Study of the Photon Exit Window with GEANT Simulations

Keewee Tan, Columbia University

Aug 22, 2001

Contents

1 Random Number Generator

For the first 2 weeks or so, I spent most of my time learning to use GEANT and PAW, both of which were new to me. Then I started to do some simulations with GEANT. However, the first issue I need to resolve is to determine how the random number generator in GEANT will affect my simulations. GEANT, when given a certain setup with several parameters (which can include the entire geometry of the detector, the thickness and composition of the window, the strength of the magnetic field present, etc), simulate for each event all the physical processes that can occur. For example, a photon (if it has enough energy) can undergo pair production to give a e^+/e^- pair in a particular medium with a particular probability. GEANT assigns to each process an outcome that depends on a number given by the random number generator. That number is generated by GEANT from a set of operations on 2 default numbers, following a certain algorithm. If we don't change these 2 default numbers, given the same setup, GEANT will produce identical results each time. My first task is to figure out how much our simulation is affected by the random number generator, which can be reset by the command:

RNDM [A B]

where A,B are two different 9 digits numbers. We want to know how the change in the random number generator affects the result of our simulation, and we want to know whether running more events will make our simulations more accurate(i.e. a smaller uncertainty)

What I did was to run several simulations (each with a different set of 'seeds' for the random number generator), and look at the conversion rate of 20GeV photons in our custom window with thickness 9.883mm and the following composition:

Element	Percentage	Density(g/cm^3)
Al	86.22%	2.7
Si	10.8%	2.33
Cu	2.5%	7.87
Fe	0.21%	8.96
Mg	0.27%	1.74

Following the suggestion of Allen, I did 100 runs of 10,000 events (i.e. I bombard the window with 10,000 20GeV photons), and 20 runs of 500,000. The results are shown in Figure 1:

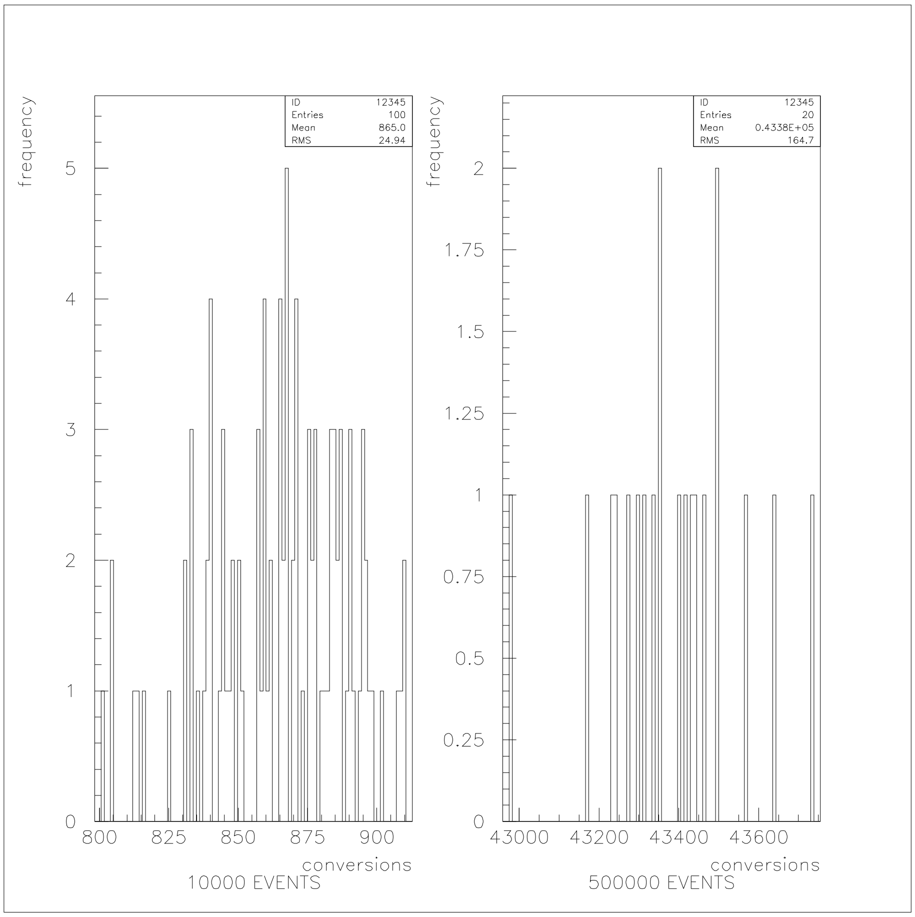

Figure 1: Random Number Generator.

The 10,000 events runs suggested that the conversion rate is 8.65% while the 500,000 events runs suggested that the conversion rate is 8.68%. What does theory predict? When a photon hits the window, it either converts (i.e. decay into a e^+/e^- pair via pair production) or it doesn't. Let's call the probability of conversion p. Then the probability that it doesn't convert is just q=1-p. We can see that conversions of photons must obey the Binomial distribution:

$$f(x) = \binom{n}{x} p^x q^{n-x}$$

where n is the number of events in the run. In fact, from Figure 1, we can see that the 10,000 events runs do look like an asynmetric Binomial distribution, though it is not so obvious in the case of the 500,000 events runs, for sheer lack of statistics. For large number of events, which can be as big as half a million in our case, we can use the Normal (or Gaussian) distribution:

$$f(x) = \binom{n}{x} p^x q^{n-x} \sim \frac{1}{\sqrt{(2\pi npq)}} e^{-\frac{(x-np)^2}{2npq}}$$

to get a (very good) approximation of the Binomial distribution, since it becomes very cumbersome to compute the binomial distribution for large number of events.

For the normal distribution we see that for the number of events n, we have the following relations:

$$\begin{aligned} \mu &= np \\ \sigma &= \sqrt{n}pq \end{aligned}$$

where μ is the mean number of conversions, and σ is the standard deviation. We use the probability of conversion, p=0.0868, from Figure 1. We see the experimental RMS we get from the simulations fall within the predicted theoretical values[1]:

$$\begin{aligned} RMS(10,000 events) &= 24.94 < \sigma = \sqrt{1}0,000 * 0.0868 * 0.9132 = 28.15 \\ RMS(500,000 events) &= 164.7 < \sigma = \sqrt{5}00,000 * 0.0868 * 0.9132 = 199.08 \end{aligned}$$

We also see that the percentage uncertainty (due to the random number generator) in the results of our simulation is

$$\frac{\sigma}{\mu} = \frac{\sqrt{n}pq}{np} = \sqrt{\frac{q}{np}}$$

For the 10,000 events runs, we have

$$\sqrt{\frac{q}{np}} = \sqrt{\frac{0.9132}{10,000 * 0.0868}} \sim 3.24\%$$

[1]This might sound like circular reasoning, since we are using the p,q from the simulations to show that the simulations are within theoretical errors. However, the result holds even for p ranging from 0.08 to 0.09, which is a very large, and thus unlikely, margin of error for GEANT.

which is obviously unacceptable for our purpose. For the 500,000 events runs, we have

$$\sqrt{\frac{q}{np}} = \sqrt{\frac{0.9132}{500,000 * 0.0868}} \sim 0.46\%$$

This is much better. Therefore, we see that we do indeed get a smaller percentage uncertainty if we increase the number of events in our simulations. If we want to get within 0.1% percentage uncertainty, we need

$$\sqrt{\frac{q}{np}} < 0.001$$

$$\Rightarrow n > \frac{q}{p(0.001)^2} = 10.52 * 10^6$$

which is more than 10 million events! Given that running 500000 events take approximately half an hour, 10 million events will take around 10 hours! If we are going to run many simulations, then it is obviously impractical to run so many events. So we concluded that running 500,000 events should be good enough for our purposes. And in future we shall take our conversion rate (along with the uncertainty due to the random number generator) to be 8.68% +/- 0.04% (0.46% of 8.68% is just 0.04%)

2 Effects of the Thickness of the Window on Conversion Rate

The next thing I investigate is the effect of the thickness of the window on the conversion rate. The thickness of the window is given as 9.883 +/- 0.003mm. We are not quite sure if we can trust this degree of accuracy. To be absolutely sure that any uncertainty in the thickness of the window is not going to adversely affect our experiment, we want to know how much the uncertainty, if any, in the thickness of the window will affect our conversion rate.

To do that, I run several simulations, each with a different thickness of the window (with everything else unchanged), ranging from the given thickness plus or minus 50 σ (σ=0.003mm). The results are shown in Figure 2:

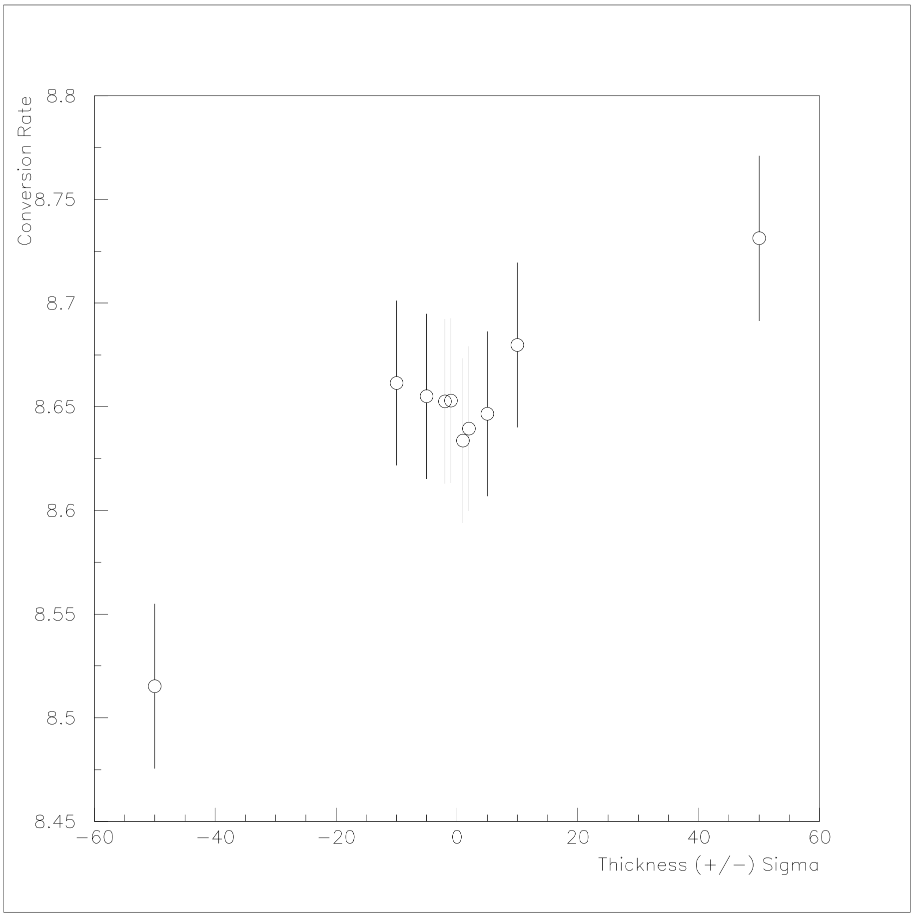

Figure 2: Conversion Rate vs Thickness.

We see that the relationship between the conversion rate and thickness of the window is linear. I did a vector fit of the line (Figure 3), and the gradient of the line is 0.0020715 with a chi square of 0.2801.

Thus we have the following relationship:

$$\Delta(Conversion\ Rate) = 0.0020715 * \Delta(Sigma)$$

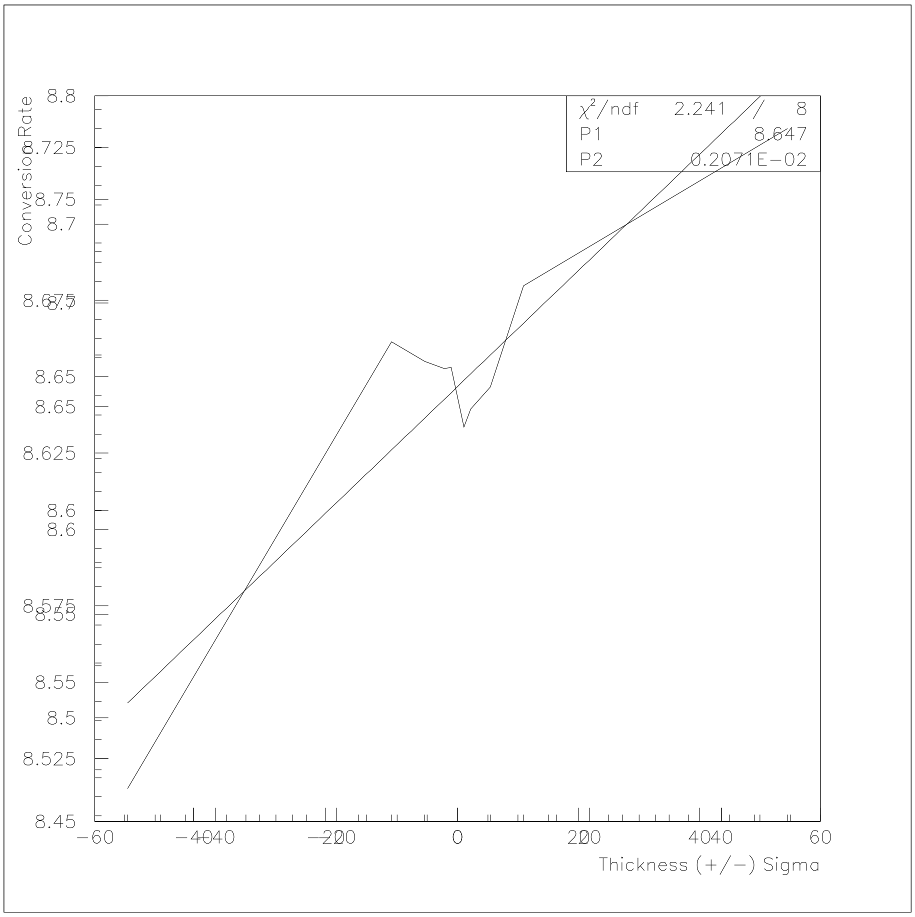

Figure 3: Chi Square Fit of Conversion Rate vs Thickness.

3 Effects of the Composition of Window on the Conversion Rate

We are also interested in finding how the composition of the window affects the conversion rate. As a reminder, the composition of the window is given as follows[2]:

[2]Note: The person who made the window has retired. This is what we think is the correct composition of the window, from what little information provided to us. That's why it is critical that we know exactly how the composition will affect our conversion rate.

Element	Percentage	Density(g/cm^3)
Al	86.22%	2.7
Si	10.8%	2.33
Cu	2.5%	7.87
Fe	0.21%	8.96
Mg	0.27%	1.74

The determining factor is the density of the material that makes up the window. We see that of the 5 elements that make up the window, the two most abundant elements, Al and Si, have very similar atomic numbers and densities, being very close together in the periodic table. Of the 3 remaining elements, Fe and Cu are a lot denser than Mg (or Al or Si), but only Cu is present in sizable proportion. There are of course an infinite number of permutations of varying the composition, but it makes the most sense to vary the proportion of Cu, at the expense of the most abundant element in the mixture, Al. I did several simulations, with the proportion of Cu varying from 0 to 10%. The result is shown in Figure 4.

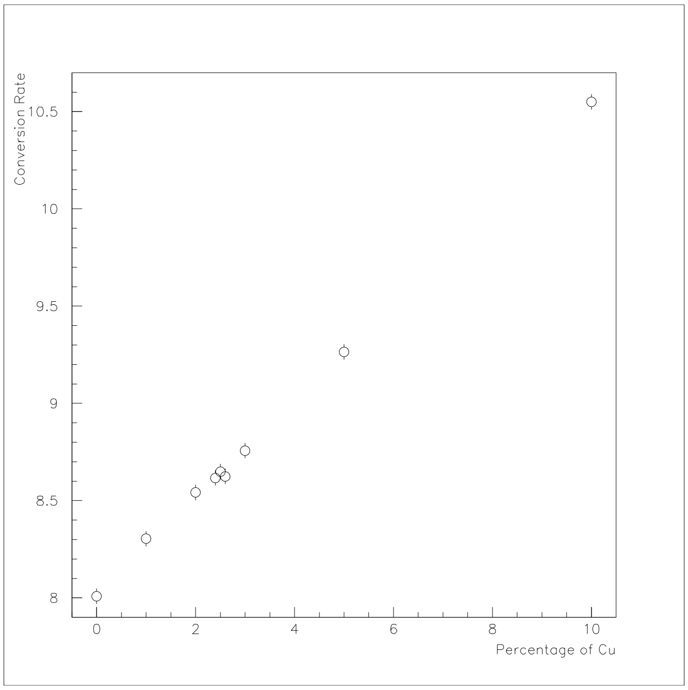

Figure 4: Conversion Rate vs Percentage of Cu

We see that the conversion rates increases linearly with the composition of Cu, which is not surprising perhaps, since Cu is a lot heavier and denser than either Al or Si. Doing a vector fit, one gets (Figure 5) a chi square fit of 0.412 and a gradient of 0.25236.

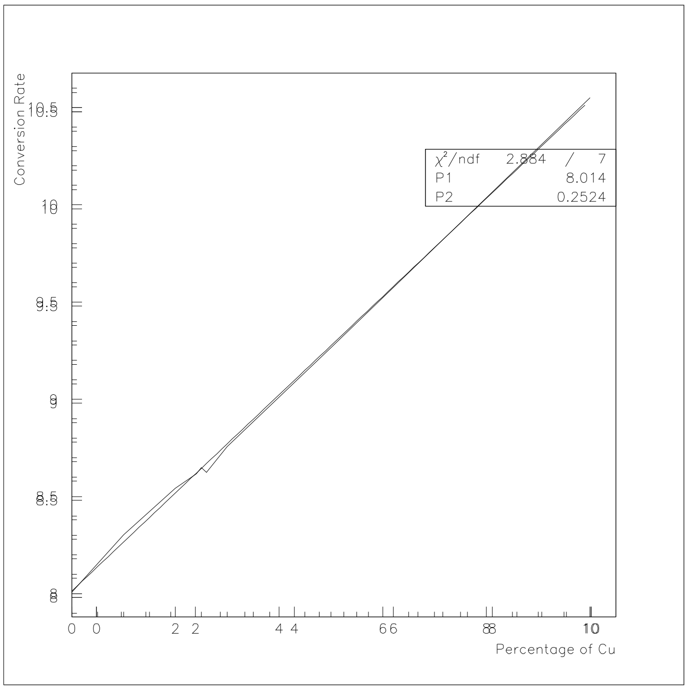

Figure 5: Chi Square Fit of Conversion Rate vs Percentage of Cu

Thus, we have

$$\Delta(Conversion\ Rate) = 0.25236 * \Delta(Percentage\ of\ Cu)$$

4 Effects of Photon Energies on the Conversion Rate

The next question to ponder is whether the conversion rate depends on the energy of the photons. For that, I generated photons of 8, 10, 12, 15, 20, 25 and 30 GeV, and see how many of them converted as they passed through the window. To see how valid my results are, I also calculated the theoretical conversion rate. This study has already been done before by Miro. I repeated his calculations, and got very similar theoretical predictions to his. Basically, what we had to do was to go to a webpage maintained by the National Institute of Standards of Technology [3], enter the composition and density of our window, and the database will generate the attenuation lengths for a range of photon energies. For the mixture that we have for the window, we sum the attenuation lengths using the formula:

$$\frac{1}{\lambda_{mixture}} = \sum \frac{\omega_i}{\lambda_i}$$

where ω_i is the weight fraction of the elements,and λ_i is the respective attenuation length of the elements. From that, we calculate the conversion rate using

$$ConversionRate = 1 - e^{-\frac{t}{\lambda}}$$

where t is the thickness of the window. At 20 GeV, the theoretical calculations yield a conversion rate of 8.77%. as compared to our value of 8.68% (from Figure 1). The simulation results are close to, but not exactly what we would expect from the theoretical calculations. Basically. the theories predicts a series of steps, but somehow we got something which doesn't exactly looks like a straight line and which fluctuates quite a bit. The simulation (with error bars) and the theoretical results are plotted on the same graph (Figure 6):

[3]The URL is http://physics.nist.gov/PhysRefData/Xcom/html/xcom1.html

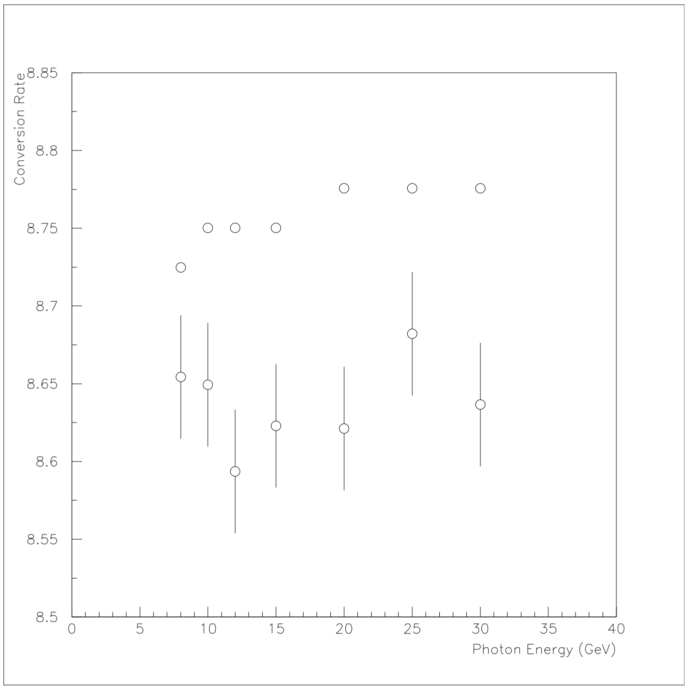

Figure 6: Conversion Rate vs Photon Energy

5 Effects of Smearing in the window

The last project I worked on was the smearing effect in the window. As the photon passes through the window, it has a probability of converting into a e^+/e^- pair somewhere along the thickness of the window. These e^+/e^- pair can undergo *bremstraahlung* and emit photons. The photons can sometimes have high enough energies that they too can convert to a e^+/e^- pair, which in turn can undergo *bremstraahlung*, and emit photons, which can in turn convert, and so on. Effectively, we have a cascade of secondary particles from the initial 20 GeV photon. We want to know the energies of the most energetic of the secondary e^+/e^-, as well as the angles they made (so that we know how many will hit our detector), etc. What I did was to use the NTloop command, and write a program to extract the information I want from the Ntuple [4].

[4] Special thanks must go to Allen for coming out with the algorithm to do it. For anyone who wants to do similar things, maybe with slight modification, the kumac file

If we consider only the secondary electrons and positrons, they lose on average 10% of their initial energies, with E'/E~0.9 for both positrons and electrons. When we consider all electrons and positrons, i.e. including those that were produced by the conversion of the 20GeV via pair production, and which went right through the entire length of the window without any interaction, one gets the following relation:

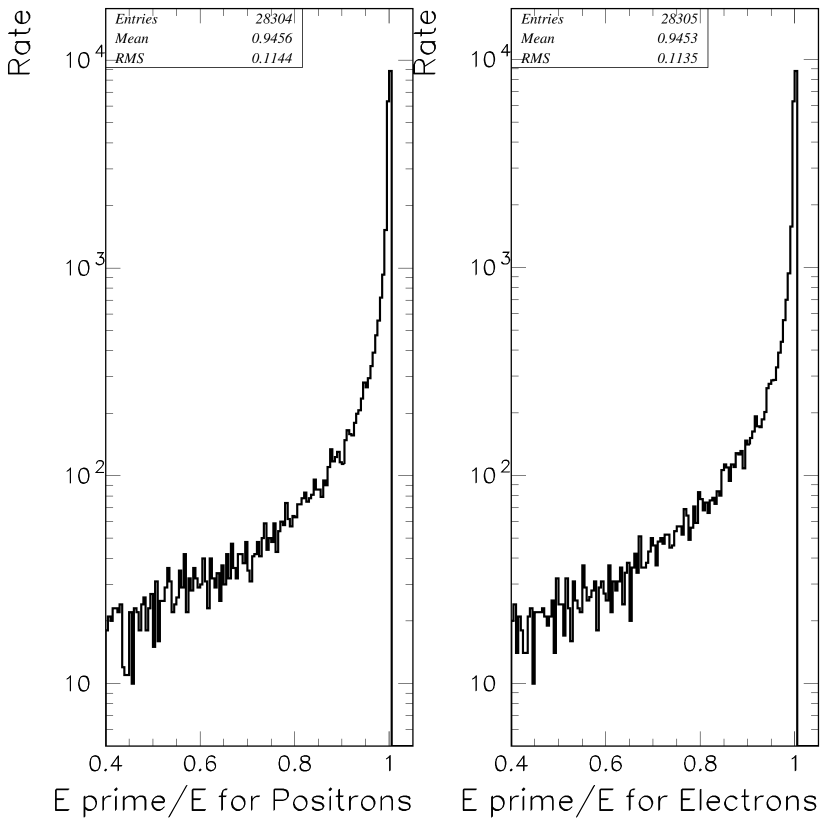

Figure 7: Energy Loss of Positrons and Electrons.

used is smloop.kumac. To make changes, simply edit sm.f. All these files are located in /nevis8/keewee/lumiGeant/wrk/.

We see that on average, $\sim 5.5\%$ of the energies of the electrons and positrons is lost in the window, with E'/E $\sim$ 0.9456 (for positrons) and $\sim$ 0.9453 (for electrons). This is a reasonable number, since most of the energy loss of the electrons (positrons) is through radiation loss, which is governed by the equation:

$$E = E_0 exp(-\frac{x}{L_{rad}})$$

where E_0 is the initial energy, E is the energy after traveling a distance x, and L_{rad} is the radiation length. Our window has a thickness which is equivalent to 0.12 radiation length. However, not all of the photons converted immediately upon entering the window. So the mean distance traveled by the electrons (positrons) is approximately half the thickness of the window, or 0.06 radiation length. Putting all these into the above formula, one gets

$$E = E_0 exp(-\frac{x}{L_{rad}}) = 0.9418 E_0$$

which is in good agreement with our value of $0.9456 E_0$ (for positrons) and $0.9453 E_0$ (for electrons).

If we plot the angle distribution of the positrons, we see that it resembles a Gaussian peaked around 0. We do a Gaussian fit in PAW (Figure 8 and 9) to see how good the fit is.

From the angle distribution, we can find out, in the presence of magnetic fields, how many electrons (positrons) will hit our detectors. This, and many other interesting things, can be easily done in a more in-depth study, by simply making slight modifications to the sm.f file. All these would give us a better understanding of the acceptance of the window.

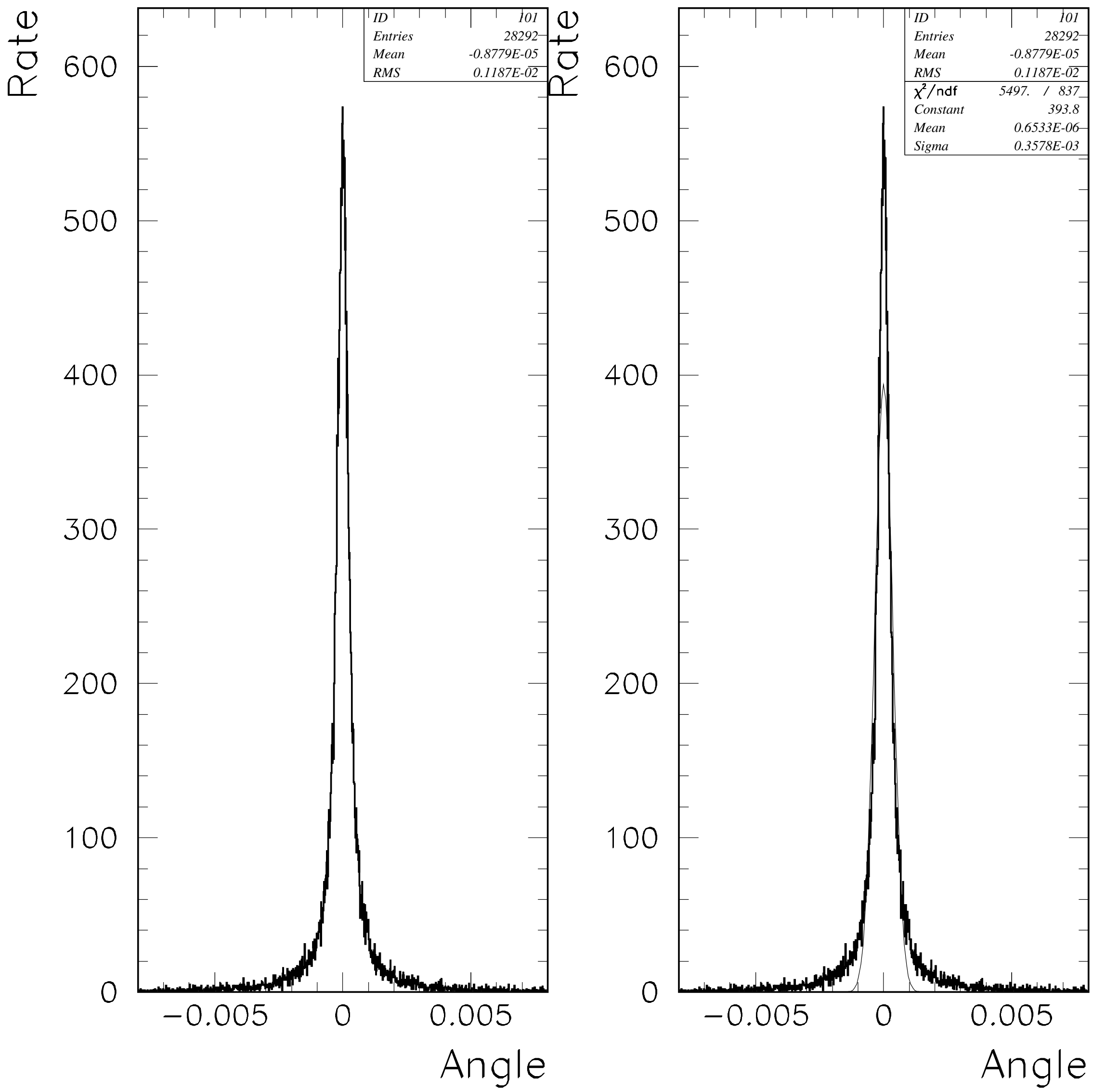

Figure 8: Angle Distribution of Positrons and its Gaussian Fit.

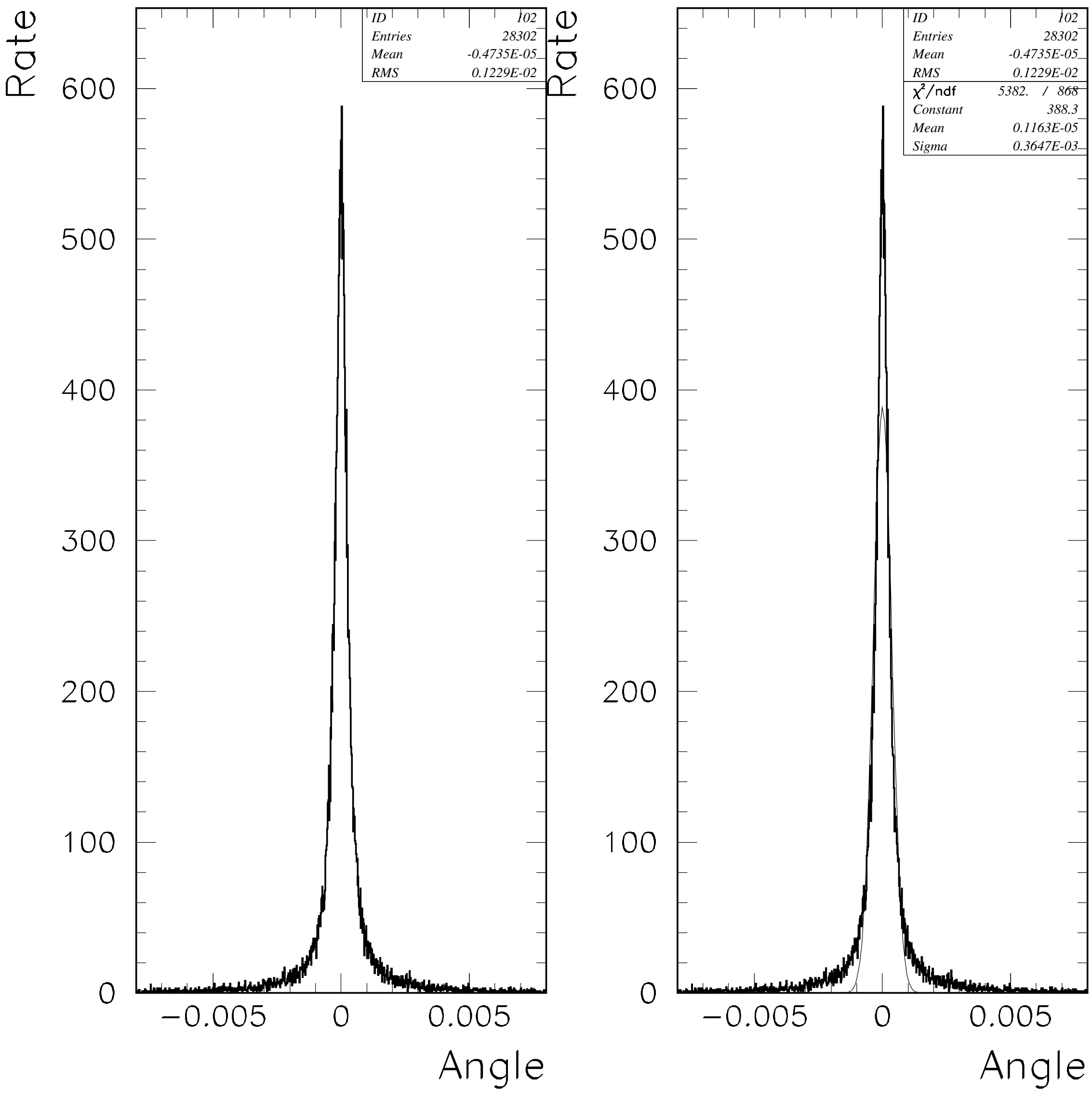

Figure 9: Angle Distribution of Electrons and its Gaussian Fit.