline $x(t)$. The frequency of oscillation is determined by the how rapidly the action $S$ changes along the path.

- The probability that a particle will take a given path, $x(t)$, (up to some overall multiplication constant) is:

$$P \propto |\psi|^2 = \psi^* \psi$$

$$\propto e^{-iS[x(t)]/\hbar} e^{iS[x(t)]/\hbar}$$

$$= e^{iS[x(t)]/\hbar - iS[x(t)]/\hbar} = e^0$$

$$= 1$$

- This is the same for every worldline. According to Feynman, the particle is equally likely to take any path through space and time!
In physics, exhibition of *symmetry* under some operation implies some *conservation law*

<table>
<thead>
<tr>
<th>symmetry</th>
<th>invariant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Space translation</td>
<td>momentum</td>
</tr>
<tr>
<td>Time translation</td>
<td>energy</td>
</tr>
<tr>
<td>Rotation</td>
<td>Angular momentum</td>
</tr>
<tr>
<td>Global phase; $\Psi \rightarrow e^{i\theta} \Psi$</td>
<td>Electric charge</td>
</tr>
<tr>
<td>Local phase; $\Psi \rightarrow e^{i\theta(x,t)} \Psi$</td>
<td>Lagrangian + gauge field (→ QED)</td>
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There are carefully chosen sets of transformations for $\Psi$ which give rise to the observable gauge fields:

- That's how we get electric, color, weak charge conservation !!!
QED from local gauge invariance

- Apply local gauge symmetry to Dirac equation:

\[ \bar{\Psi} \rightarrow e^{i\theta(x,t)}\bar{\Psi}, \quad \Psi \rightarrow e^{-i\theta(x,t)}\Psi \]

This type of transformation leaves quantum mechanical amplitudes invariant.
QED from local gauge invariance

- Apply local gauge symmetry to Dirac equation:
  \[ \Psi \rightarrow e^{i\theta(x,t)}\bar{\Psi}, \quad \Psi \rightarrow e^{-i\theta(x,t)}\Psi \]

- Consider very small changes in field:
  \[ \Psi \rightarrow \Psi + \delta\Psi = \Psi - i\theta(x,t)\Psi \quad \text{ie.} \quad \delta\Psi = -i\theta(x,t)\Psi \]

  The effect on the Lagrangian is:

  \[ L = \bar{\Psi}(i\gamma^\mu\partial_\mu - m)\Psi \Rightarrow \delta L = \bar{\Psi}\gamma^\mu\partial_\mu\theta(x,t)\Psi \]

- If Lagrangian is invariant, then \( \delta L = 0 \)
QED from local gauge invariance

To satisfy $\delta L = 0$, we “engineer” a mathematical “trick”:

1) Introduce gauge field $A_\mu$ to interact with fermion, and $A_\mu$ transform as: $A_\mu + \delta A_\mu = A_\mu + \frac{1}{e} \partial_\mu \theta (x,t)$

2) In resulting Lagrangian, Replace $\partial_\mu \rightarrow D_\mu = \partial_\mu + ieA_\mu$

In that case, L is redefined:

$$L = \bar{\Psi} (i\gamma_\mu D_\mu - m) \Psi$$

The new Lagrangian is invariant under local gauge transformations.
Need to add kinetic term for field (field strength):

Define $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Add term $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ (Lorentz invariant, matches Maxwell’s equations)
QED Lagrangian

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Define \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \)

Add term \(-1/4 F_{\mu\nu} F^{\mu\nu}\) (Lorentz invariant, matches Maxwell’s equations)

Final lagrangian (for QED):

\[
L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi
\]

Note: No mass term for \( A_\mu \) allowed; otherwise \( L \) is not invariant.

\( \rightarrow \) gauge field is massless!
QED Lagrangian

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We have mathematically engineered a quantum field which couples to the fermions, obeys Maxwell's equations, and is massless!

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PHOTON

Final lagrangian (for QED!):

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→ gauge field is massless!
Follow similar reasoning, but allow for self-interaction of gauge bosons (jargon: QCD and weak interactions based on non-abelian theories)

In non-abelian theories gauge invariance is achieved by adding \( n^2 - 1 \) massless gauge bosons for SU(n)

(SU(n): gauge group)

- SU(2) –3 massless gauge bosons \((W_1, W_2, W_3)\) for weak force
- SU(3) –8 massless gluons for QCD
QCD and Weak Lagrangians

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Higgs Mechanism

- A theoretically proposed mechanism which gives rise to elementary particle masses:
  - W, Z, H bosons and standard model fermions

See Lectures by T. Andeen for more details!
SM: How simple, really?

- **SM does not predict:**

  | 3 Couplings | $g_s, e, \sin \theta_W$ |
  | 4 CKM parameters | $\vartheta_1, \vartheta_2, \vartheta_3, \delta$ |
  | 2 Boson masses | $m_Z, m_H$ |
  | 3 Lepton masses | $m_e, m_\mu, m_\tau$ |
  | 6 Quark masses | $m_u, m_d, m_s, m_c, m_t, m_b$. |

- **Determine experimentally:**

  - $m_W^2 = \frac{1}{2} g^2 \rho_0^2$  
  - $m_Z^2 = \frac{1}{2} (g^2 + g'^2) \rho_0^2$  
  - $m_W^2 = 4 \lambda \rho_0^2$  
  - $g = e / \sin \theta_W$  
  - $g' = e / \cos \theta_W$  
  - $m_f = c_f \rho_0$

- **These values must be added by hand (experimental measurements)**
SM predicts relationships

- All observables can be predicted in terms of 26 free parameters (including neutrino masses, mixing parameters).
- If we have > 26 measurements of these observables, we overconstrain the SM. Overconstrain ⇒ we don’t have any more ad hoc inputs AND we can test the consistency of the model.
- In practice:
  Pick well measured set of observables.
  Calculate other observables in terms of these well known quantities.
  Test predictions; measure observable, compare to theory.
Where can the SM be tested?

- Particle physics experiments designed to test specific aspects of SM
  - Major historical experiments:
    - @LEP (ALEPH, DELPHI, L3, OPAL)
      - Electroweak, qcd
    - @Fermilab (CDF, D0)
      - Electroweak, qcd, quark mixing
  - Major running experiments:
    - CMS, ATLAS @LHC
Higgs production at the LHC

1. Gluon fusion

2. Vector boson fusion

3. t\bar{t}-fusion

4. Associated production
Higgs production at the LHC

\[ \sigma(pp \rightarrow H+X) \]

\[ \sqrt{s} = 14 \text{ TeV} \]

\[ m_t = 175 \text{ GeV} \]

CTEQ4M

\[ \sigma (\text{pb}) \]

\[ \text{events for } 10^5 \text{ pb}^{-1} \]

M. Spira et al.
NLO QCD

\[ M_H (\text{GeV}) \]
Higgs decay modes

For $M < 135$ GeV: $H \rightarrow bb, \tau \tau$ dominant

For $M > 135$ GeV: $H \rightarrow WW, ZZ$ dominant

Tiny but also important: $H \rightarrow \gamma\gamma$
Higgs searches -- examples
Summary

- SM unites electromagnetic, weak, strong forces
- SM predicts cross-sections, couplings
- SM incomplete
  - 26 free parameters
  - Relations between some free parameters are predicted; allow for experimental tests.
The Standard Model: Limitations and Challenges

[In brief]
Grand unification scale: $10^{16}$ GeV

The Standard Model provides no explanation for what may happen beyond this unification scale, nor why the forces have such different strengths at low energies.
In the 1970’s, people started to think a lot about how to combine the SU(3), SU(2), and U(1) gauge symmetries of the Standard Model into a more global symmetry.

The first such Grand Unified Theory, or GUT, was a 1974 model based on SU(5) symmetry.

This model grouped all of the known fermions – i.e., the leptons and quarks – into multiplets.

Inside the multiplets, quarks and leptons could couple to each other and transform into one another. In essence, this theory imposed a grand symmetry: all fermions, whether quarks or leptons, were basically the same.
In this early model, interactions between the quarks and leptons were mediated by two new massive bosons, called the X and Y.

To conserve electric and color charge, the X and Y had odd properties: charges of \(-\frac{4e}{3}\) and \(-\frac{e}{3}\), and one of three possible colors. They were also incredibly massive, close to the grand unification scale of \(10^{16}\) GeV.

Hence, including both particles and antiparticles, the model predicted 12 types of X and Y.

In addition to these 12, there are also 8 gluons, 3 weak bosons, and 1 photon, for a total of 24. This makes sense, for recall that a theory exhibiting SU(n) gauge symmetry requires the existence of \(n^2-1\) gauge bosons.
Aside: Experimental consequences

- The SU(5) GUT implies that it would take huge energies to even hope to see an X or Y particle “in the wild”.
- However, even at “normal” (i.e. low) energies, virtual X and Y exchanges could take place.
- This is major: if quarks can decay into leptons via virtual X and Y exchange, then “stable” particles might actually be unstable!
- Example: the proton could possibly decay via exchange of a virtual X.
Aside: Proton decay

- The instability of the proton is one of the few tests of GUT physics that would be manifest at everyday energies.

- Computations show that relative to most elementary particles, the proton is very stable; its lifetime according to the SU(5) GUT is $10^{30}$ years!

- How can we detect such an effect? Put many protons together —e.g., in a huge tank of water—and wait for some to decay... Super-Kamiokande water Cerenkov detector, Kamioka, Japan. The detector was built to look for proton decay, although it has made major contributions to understanding neutrino oscillations!
Aside: Proton decay

- Although the proton lifetime is quite long (!), a kiloton of material contains about $10^{32}$ protons, meaning that about one decay per day should occur in such a sample.

- The Super-Kamiokande detector holds 50,000 tons of water viewed by 11,000 photomultiplier tubes. It is located underground, shielded from contamination by radiation near the Earth’s surface.

- No examples of proton decay have been found despite more than two decades of searching, leading to lower limits on the proton lifetime of about $10^{34}$ years.

- Is this a disaster for the theory? Not necessarily. GUT physics can be saved if we introduce supersymmetry...
Aside: Supersymmetry: Improved GUTs

- The idea behind GUTs like SU(5) symmetry is to present different particles as transformed versions of each other.
- The SU(5) GUT treats quarks and leptons as the same underlying “something”, unifying the fermions in the Standard Model.
- However, the early approaches to grand unification failed to incorporate the gauge bosons and Higgs scalars of the Standard Model into a unified scheme.
- Hence, some eventually suggested that the **bosons and fermions** should be united somehow by invoking a new kind of symmetry: a so-called “supersymmetry” (or SUSY for short).
- Aside: theorists did not develop supersymmetry explicitly for the Standard Model; the field started obscurely in the early 1970’s during investigations of the spacetime symmetries in *quantum field theory.*
Aside: Supersymmetry

- So what, exactly, does supersymmetry predict?
- It says that for each known boson, there should be a fermion of identical mass and charge, and similarly for each known fermion, there exists a boson of identical mass and charge.

particle $\leftrightarrow$ sparticle
Aside: Supersymmetry

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- If this is the case, why haven’t we observed a selectron of mass 0.511 MeV?
Aside: Supersymmetry

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  \[
  \text{particle} \leftrightarrow \text{sparticle}
  \]
- If this is the case, why haven’t we observed a selectron of mass 0.511MeV?
- Answer: supersymmetry is a broken symmetry at our energy scale.
- Hence, all of the superpartners are much more massive than their Standard Model partners.
Aside: SUSY at the LHC

- If SUSY is a symmetry of nature, at least some of the sparticles should have a mass of less than about 1,000 GeV.
- We hope to see evidence for SUSY at the LHC.
- There are good reasons to think that SUSY particles may also comprise the dark matter in the Universe!
What is dark matter? Is it related to supersymmetry? Do the phenomena attributed to dark matter point not to some form of matter but actually to an extension of gravity?

What is the cause of the observed accelerated expansion of the Universe? Why is the nature of the energy density (dark energy) that drives this accelerated expansion?
What’s with all the different masses?

“There remains one especially unsatisfactory feature [of the Standard Model of particle physics]: the observed masses of the particles, $m$. There is no theory that adequately explains these numbers. We use the numbers in all our theories, but we do not understand them – what they are, or where they come from. I believe that from a fundamental point of view, this is a very interesting and serious problem.”

Richard Feynman
(3) What’s with all the different masses?

- Is there a theory that can explain the masses of particular quarks and leptons in particular generations from first principles (a theory of Yukawa couplings)?

- Related to this, are there more than three generations of quarks and leptons? Why are there generations at all?
(4) And what about the neutrinos?

- What is the mechanism responsible for generating neutrino masses?

On “The origin of neutrino mass”, Hitoshi Murayama
(4) And what about the neutrinos?

- What is the mechanism responsible for generating neutrino masses?

---

On “The origin of neutrino mass”, Hitoshi Murayama
Other shortcomings:

- Does not accommodate gravitation. There is no known way of describing general relativity, the canonical theory of gravitation, within quantum field theory.
Summary: Moving forward

- All those are areas of theoretical and experimental research which are actively being explored.

- Exploring new ideas, pushing beyond experimental limits, and pushing the boundaries of our understanding of the cosmos is what has driven past experimental and theoretical triumphs—those which lead to the development of the Standard Model (first self-consistent theory of fundamental particles and their interactions).

- It is also what will continue to drive the field forward by answering questions, fixing deficiencies in our presently limited understanding, and revolutionizing our thinking.

- The goal is simplicity. This is what particle physics is after.
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