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Photon Exit Window

1 Basic parameters

Photons from the bremsstrahlung process at the interaction region travel in the vacuum inside the photon beam pipe. The beam pipe ends 92.3m away from the interaction region by the Exit Window where fraction of the bremsstrahlung photons converts to an e^+e^- pair.

Window is made of Al 86.22%; Si *10,8%; Fe 0,21%; Cu 2,5%; Mg 0,27%*. The thickness of the window has been measured at 10 different positions with high precision. Measurements are summed up in the table 1. This results in average thickness of $9.883 \pm 0.003mm$ with spread 0.0054mm. So the non-uniformity across the surface is $\approx 0.06\%$ which allows us to determine the conversion rate very precisely.

1	9.8791
2	9.8791
3	9.8828
4	9.8801
5	9.8906
6	9.8780
7	9.8790
8	9.8792
9	9.8877
10	9.8939

Table 1: Measurements of the thickness of the exit window.

The exit window represents a dead material in the beam line. To quantify this effect its thickness is given in the terms of radiation length *X0* which is defined as the length after which an electron loses $1/e$ of its energy by the bremsstrahlung process. If the material is a mixture, as it is in this case, radiation length is given by

$$\frac{1}{X0} = \sum \frac{w_i}{X0_i}, \tag{1}$$

where $w_i, X0$ are the weight fractions and radiation lengths in gcm^{-2} of single constituents. Using this equation and properties of used materials given in the table 2, the radiation length of the mixture is $23.215gcm^{-2}$. Dividing it by the density

$$\rho_{mixture} = \sum w_i * \rho_i = 2.82gcm^{-3} \tag{2}$$

the $X0 = 8.232cm$. Taking into account the window thickness leads to the conclusion that exit window represents *0*.12X0 of material.

Material	weight fraction w_i [%]	ρ $[gcm^{-3}]$	$X0$ $[gcm^{-2}]$	$1/\lambda_{\gamma\to e^+e^-}$ at 20GeV $[cm^2/g]$
Al	86.22	2.70	24.01	3.1810^{-2}
Si	10.8	2.33	21.82	3.5010^{-2}
Fe	0.21	7.87	13.84	5.5710^{-2}
Cu	2.5	8.96	12.86	5.9710^{-2}
Mg	0.27	1.74	25.27	3.0410^{-2}
Exit Window	-	2.825	23.215	3.2910^{-2}

Table 2: Properties of materials used in the exit window.

Pair production is mediated by the soft photon exchange generated by the electric field of either nucleus or electrons. The cross-section is energy dependent and starts at $E = 2m_e$ then grows rapidly until it saturates at high energies where it becomes the dominant process. The parameter of interest is the attenuation length, which is the mean distance the photon travels before it interacts with the medium. It is conveniently expressed in the terms of g/cm^2 where for the mixture

$$\frac{1}{\lambda_{mixture}} = \sum \frac{w_i}{\lambda_i}. \tag{3}$$

In order to express the attenuation length in centimeters it must be divided by the density. Attenuation lengths for photon-window material interactions are shown in the figure 1. At energies of our interest the cross section is dominated by pair production and it has a very slow dependence on the photon energy. The attenuation length changes from $30.79g/cm^2$ at 5GeV to $30.25g/cm^2$ at 20GeV resulting to $\approx 1\%$ change over this range. Within the energy window used for luminosity measurement the energy dependence is negligible. One can also notice, that $\approx 92\%$ of conversions happens in the nuclear rather than electron field. The data are from *http://physics.nist.gov/PhysRefData*, through the courtesy of John H.Hubbell (NIST).

All this information allows a calculation of an energy-dependent conversion rate. At $E_\gamma = 20GeV$, the center of the spectrometer acceptance, the attenuation length is $\lambda = 10.76cm$. This gives the conversion rate

$$conversion\ rate = 1 - exp(-t_{window}/\lambda) = 8.77\%. \qquad (4)$$

2 Systematic Errors

Int this section the systematic errors induced by the uncertainties on the window parameters are examined. It doesn't take into account the effects of secondary interactions in the window which are studied later.

- **Window Thickness**

  The effect on the conversion rate (4) is approximately proportional to the window thickness

  $$conversion\ rate = 1 - exp(-t/\lambda) \approx t/\lambda \qquad (5)$$

  Therefore the uncertainty due to thickness is $\Delta t/t = 0.03\%$ which is negligible.

- **Attenuation Length**

  Attenuation length varies less than 1% over the accepted energy window of the bremsstrahlung photons. On the other hand, the theoretical uncertainty is $\approx 2 - 3\%$ which translates directly into the luminosity measurement.

  The problem is illustrated in the figure 2. The conversion rates generated by GEANT are systematically lower than those given by NIST[1]. The difference in the predicted conversion rate is **1.7%!**. Hopefully the acceptance can be normalized by the use of 6mt tagger.

- **Window Composition**

  For simplicity assume a case of changing only weight fractions of Copper and Aluminum in the material. This affects the density and the attenuation length of the exit window. Propagating errors from the equations (2,3,4) gives the result

  $$\frac{\Delta conv}{conv} = \Delta w_{Cu}(\frac{\rho_{Cu} - \rho_{Al}}{\rho} + (\frac{1}{\lambda_{Cu}} - \frac{1}{\lambda_{Al}})\lambda) \approx 3.1 \Delta w. \qquad (6)$$

  Requiring the conversion rate to be known to 1% level, the uncertainty on the copper fraction must be $\Delta Cu < 0.33\%$ or in other words the copper fraction must be known to 13%. The Monte Carlo studies by GEANT are in agreement with these results.

  The corresponding change in the density of the window is $\Delta\rho = 0.02gcm^3 = 0.7\%$. If the density can be measured with this precision, one could restrict the systematic error due to uncertainty in the window composition to be negligible.

[1] See section 1 for reference

3 Angular Distributions

There are two effects giving opening angle to the outcomming e^+e^- pair. One of them is the opening angle directly from the kinematics of the photon conversion and the second one is multiple scattering in the window.

Opening angle distribution function can be approximately written as [1]

$$f(u) = \frac{9a^2}{9+d}(ue^{-au} + due^{-3au}), \tag{7}$$

where a=0.625 and d=27.0 and the scaling variable is

$$u = \frac{E_e}{m_e}\theta \tag{8}$$

Mean value is $< u > \approx 1.44$ leading to a typical angle of $\theta = 1.44\frac{m}{E} = 7.210^{-5}$ at E=10 GeV which translates to the typical displacement at the spectrometer to be $\approx 0.9mm$.

GEANT simulation shows that including multiple coulomb scattering increases the deflection to $\theta_x \approx 2.810^{-4}$ leading to displacement of $\Delta x \approx 3.4mm$ at the spectrometer. The probability distributions for deflection angle and its projection on x-axis are shown in fig.3.

The opening angle degrades the resolution on the beam profile measurement. If only non-zero spectrometer energy and position resolution would smear the data, the resolution on the profile would be $\sigma(x) = 0.5mm$ and $\sigma(y) = 3.2mm$. Including opening angle and multiple coulomb scattering the resolution is

$$\sigma_{x_\gamma} = 3.4mm \quad \sigma_{y_\gamma} = 4.2mm. \tag{9}$$

The correction to the luminosity measurement is approximately -2.5% which is small and hopefully can be simulated properly. The beam profile can be measured using coincidences and therefore we can check that the simulation is correct.

4 Radiation losses

The exit window itself represents 0.12X0 of dead material. Electrons traveling in the material may radiate hard photons which can consequently convert to further e^+e^- pairs. The electromagnetic shower develops. The average energy loss is given by

$$\frac{\Delta(E)}{E} = 1 - e^{-t} \tag{10}$$

where t is the material thickness in terms of X0. However, since the conversion may happen inside the window, the electrons travel on average shorter distances. Proper folding of distributions leads to average loss of 5.85% which is consistent with GEANT simulation as shown on figure 4. The reconstructed photon energy is on average lower by the same amount as well. The energy loss distribution has a large tail with $1/\nu$ fall-off typical for Bremsstrahlung where $\nu = \Delta E/E$. GEANT simulation shows that $\sim 31\%$ of electrons loose more than 0.5% of their energy. These numbers are related to the $1/\nu$ shape of the radiation and the mean energy loss by

$$frac = \frac{-<\nu>}{1-\nu_{min}} * ln(\nu_{min}). \tag{11}$$

Putting $<\nu> = 5.85\%$ for the average energy loss and $\nu_{min} = 0.5\%$ for the treshold the fraction of events that radiate is 31% in perfect agreement with GEANT simulation.

As the shower develops the secondary electrons might be the ones with the highest energy. Only $\sim 0.1\%$ of the electrons with the highest energy are not the ones originating directly from the bremsstrahlung photons which makes this effect negligible.

The acceptance is affected in two ways. Either the photon is reconstructed to have lower energy and migrates to or out of the energy window or the electron (positron) loses part of its energy is more bent by the magnetic field. The high energy electron migrate into and low energy ones out of the acceptance. Both of these effects play role in the acceptance calculation.

Expected rate measured by the spectrometer has been determined by fast MC simulation to be $-10.5 \pm 0.2(stat)\%$ lower compared to the case with no radiation. This result depend on the thickness of the window as

$$Rate = (1 - a < \frac{\Delta E}{E} >) Rate(no\ radiation) \tag{12}$$

where a is determined to be $a = 1.83 \pm 0.03(stat)\%$. Under assumption that the radiation thickness of the window is know to 2% the total correction to the acceptance is $-10.5 \pm 0.2(stat) \pm 0.2(syst)$. This is quite large, however the uncertainty associated with this correction stays well bellow 1%. Variation of the treshold (11) in the simulation shows negligible effects and the $1/\nu$ shape of the radiation loss is assumed to be exact. The correction itself depends of course on the actual geometry and the cuts used in the analysis so its value given in this section should be interpreted with this in mind.

There is a possibility to check the simulation by looking at the low energy tails of the E_γ spectrum from the radiation by looking at the coincidences with the 6mt tagger which operates in the small energy window. This will allow us to check the systematic uncertainty on this large correction.

References

[1] GEANT: Detector description and simulation tool, CERN Program Library Long Writeup W5013 (1993)

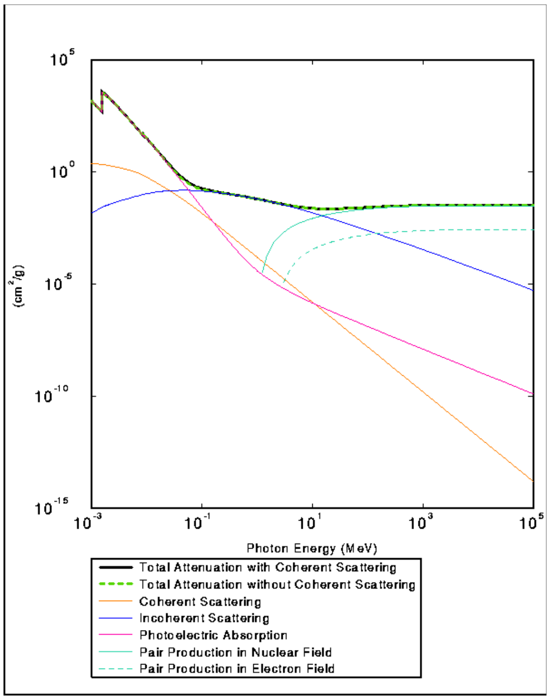

Figure 1: 1/(attenuation length) as a function of energy for various photon - medium interactions.

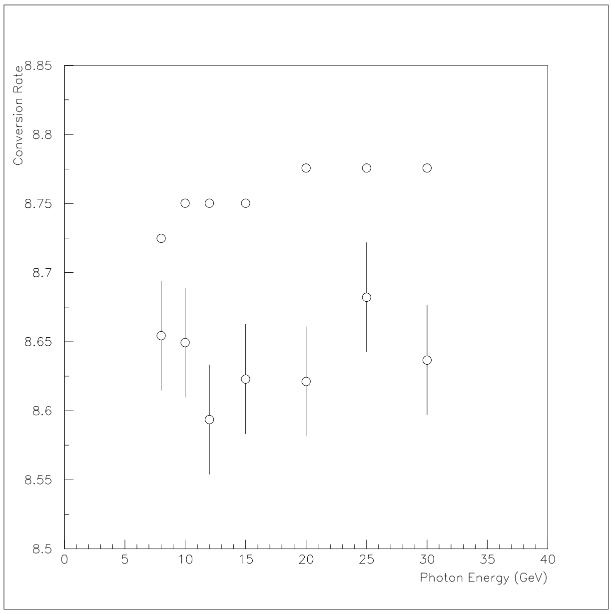

Figure 2: Comparison of energy dependent conversion rate from NIST and GEANT. The difference is 1.7%.

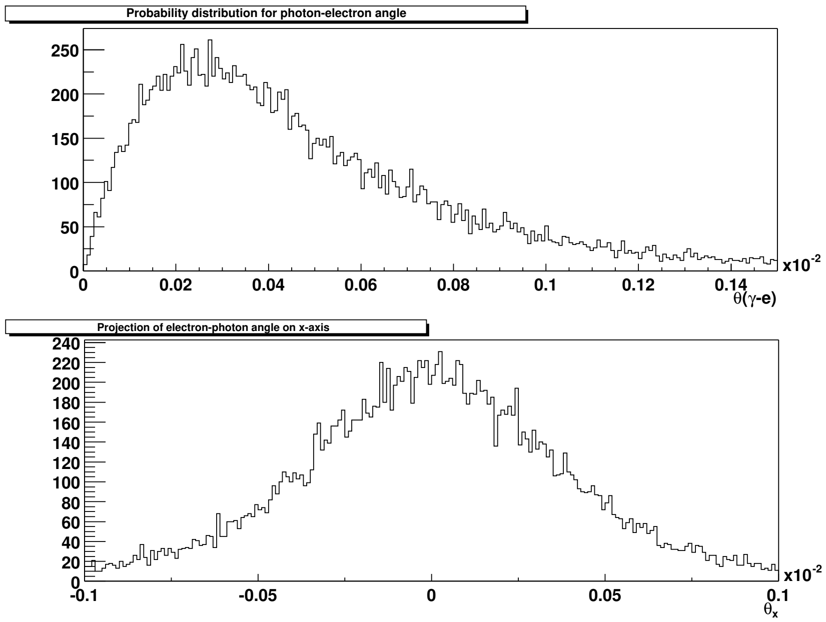

Figure 3: Probability distribution functions for the angle between bremsstrahlung photon and outgoing electron. The lower plot shows the projection of that angle on the x-axis.

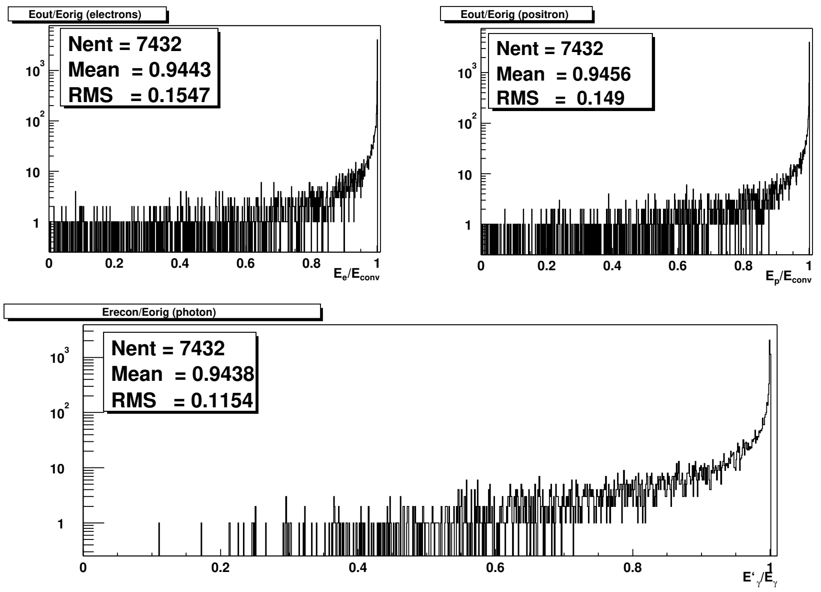

Figure 4: Energy loss distribution of outgoing $e^{\pm}$ and impact on reconstruction of photon energy.