Reconstruction and Calibration of Energy and Position

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In the last ten weeks, I tried to develop techniques for calibrating the position and energy of the spectrometer. In this project, only simulation data from Geant were used to make analysis. But the preliminary results show that the methods are very promising. In this article, I shall present the results, and explain what I have done. I hope it is helpful for someone who continue this study.

1 Position and Energy Reconstruction

Before we talk about the calibration, it would be better to take another look at the reconstruction procedure. The reconstruction routine was written by Miro and Stathis last summer. The procedure is shown in Fig. 1 schematicaly.

In Fig. 1, `t_event`, `BPC_RAW`, `BPC_EVENT` and `t_spec_result` are data structures used in different stages of the reconstruction procedure.

Note that in real data taking process, `t_event` is prepared by some other routine. `BPC_RAW` only contains unreconstructed information directly from `t_event`. These information will be converted to reconstructed positions and energies in structure `BPC_EVENT`. Note also that some parts are not finished yet. In structure `t_spec_result`, `t_spec_todisplay` and `t_spec_tofile` are still not defined. Substructure `t_spec_toevb` also needs to be clarified.

The reconstruction of energy is straight forward—we sum up all the energies deposited in every strip of the BPC. But we should worry about the behavior of the shower at the edge of the BPC's. The corrections made to the reconstructed energy are transverse leakage and attenuation. The study of the leakage and attenuation is part of Anne's work, so I will not reiterate it here. The position reconstruction needs more efforts. The method that we currently use is *logarithmic-weighting technique,* taken from [1].

The logarithmic-weighting technique is based on the fact that the shower profile in the BPC takes an overlayed exponential form. It is shown in [1] that a logarithmic-weighting of the energy depositions gives a smaller intrinsic position bias as well as a better position resolution. The position measured using the logarithmic-weighting method is given by

$$X = \frac{\sum_{i=1}^{N} w_i X_i}{\sum_{i=1}^{N} w_i} \tag{1}$$

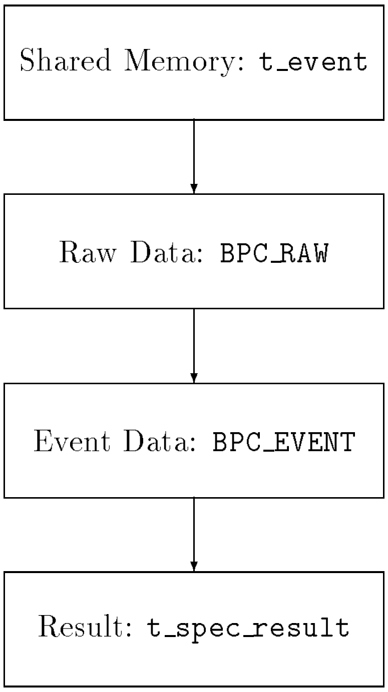

Figure 1: Schematic procedure of the reconstruction

where

$$w_i = \begin{cases} W_0 + \ln\left(\frac{E_i}{E}\right) & \left(\frac{E_i}{E}\right) > e^{-W_0} \\ 0 & \left(\frac{E_i}{E}\right) \leq e^{-W_0} \end{cases} \tag{2}$$

and

$$E = \sum_{i=1}^{N} E_i \tag{3}$$

In equation 2, W_0 is a positive dimensionless parameter, which serves as a threshold on the fraction of the total energy E that strip i must exceed in order to be included in the calculation. There is a detailed discussion on the choice of W_0 also in [1]. The W_0 that we used in the program is 2.80, which is supposed to give the best result in the old system. But we need to find the best value of W_0 for our new system.

So far, we have the reconstruction software, and we need to test its performance. Using Geant, Stathis built a system which is able to simulate the photon conversion in the window, the magnetic field, and the shower in the two BPC's. First Geant generates events using $20GeV$ single energy photons. Then the reconstruction program read the energy deposited in each channel of the two mock BPC's, just as they did in the real case, and calculate the positons and energies of the hits. In front of each of the two mock BPC's, there is also a 'cheating layer', which records all the particles passing through, and write them into a ntuple. This enables us to compare the result of the reconstruction program with the output directly from the Geant ntuple. This kind of comparision has been made, and they are in good aggrement. Fig. 2- 4 are plots from the results of the reconstruction code. They contain important information of the hits and the detectors. We will go back to some of them in later sections.

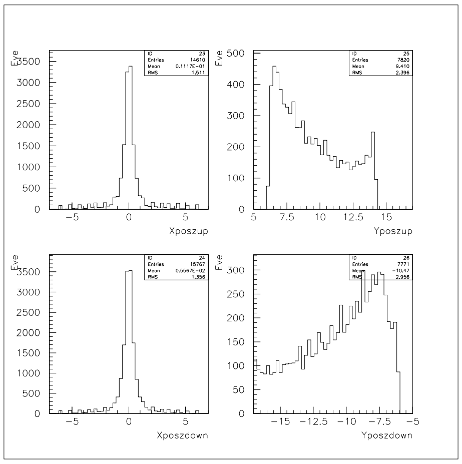

Figure 2: The reconstructed position spectrum. Unit of position: cm.

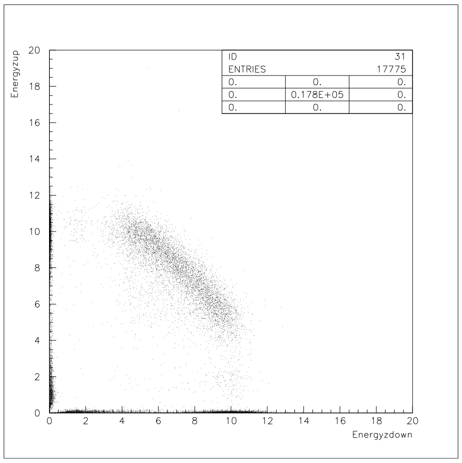

Figure 3: E_{up} v.s. E_{do}. Unit of energies: GeV.

Note there are some regular peaks in reconstructed position spectrum in Fig. 2, which don't appear in Geant's output. Further investigation showed that those are events with very low energy. Since their energies are so low that they are deposited in only one strip, the reconstuction routine put their position at the center of the strip. To avoid this, we may lower W_0 to increase the energy threshold. This should not be a problem in following analysis, because we will make energy cuts, and those low energy events will be always cut away.

Note also that in Fig. 4 there are low energy stuff at the edges of the two BPC. It is due to leakage. We will go back to this in energy calibration in section 3.

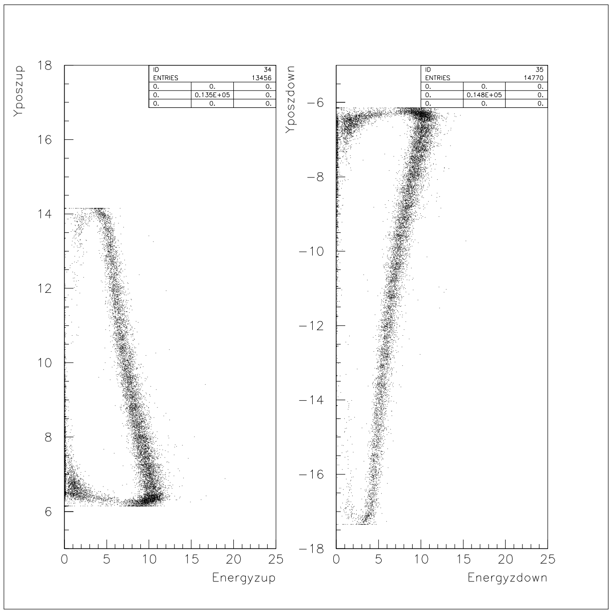

Figure 4: Energy v.s. y position for two dectors. Unit of energy: GeV, unit of position: cm.

2 Position Calibration

In order to get energy scale precisely, we need to align the calorimeters relative to the collimator and magnet axis. As described in [2], there are two possible schemes to find this relative alignment. The first one employs the symetric distribution of the events in the upper and lower BPC's. It would be able to make *in situ* alignment. In the following discussion I shall only concentrate on the first method.

The basic idea under this method is: The rate of e^+/e^- at a given distance from the collimator axis should be identical for the upper and lower BPC's, provided the magnetic field is symmetic about the axis and provided the photon beam distribution across the slit of collimator is symmetric. If we know the distance, D, between the two BPC's (It can be measured easily), the alignment can be found by requiring the rates to be equal at a given distance from the collimtor axis. Define Y_{up}, Y_{do} to be the distances of the edge of the upper BPC and lower BPC from collimator axis, and y to be the distance of the hit in the BPC from the collimator axis, then we calculate the offset η by requiring equal rates in the two BPC's:

$$R_{up}(y+\eta) = R_{do}(y) \tag{4}$$

We solve the following equations for relative alignments:

$$Y_{up} + Y_{do} = D \tag{5}$$

$$Y_{up} + \eta = Y_{do} \tag{6}$$

So the important questions are to obtain the relative shift η , and to estimate the error in obtaining η . There are two possible methods which would be able to give the value of η as well as the uncertainty estimation.

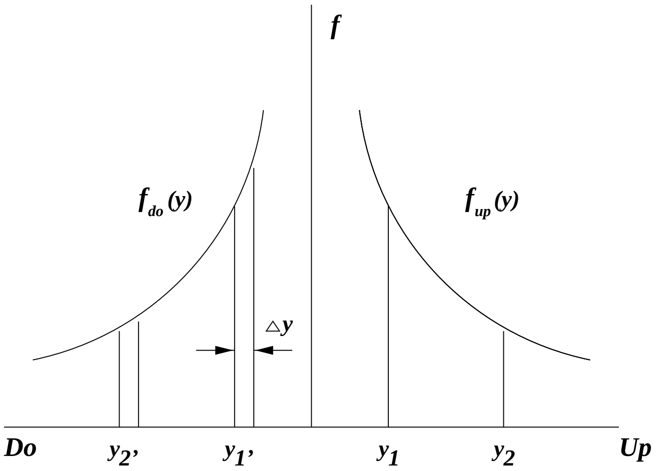

Figure 5: Shematic illustration of $f(y)$.

Method I is simple and straight forward. Assume the density of the event distribution is a function of positon, and the two functions, $f_{up}(y)$ and $f_{do}(y)$ are symmetric(both are donoted as $f(y)$), as shown in Fig. 5. Note that different coordinate systems are used for upper and lower detectors. If we pick up a band $[y_1, y_2]$ in the upper detector, the number of entries in the band would be:

$$N = \int_{y_1}^{y_2} f(y)dy \tag{7}$$

Within uncertainty, this number should be identical to the number of entries, N', in the band $[y_2', y_1']$ of the lower detector, which is symmetric to $[y_1, y_2]$.

$$N' = \int_{y_2'}^{y_1'} f(y)dy \tag{8}$$

The error of N is:

$$\delta N = \sqrt{(\frac{\partial N}{\partial y_1})^2 \delta {y_1}^2 + (\frac{\partial N}{\partial y_2})^2 \delta {y_2}^2} \oplus \sigma_{stat} \tag{9}$$

where δy_1 and δy_2 are systematic errors of the positions, which come from mechanical measurement and position reconstruction algorithm. If we take $\delta y_1 = \delta y_2 = \Delta d$, as a rough estimation,$d \sim\ 200 \mu m$. σ_{stat} is the statistical error. Assume we have infinitely many statistics, the systematic error becomes dominant, and σ_{stat} will dissappear from Eq. 9. Then the error of N is simply

$$\delta N = \sqrt{f(y_1)^2 + f(y_2)^2} \cdot \Delta d \tag{10}$$

This equation will hold if we take data for a long enough period. In method II, we will see how to estimate the length of this period.

Next let's try to move $[y_2', y_1']$ by a small distance Δy. Then the difference between N and N' caused by this movement is

$$\Delta N = |N - N'| = |f(y_1) - f(y_2)| \cdot \Delta y \tag{11}$$

This difference should exceed δN in order for us to tell if the band is really moved. Thus the smallest relative shift that we can recognize can be found using

$$\Delta N \geq \delta N \tag{12}$$

i.e.

$$\Delta y \geq \frac{\sqrt{f(y_1)^2 + f(y_2)^2}}{|f(y_1) - f(y_2)|} \cdot \Delta d \tag{13}$$

Armed with these knowledges, method I can be simply put as: We fix the band $[y_1, y_2]$ in the upper detector, shift the band $[y_2', y_1']$ in the lower detector. For each shift s, we calculate the difference ΔN, then the curve for ΔN v.s. s should be like what is shown in Fig. 6.

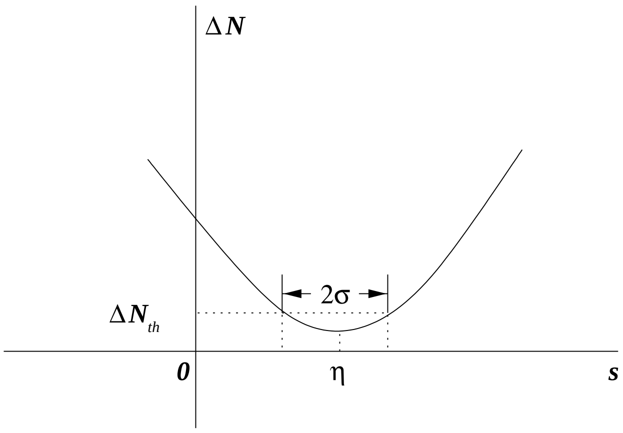

Figure 6: Schematic illustration of algorithm of method I.

The relative shift for minimum ΔN is just what we dreamed for in Eq. 4, while Eq. 13 tells us the uncertainty of the calibration using this method, provided we have enough statistics. The advantage of method I is its simplicity, where we can clearly see the effect of systematic error. The disadvantage is also its simplicity. In this method we only considered the integral of the rate, but ignored the shape of the event distribution.

Method II is more subtle than method I. It relys more on statistics theory. A nice discussion about some results of the statistics theory can be found in [3].

Our method comes naturally from the theory. First we fill two histograms for the upper and lower detectors with positions of the hits. Instead of calculating the magnitude of ΔN, we 'fit' the two histograms. By 'fit', I mean we calculate the χ^2 for each relative shift. Eg. If we fit the two histograms bin by bin, first we pick up a band in the histogram of the upper detector, then chose a band with the same width and relative shift s in the histogram of the lower detector. χ^2 of the fitting is given by:

$$\chi^2 = \sum_{i=1}^{N} \frac{(u_i + d_i)^2}{\sigma_{u_i} + \sigma_{d_i}} \tag{14}$$

where N is the number of bins contained in the band, u_i and d_i are number of events in bin i, σ_{u_i} and σ_{d_i} are errors of u_i and d_i. Usually, $\sigma_{u_i}/\sigma_{u_i}$ contains statistical error and systimatic error. How one consider the systematic error in this formula is still not clear. In the following discussion, we negelect that factor and only consider statistical error. Then $\sigma_{u_i} = u_i$, and $\sigma_{d_i} = d_i$. The χ^2 in Eq. 14 gives us the information about how well the two bands fit with each other. Since the two histograms are symmetric, the χ^2 will be minimized if the relative shift is exactly η. We expect the curve χ^2 v.s. s should be like

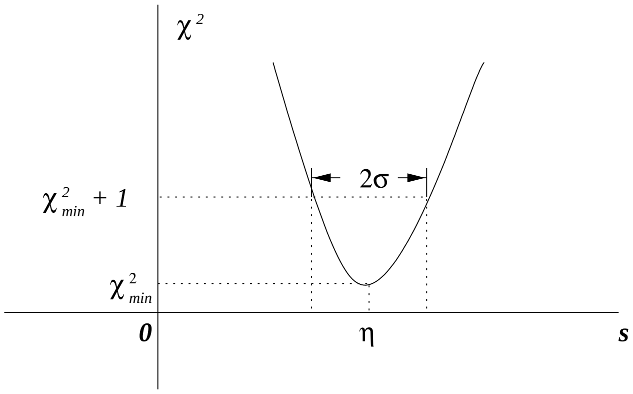

Figure 7: Schematic illustration of the algorithm of method II.

Note that at the points where $\chi^2 = \chi^2_{min} + 1$, the separation between the two points would be just $2\sigma_{calib}$, where σ_{calib} is the statistical error of this calibration procedure.

To make this procedure even more subtle, we can use linear interpolation. Eg. for points in the histogram of lower detector, the corresponding 'event number' was linear interpolation from its nearest two bins. It's error is propagated also from these two bins. This technique enable us to calculate χ^2 v.s. s using steps smaller than one bin. When χ^2 increases dramatically with s, it would be very useful if we want to find where χ^2 increases by one unit.

To evaluate this method, we tried to plot χ^2 v.s. s using the simulation data. Since the statistics that we need is too large for Geant, what we do is just filling the first histogram with a linear function, the second with Poisson random numbers centered around the cooresponding value of the same linear function, as shown in Fig. 8 and Fig. 9 respectively. The reason why we assume linear distribution is that, in Geant's simulation the distribution of the events is roughly linear. In order to make more sense of the linear filling, the slope that we used in the linear function is close to that in Geant's simulation.

The plot χ^2 v.s. s is shown in Fig 10. From Fig. 10, we know that one σ_{calib} is about $100\mu m$. The number of events contained in Fig. 10 is about 36×10^7. Suppose the rate of the real data taking is $10kHz$, this amount of data will take about 10 hours. So if we want the statistical error smaller than $100\mu m$, we should take one or two runs to make the calibration.

Further study is needed to realize this technique. In order to get a better estimation of how long it will take to calibrate the position, σ_{calib} v.s. number of events N can be produced.

A possible improvment is to include systematic error into Eq. 14. If we include systematic

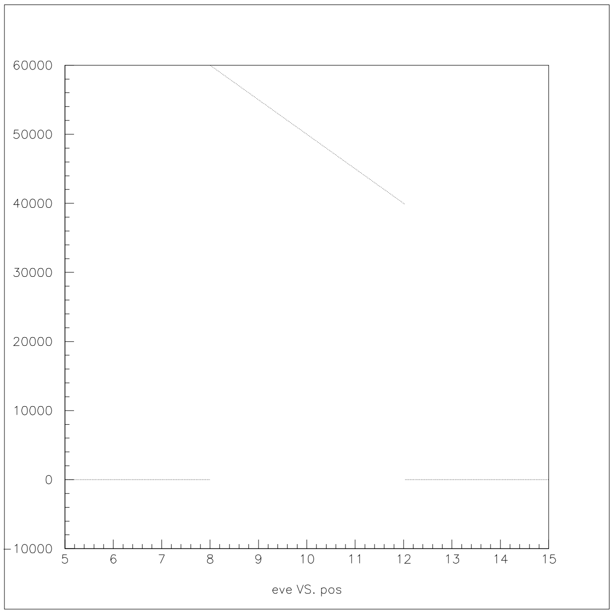

Figure 8: Filling of the histogram for upper detector using linear function. Unit of position is cm.

error, then

$$\sigma_{d_i} = d_i \oplus \sigma_{sys} \tag{15}$$

$$\sigma_{u_i} = u_i \oplus \sigma_{sys} \tag{16}$$

The simplist assumption is that σ_{sys} is constant (although this may not work). Then the above equations become

$$\sigma_{d_i} = \sqrt{{d_i}^2 + {\sigma_{sys}}^2} \tag{17}$$

$$\sigma_{u_i} = \sqrt{{u_i}^2 + {\sigma_{sys}}^2} \tag{18}$$

Plug these two equations into Eq. 14, we could make $\frac{\chi^2}{n}$ v.s. σ_{sys} plot. Statistics theory tells us that

$$\frac{\chi^2}{n} \approx 1 \tag{19}$$

It would be possible that we use this relationship to get a 'reasonable' systematic error on $\frac{\chi^2}{n}$ v.s. σ_{sys} plot.

Another idea of possible improvement came from Miro, who tries to take advantage of the fact that the fractional cross section becomes very steep at the low energy region. Thus the slope of the events distribution will be very large. From Eq. 13, we can see the systematic error would be decreased significantly.

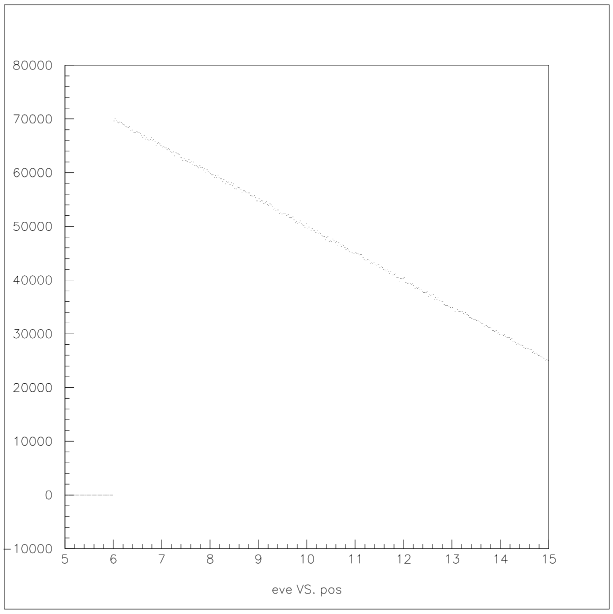

Figure 9: Filling of the histogram for lower detector using Poisson random number. Unit of position is cm.

3 Energy Calibration

The Energy calibration goes after the position calibration, because in the real data analysis we need precise position to get energy to a desired precision. The energy calibration procedure for real data is discussed in [2], where strip-to-strip energy scale factors are calculated. In this preliminary study, we try to calibrate the 'mock BPC's' using the simulation data from Geant. Since the 'mock BPC's' don't have attenuation and strip-to-strip variation, what we do is just to find the overall energy scale factor S such that

$$E_i = S \cdot ADC_i \quad for \;\; i = 1 \ldots N \tag{20}$$

where ADC_i is the digitized ADC value from the detectors, E_i is the calibrated energy, and N is the number of channels. In the simulation, we used photon beam with single energy $20GeV$. So for the coincidences, if E_{up} is defined to be the energy deposited in the upper BPC, E_{do} to be the energy deposited in the lower BPC, and E to be the energy of the incident photon, then

$$E_{up} + E_{do} = E \tag{21}$$

In our case, $E = 20GeV$. Although this situation is simple enough, it still gives us some sense about problems in the real case, which are 1) how to pick up the pure coincidences 2) the leakage at the edges of the BPC's. More detailed study shows that these problems can be solved by making appropriate cuts.

In the selection of coincidences, we need to cut away the single hits. Note in Fig. 3 the single hits cluster near the x and y axis, so we can make cuts like $E_{up} > E_{cut\,up}$ and

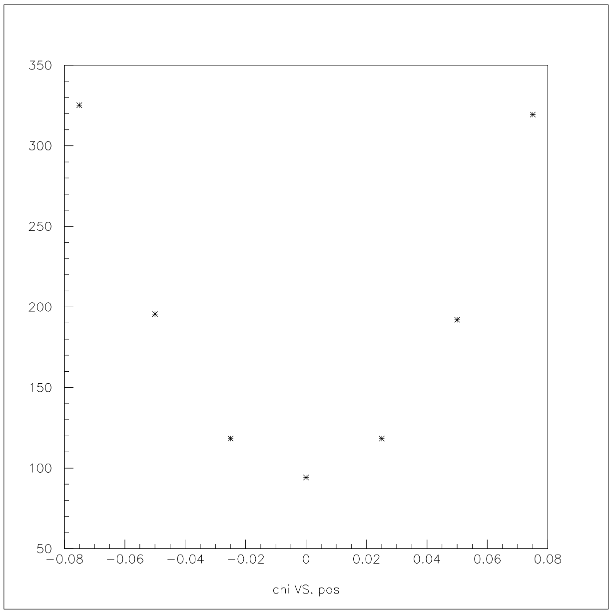

Figure 10: χ^2 v.s. relative position (in cm).

$E_{do} > E_{cut\,do}$. The actual cuts that we made are

$$E_{up} > 8.09GeV \tag{22}$$
$$E_{do} > 8.09GeV \tag{23}$$

The effect of leakage is shown in Fig. ??. Note that we should get rid of the low energy stuff at the edges in order to get correct energy of coincidences. So we should make energy-dependent fiducial volume cut on positions. The actual cuts that we made are

$$-5.6cm < x_{up} < 5.6cm \tag{24}$$
$$6.55cm < y_{up} < 13.75cm \tag{25}$$
$$-5.6cm < x_{do} < 5.6cm \tag{26}$$
$$-16.95 < y_{do} < -6.55cm \tag{27}$$

The Eve v.s. $ADCvalue$ plot after the calibration is shown in Fig 11. We can see that the result is pretty satisfactory.

4 Other Stuff

I have been worked on the reconstruction code for about a month, and many changes have been made. Now the program is working well for mock data coming from Geant. But there are still a lot of work to do to get prepared for the real data taking. For anyone who will construct the code, maybe my update of the code can be of some help. Now the updated code is in directory `/home/zhang/tmp/src_working/` on server `luxury2` (but sometimes it is called `luxury`). The changes that I made are mainly in `/home/zhang/tmp/src_working/recon/recon_spec`.

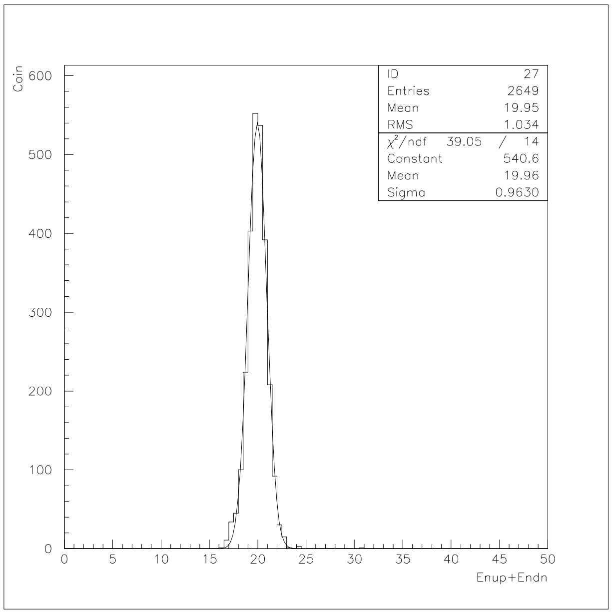

Figure 11: The Energy of coincidences after calibration.

I have also worked on the position and energy calibration for about a month. The programs that I wrote for the analysis are mainly in directory `/nevis8/yuanbo/data_anal/` on server `znev02`. Especially, the position calibration codes are in `/nevis8/yuanbo/data_anal/pos_cal`. Of course, rewriting these programs is not a big job. But I would be very happy if it helps whoever taking over this job in any way.

References

[1] B. Surrow. *Measurement of the Proton Structure Function F_2 at Low Q^2 and Very Low x with the ZEUS Beam Pipe Calorimeter at HERA*. March 1998, DESY-THESIS-1998-004.

[2] Columbia Uinversity. *Spectrometer - calibration, monitoring, luminosity Draft 0.0.* August 2001.

[3] Caso et al. *The European Physical Journal C. Review of particle physics. Volume 3.* 1998.