Study of the magnetic field for the ZEUS Luminosity Spectrometer

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1 Introduction

The luminosity spectrometer [1] is a detector to measure luminosity ($\mathcal{L}$) of high-energy ep collisions at HERA collider. This is done by measuring the rate of the bremsstrahlung photons approximately $100m$ downstream from the interaction point. In this setup, all photons are not detected directly but rather a known fraction are converted into e^+e^- pairs in the exit window of the vacuum pipe. Each electron-positron pair is then split by the dipole magnet and detected in two small calorimeters. Simulation studies show that 1% uncertainty on the magnetic field transforms to $\approx 1\%$ error on the acceptance so that the error on the luminosity scales with magnetic field. Therefore a precise knowledge of the magnetic field, especially in the central region is essential for the precise determination of the luminosity.

In our defined geometry the magnetic field is primarily in the x-direction, the beam direction is $+z$ and the electron (positron) is deflected in $\pm y$. The deflection of the electron in the magnetic field can be approximated as

$$\Delta y = \frac{0.3 \int B_x dz L}{p} \tag{1}$$

where L is the distance from the center of the magnet to the spectrometer, $\int B_x dz$ is in Tesla-meters and the electron momentum p is given in GeV/c. This approximation holds only if the deflection angle is small which is true in this setup. When using directly this formula the transverse deflection inside the magnetic field must be small enough so that the change in $\int B_x(x,y)dz$ over that distance is small. For electrons hitting the spectrometer the typical deflection inside the magnetic field is $\approx 3mm$ over which the field changes by less than 1%. The maximum deflection can be as large as 6mm. Because of asymmetries in the field, deflections from B_y are important for this setup while the deflections from B_z are not. The discussion here describes how the field map of B_x and B_y were obtained from a finite set of measurements of B_x.

2 Data set

The spectrometer uses a 60cm long dipole magnet with a 10cm bore and maximum field strength of $B_x \approx 0.5T$. In addition shielding is arranged around the proton beam-pipe made of high permeability material, whose purpose is to reduce the magnetic field inside the proton pipe so that the magnet doesn't affect the proton beam operation. The shielding is a cylinder centered at x=17mm,y=126.7mm with respect to the center of the magnet with outer diameter of 85mm. Its position was 11mm different from this value when the measurements were taken; this effect is studied in section 5. This is the only significant asymmetric part of the setup.

The magnetic field was measured in a set of points in the x-y plane along the z-axis in 10mm steps from -800mm to 800mm for three different currents 137A, 233A and 290A for ~ 38 fixed values of x,y. Only the major B_x component of the field was measured and the other components are calculated by the means of Maxwell's equations as described in section 3. The reproducibility of the magnetic field measurement was about 0.01%; the calibration precision of the probe was quoted to be of similar order. This error is negligible in comparison to the systematic errors due to interpolations. In addition another set of measurements at the center of the magnet was taken in order to determine the dependence of the magnetic field on the current.

3 Method of interpolations

The measured points do not cover all possible points in x-y plane and the differences between two neighboring points are significantly larger than the required 1% precision. Therefore an accurate method of interpolating among measured values is needed. In addition, the B_y component of the magnetic field is calculated from the Maxwell's equations and its impact on the electron trajectory is studied.

Simple polynomial fits to the measured data proved not to be a suitable tool for doing the interpolations. We make direct use of Maxwell's equations in free space

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = 0 \tag{2}$$

to find a differential equation, which is then solved by the numerical methods described below. From (2) it follows that each component of the magnetic field satisfies the 3-dimensional Laplace equation

$$\nabla^2 B_i = 0. \tag{3}$$

To simplify the problem we make use of equation (1) noting that it is sufficient for our purposes to work only with the magnetic fields integrated over z. Integrating equation (3) gives

$$(\partial_x^2 + \partial_y^2) \int_{-\infty}^{\infty} B_x(x, y, z) dz + \partial_z (B_x(x, yz = +\infty) - B_x(x, y, z = -\infty)) =$$

$$= (\partial_x^2 + \partial_y^2) \int_{-\infty}^{\infty} B_x dz = 0 \tag{4}$$

where the last step uses the fact that the fields and its derivatives are zero very far from the magnet. This means that integrated B_x and B_y components satisfy the 2-dimensional Laplace equation.

This type of differential equation can be solved by the Jacobi relaxation method [2]. It is an iterative procedure which uses the fact that for the solution satisfying the Laplace equation (4) the value at a given point is equal to the average of the values around it. This can be implemented by defining a rectangular grid of points with values $B_{i,j}$ with an iterative step

$$B^{n+1}_{i,j} = \frac{1}{4}\omega(B^n_{i+1,j} + B^n_{i-1,j} + B^n_{i,j+1} + B^n_{i,j-1}) + (1-\omega)B^n_{i,j} \tag{5}$$

where n is the iterative step and ω can have a value changing with n to speed up the convergence, however was set to constant $\omega = 1/2$ for this analysis. All measured values were not allowed to change during the iteration, the non-measured points were changed until the solution was found. There is one more problem left to be solved. This method is designed to find solutions inside the region if the boundary is known. However, not all the points on the boundary were measured. For this reason, the extra boundary condition that the first derivative of the field normal to the surface is equal to zero was chosen. This is of course not true for the real field but this affects only points close to the boundary. The consequences of this choice on the integrated fields are acceptable as discussed in more detail in section 5.

After the $\int B_x(x,y,z)dz$ is found, we can approximate the z dependence as

$$B_x(x,y,z) = f(z)\int B_x(x,y,z)dz \tag{6}$$

where $f(z)$ is the measured functional dependence of magnetic field on z normalized so that its integral is equal to one. The physical assumption is that the z-dependence is weakly dependent on x and y, confirmed by the measurements.

The last step is required so as to calculate the B_y component of the magnetic field. Because B_x changes with y, B_y must change as well, hence cannot be zero. Integrating equation (4) over z leads to

$$\partial_x \int B_y dz = \partial_y \int B_x dz \tag{7}$$

from which B_y can be calculated when $B_x(x,y)$ is known as

$$\int B_y(x,y)dz = \int_0^x \partial_y \int B_y(x',y)dzdx' + \int B_y(x=0,y)dz \tag{8}$$

The last term represents the boundary condition of the problem. For a magnet being symmetric with respect to the plane defined by x=0, B_y would be zero and the last term in (8) disappears. In the real setup the symmetry is broken only slightly by the shielding of the proton beam-pipe so in this study we still assume that $B_y(x=0,y)=0$.

The B_z component is not important for the luminosity measurement since the electrons travel parallel to z. The effect of small p_t coming from the beam divergence and the deflection of electrons inside the magnet is negligible.

4 Results

The results of the relaxation program for $\int B_x dz$ and $\int B_y dz$ for the current $I = 233A$ for which the grid of points was measured are shown in figures 3,4. Note that the ordinate scales for the B_y are smaller by about a factor if 10 compared to B_X. The variations over the acceptance region are of the order of 15% relative to the typical $\int B_x dz$. The grid was chosen to be $2.5mm \times 2.5mm$ so that the difference between two neighboring points is less than 0.5%.

The B_y component, up to 5% of B_x in some regions, will cause deflections of electrons at the detector by a few millimeters. This can be checked by studying the $x_{up} - x_{dn}$ distribution for the coincidence in the spectrometer.

The dependence of the magnetic field on the current at the point x=y=0 is shown in the figure 2. Except for the very highest current the dependence is linear. Beyond I=250A the saturation of magnetic materials starts to appear. The magnetic field map at a given current is then obtained by interpolation with this curve. The total rms of the distribution of deviation from the linearity for every point on the grid for currents I=233A and I=137A is only 0.05%. This shows that the relaxation method reproduces the shape of both B_x and B_y very well for completely independent measurements and the field has the same shape at different currents. Similar result for the rms is obtained for the I=290A data.

The z dependence is obtained by the use of (6). The function f(z) in the center of the magnet is shown in figure 1. The differences in shape for different positions and currents are at the level of few per-cent (negligible), so the same function was used for all data.

5 OPERA

OPERA-2d [3] is a program designed for the analysis of vector fields. The input is the geometry and the material definitions including the hysteresis curves from which the program calculates the magnetic fields. It is in general extremely difficult to calculate the magnetic fields with high precision. For this reason the program was not used for absolute comparison but rather to check the relative effect of small changes in the geometry during magnet installation.

In a first study the program was used to check the interpolation method. From the calculated magnetic fields the points corresponding to measurements were used as inputs to the relaxation program. The results obtained were then compared to the "true" values from OPERA and the comparison is shown in the figure 5. It is clear that the data from relaxation program underestimate the field as one extrapolates out from the measured region and the difference reaches 4-5% in the corners of the considered volume. This method cannot therefore be used for substantial extrapolations. Furthermore there are localized deviations at the measured points on the boundary. These are however only at the level of $\approx 1\%$, hence not affecting the measurement. In any case, few photons are expected in the region this far from the center during stable beam operation with the photon beam centered, giving less importance to the problems close to boundary.

In the central region, the maximum error is $\approx 1\%$. For a Gaussian beam profile centered on the magnet aperture (x=y=0) $\sigma_x = 20mm$ and $\sigma_y = 7mm$, the average uncertainty on the luminosity from field interpolation is 0.4%.

During the installation in the HERA tunnel, the beam-pipe shielding was moved by 11mm compared to the setup when the measurement was taken. Study with OPERA showed that the difference in the magnetic field due to this change is less than 1%.

6 Conclusions

The magnetic field of the dipole magnet for the luminosity spectrometer was studied. The interpolations of measured data are obtained by the Jacobi relaxation method. The comparisons with the OPERA program show that uncertainties are below 1% in the acceptance region and the average uncertainty is below 0.5% for the beam located at magnet center. This satisfies the requirements on the precision of the luminosity measurement.

References

[1] A.Caldwell,S.Paganis,R.Sacchi,F.Sciulli A Luminosity Spectrometer for ZEUS, HERA-ZEUS Document (oct.1999)

[2] Numerical Recipies in C, Press et al, Cambridge University Press

[3] Opera 2-d, Vector Fields Ltd, Oxford

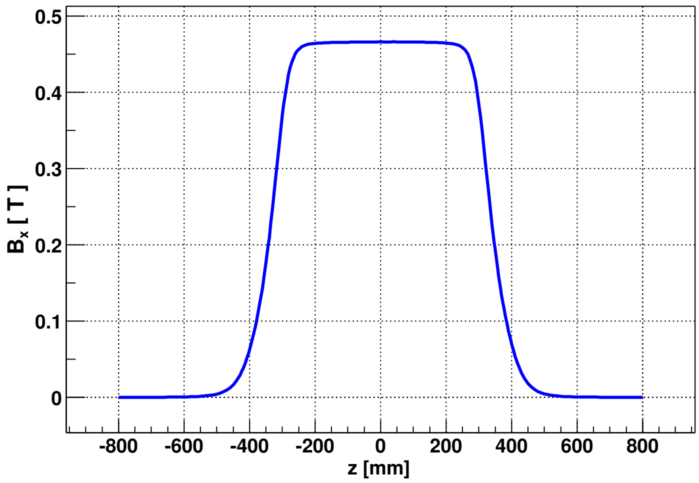

Figure 1: B_x as a function of z at the center of the magnet x=y=0.

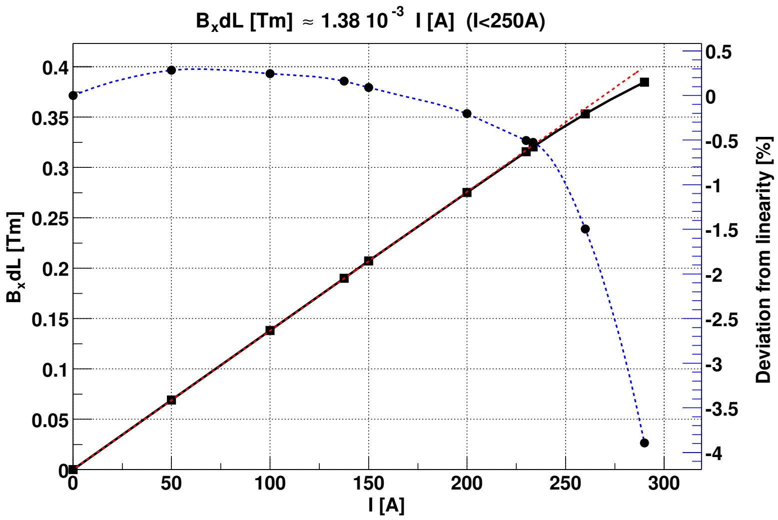

Figure 2: Dependence of the magnetic field on the current. Significant deviations from the linearity (right axis) take place only for currents $I > 250A$.

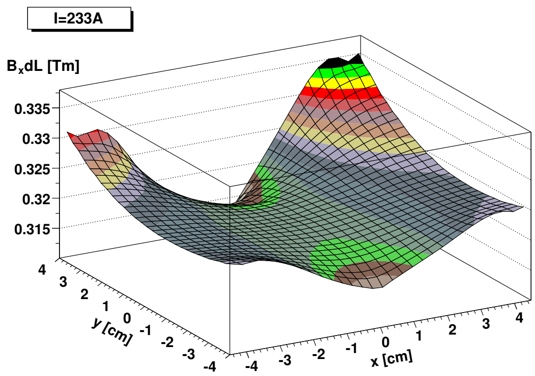

Figure 3: Results for $\int B_x(x,y)dz$ as obtained by the relaxation method for I=233A

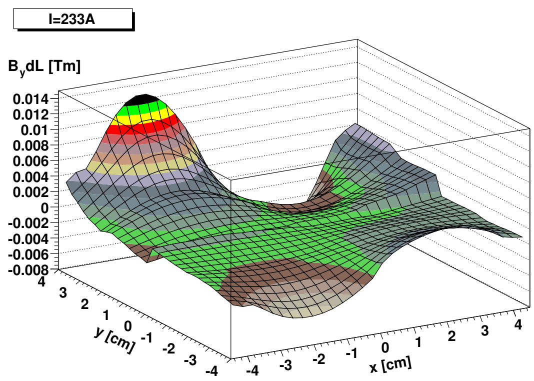

Figure 4: Results for $\int B_y(x,y)dz$ as obtained by the relaxation method for I=233A

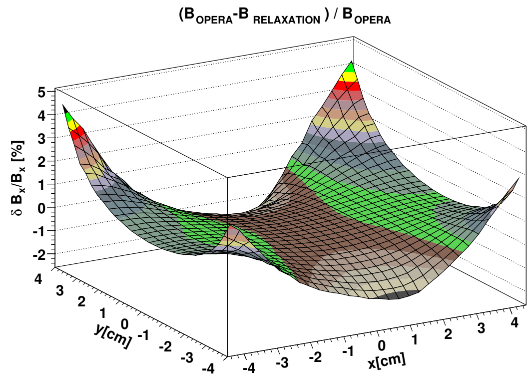

Figure 5: Comparison of the magnetic field from the Opera 2-d and the relaxation method as described in section 5. The maximum difference in the corners of considered region goes up to $\approx 4\%$. In the central region the difference is below 1%.