ELSEVIER
Cryogenics 41 (2001) 513–520
Cryogenics
www.elsevier.com/locate/cryogenics

Thermodynamic analysis of GM-type pulse tube coolers ☆

Y.L. Ju *,[1]

Cryogenic Laboratory, Chinese Academy of Sciences, P.O. Box 2711, Beijing 100080, People's Republic of China

Received 29 January 2001; accepted 28 June 2001

Abstract

The thermodynamic loss of rotary valve and the coefficient of performance (COP) of GM-type pulse tube coolers (PTCs) are discussed and explained by using the first and second laws of thermodynamics in this paper. The COP of GM-type PTC, based on two types of pressure profiles, the sinusoidal wave inside the pulse tube and the step wave at the compressor side, has been derived and compared with that of Stirling-type PTC. Result shows that additional compressor work is needed due to the irreversible entropy productions in the rotary valve thereby decreasing the COP of GM-type PTC. The effect of double-inlet mode on the COP of PTC has distinct improvement at lower temperature region (larger T_H/T_L). It is also shown that the COP of GM-type PTC is independent of the shape of the pressure profiles in the ideal case of no flow resistance in the regenerator.

Keywords: Pulse tube cooler; GM-type; Thermodynamic analysis

1. Introduction

In general pulse tube cooler (PTC) requires a phase shifter system, located at the hot end of the pulse tube, to achieve an optimum phase angle θ between the gas flow rate and the pressure oscillation inside the pulse tube to increase the cooler performance. The so-called orifice [1] and double-inlet [2] modes are two of the most well-known configurations. Other innovations are multi-bypass [3], two-piston [4], four-valve [5], inertance tube [6], active-buffer [7], inter-phasing [8], and double-orifice [9].

In recent years multi-stage 4 K-PTCs [9–11] have been reported with multi-layered hybrid magnetic materials in the coldest regenerator region. Two-stage PTCs can provide more than 0.5 W cooling power at 4.2 K and meet the cooling requirements of superconducting devices operating at 4 K [12]. By using ^{3}He as the working fluid the lowest temperature below 1.8 K has been achieved [13]. All these machines use a compressor and a valve system to produce pressure oscillation in the cooler system. They are called GM-type PTCs.

A GM-type PTC, shown in Fig. 1, only differs from the Stirling-type at the compressor side of the cooler. Instead of a piston compressor the GM-type uses a rotary valve or several electromagnetic valves to generate the pressure oscillations in the cooler. In the ideal case the compressor is isothermal and reversible. The compressor heat is removed in the aftercooler. The rotary valve connects the PTC system alternatively to a constant high pressure p_H and a low pressure p_L.

This paper is a continuation of our previous papers on the thermodynamic aspects of pulse tubes [14–17]. We will discuss the thermodynamic losses of the rotary valve and the coefficient of performance (COP) of GM-type PTC in this paper. General expressions for the COP of GM-type PTC, based on two types of pressure profiles, the sinusoidal wave inside the pulse tube and the step wave at the compressor side, have been derived and compared with those of Stirling-type PTCs.

2. Thermodynamic analysis

Fig. 2 shows the compressor and the rotary valve parts of a GM-type PTC with enthalpy flows and entropy flows in it. Consider the control volume at left

☆ This work was partly carried out at Faculty of Applied Physics, Eindhoven University of Technology, P.O. Box 513, NL-5600 MB Eindhoven, The Netherlands.

* Tel.: +86-10-6262-7302; fax: +86-10-6256-4049.
E-mail address: yonglin@cl.cryo.ac.cn (Y.L. Ju).
[1] On leave from Faculty of Applied Physics, Eindhoven University of Technology, P.O. Box 513, NL-5600 MB Eindhoven, The Netherlands.

0011-2275/01/$ - see front matter
PII: S 0 0 1 1 - 2 2 7 5 (0 1) 0 0 1 2 3 - 0

Nomenclature

A	area
C	flow conductance
C_P	heat capacity at constant pressure
C_V	heat capacity at constant volume
H	enthalpy
l	length
L	length of regenerator
$\overset{*}{n}$	molar flow rate
p	pressure
Q	heat
R	ideal gas constant
S	entropy
t	time
t_c	period
T	temperature
u	velocity
U	internal energy
V	volume
$\overset{*}{V}$	volume flow rate
W	work
Z	flow impedance

Greeks

α	ratio of molar flow rate, see Eq. (25)
α_V	volumetric thermal expansion coefficient
ρ	gas density
θ	phase angle
η	viscosity
ζ_V	dissipation rate, see Eq. (11)
ψ	ratio of flow conductance, see Eq. (24)
T	notation for general parameter, see Eq. (56)
Φ	notation for general parameter, see Eq. (59)
Ψ	dissipation parameter, see Eq. (61)
τ	time
ω	angular frequency

Subscripts

0	average value
1	orifice
2	double-inlet
A	amplitude
ac	aftercooler
c	compressor
chx	cold heat exchanger
hhx	hot heat exchanger
H	hot end, high-pressure side
L	cold end, low-pressure side
m	molar quantity
r	regenerator
t	tube, total
V	rotary valve

side, which contains the compressor and the aftercooler. Applying the first law of thermodynamics to this system, the rate of increase of the internal energy $\dot{U}$ is given by

$$\dot{U} = \overset{*}{H}_{mL} - \overset{*}{H}_{mH} + \dot{W} - \dot{Q}_{ac}, \tag{1}$$

where $\dot{Q}_{ac}$ is the heat extracted in the aftercooler. The heat flow and the molar flow rates are positive when they flow into the control volume. We consistently denote flows of extensive quantities with an asterisk * on top (like $\overset{*}{H}$) and the rate of change of extensive quantities of a certain system with a dot on top (like $\dot{U}$). For heat flows and entropy production rates, which are parameters rather than changes in functions of state, we will use the dot notation.

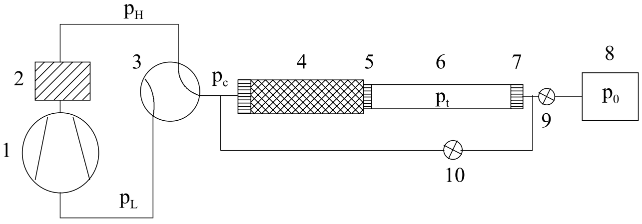

Fig. 1. Schematic diagram of a GM-type PTC: (1) compressor; (2) aftercooler; (3) rotary valve; (4) regenerator; (5) cold end heat exchanger; (6) pulse tube; (7) hot end heat exchanger; (8) buffer; (9) orifice; (10) double-inlet.

The system is in steady state unchanged over one cycle, so the average internal energy $\overline{\dot{U}} = 0$. Eq. (1) becomes

$$\overline{\dot{W}} = \overline{\dot{Q}}_{ac} + \overset{*}{H}_{mH} - \overset{*}{H}_{mL} \tag{2}$$

If the gas is an ideal gas and if it enters and leaves the control system at room temperature T_H, the enthalpy at

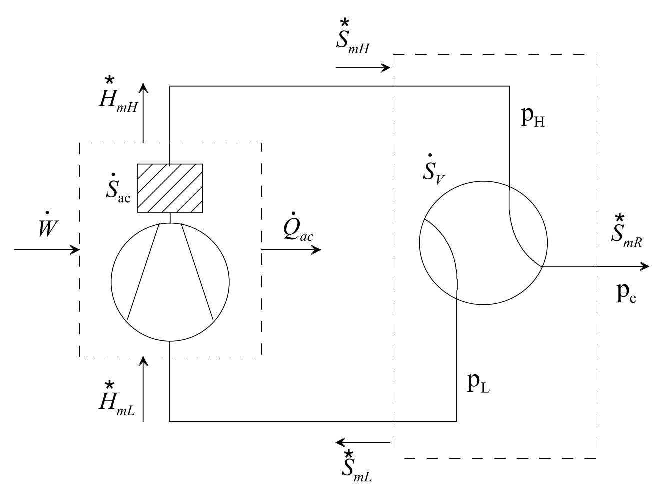

Fig. 2. Control volume of the compressor and the rotary valve parts with enthalpy flows and entropy flows in it. The control volume at the left side contains the compressor and the aftercooler; the right side contains the rotary valve and the connecting tubes.

the low- and the high-pressure sides are equal. Eq. (2) becomes

$$\overline{\dot{W}} = \overline{\dot{Q}}_{\mathrm{ac}}. \tag{3}$$

The second law gives the rate of increase of the entropy $\dot{S}$:

$$\dot{S} = -\frac{\dot{Q}_{\mathrm{ac}}}{T_{\mathrm{H}}} + \overset{*}{S}_{\mathrm{mL}} - \overset{*}{S}_{\mathrm{mH}} + \dot{S}_{\mathrm{ac}}. \tag{4}$$

The system is in the steady state over one cycle, so the average entropy production rate $\overline{\dot{S}} = 0$. Eq. (4) can be rewritten as

$$\frac{\overline{\dot{Q}}_{\mathrm{ac}}}{T_{\mathrm{H}}} = \overline{\overset{*}{S}}_{\mathrm{mL}} - \overline{\overset{*}{S}}_{\mathrm{mH}} + \overline{\dot{S}}_{\mathrm{ac}}. \tag{5}$$

For an ideal gas, we have

$$\overline{\overset{*}{S}}_{\mathrm{mH}} - \overline{\overset{*}{S}}_{\mathrm{mL}} = -\overline{\overset{*}{n}} R \ln\frac{p_{\mathrm{H}}}{p_{\mathrm{L}}}. \tag{6}$$

For the ideal case the compressor works reversibly, $\overline{\dot{S}}_{\mathrm{ac}} = 0$. Combining with Eq. (3), the work needed to compress the gas from the low pressure p_{L} to the high pressure p_{H} sides of the compressor is given by

$$\overline{\dot{W}} = \overline{\overset{*}{n}} T_{\mathrm{H}} R \ln\frac{p_{\mathrm{H}}}{p_{\mathrm{L}}}. \tag{7}$$

Consider the control volume at the right-hand side in Fig. 2, which contains the rotary valve and the connecting tubes. The system is also in steady state over one cycle. The enthalpy flows entering and leaving the system at the compressor side are equal, so there is no net enthalpy change due to this gas flow. Also there are no enthalpy flows entering and leaving this system at the regenerator side. The rotary valve works isothermally, so in the case of an ideal gas no heat has to be extracted from it.

The second law applied to this control system gives

$$\overline{\dot{S}}_{\mathrm{V}} = \overline{\overset{*}{S}}_{\mathrm{mL}} - \overline{\overset{*}{S}}_{\mathrm{mH}} + \overline{\overset{*}{S}}_{\mathrm{mR}}, \tag{8}$$

here $\overline{\dot{S}}_{\mathrm{V}}$ is the irreversible entropy production in the rotary valve. With Eq. (6), we have

$$\overline{\dot{S}}_{\mathrm{V}} = \overline{\overset{*}{n}} R \ln\frac{p_{\mathrm{H}}}{p_{\mathrm{L}}} + \overline{\overset{*}{S}}_{\mathrm{mR}}. \tag{9}$$

For the whole cooler system, the relationship between the cooling power and the work from compressor is given by [14]

$$\overline{\dot{W}} = \left(\frac{T_{\mathrm{H}}}{T_{\mathrm{L}}} - 1\right)\overline{\dot{Q}}_{\mathrm{L}} + T_{\mathrm{H}}\overline{\dot{S}}_{\mathrm{t}}. \tag{10}$$

In this expression $\overline{\dot{S}}_{\mathrm{t}}$ is the sum of the entropy produced by all irreversible processes in the cooler system during one cycle.

The modified thermodynamic loss (dissipation rate) of the rotary valve is defined as

$$\zeta_{\mathrm{V}} = \frac{T_{\mathrm{H}}\overline{\dot{S}}_{\mathrm{V}}}{\overline{\dot{W}}}. \tag{11}$$

The COP of GM-type PTC is given by

$$\mathrm{COP}_{\mathrm{GM}} = \frac{\overline{\dot{Q}}_{\mathrm{L}}}{\overline{\dot{W}}} = \frac{T_{\mathrm{L}}}{T_{\mathrm{H}} - T_{\mathrm{L}}}\left(1 - \frac{T_{\mathrm{H}}\overline{\dot{S}}_{\mathrm{t}}}{\overline{\dot{W}}}\right). \tag{12}$$

3. COP of GM-type PTC (sinusoidal function)

Based on our previous paper [15] about Stirling-type PTC we will derive below the expressions for the COP of GM-type PTC. Assuming the pressure profiles inside the tube are sinusoidal

$$p_{\mathrm{t}} = p_0 + p_1 \cos\omega t. \tag{13}$$

The volume flow rate through the orifice and double-inlet valves can be expressed as

$$\overset{*}{V}_1 = C_1(p_{\mathrm{t}} - p_0) = C_1 p_1 \cos\omega t, \tag{14}$$

$$\overset{*}{V}_2 = C_2(p_{\mathrm{c}} - p_{\mathrm{t}}), \tag{15}$$

where C_1 and C_2 are the flow conductances of the orifice and double-inlet valves, respectively. The gas velocity at the hot end of the tube can be approximated as

$$u_{\mathrm{H}} = \frac{1}{A_{\mathrm{t}}}[\overset{*}{V}_1 - \overset{*}{V}_2] = \frac{1}{A_{\mathrm{t}}}[C_1(p_{\mathrm{t}} - p_0) - C_2(p_{\mathrm{c}} - p_{\mathrm{t}})]. \tag{16}$$

The gas velocity at the cold end of the tube is given by [15]

$$u_{\mathrm{L}} = u_{\mathrm{H}} + \frac{C_{\mathrm{V}}}{C_{\mathrm{P}}}\frac{L_{\mathrm{t}}}{p_0}\frac{\mathrm{d}p_{\mathrm{t}}}{\mathrm{d}t}. \tag{17}$$

We assume here that the filling factor of the regenerator is equal to unity so that the molar flow rate is independent of the length l of the regenerator, so the pressure drop in the regenerator with flow impedance Z_{r} and length L_{r} can be expressed as

$$p_{\mathrm{c}} - p_{\mathrm{t}} = \frac{\overset{*}{n}_{\mathrm{r}} R}{p_0}\frac{Z_{\mathrm{r}}}{L_{\mathrm{r}}}\int_0^{L_{\mathrm{r}}} T\eta\,\mathrm{d}l = \frac{\overset{*}{n}_{\mathrm{r}} R T_{\mathrm{e}}}{p_0 C_{\mathrm{rH}}} \tag{18}$$

with C_{rH} being the flow conductance of the regenerator at room temperature

$$C_{\mathrm{rH}} = \frac{1}{Z_{\mathrm{r}}\eta_{\mathrm{H}}} \tag{19}$$

and the effective temperature

$$T_{\mathrm{e}} = \frac{1}{L_{\mathrm{r}}}\int_0^{L_{\mathrm{r}}} T\frac{\eta}{\eta_{\mathrm{H}}}\,\mathrm{d}l. \tag{20}$$

So the volume flow rate through the regenerator can be simply expressed as

$$V_{\mathrm{L}}^{*} = \frac{\overset{*}{n}_{\mathrm{r}} R T_{\mathrm{L}}}{p_0} = C_{\mathrm{r}}(p_{\mathrm{c}} - p_{\mathrm{t}}) \tag{21}$$

with

$$C_r = C_{rH}\frac{T_L}{T_e}.$$

The gas velocity at the cold end of the tube is

$$u_L = \frac{1}{A_t}C_r(p_c - p_t). \tag{22}$$

Combining with Eqs. (16), (17) and (22), the pressure drop across the regenerator is given by

$$p_c - p_t = \psi p_1(\cos\omega t - \alpha\sin\omega t) \tag{23}$$

with

$$\psi = \frac{C_1}{C_2 + C_r}, \tag{24}$$

here α is the ratio between the components of the molar flow rate which are in-phase and 90° phase with the pressure, and it is defined as

$$\alpha = A_t\frac{C_V}{C_P C_1}\frac{L_t\omega}{p_0}. \tag{25}$$

So the phase angle between the gas velocity and the pressure at the cold end of the tube is given by

$$\theta = \arctan\alpha. \tag{26}$$

Therefore the pressure oscillation at the compressor side is given by

$$p_c = p_0 + p_1\cos\omega t + \psi p_1(\cos\omega t - \alpha\sin\omega t). \tag{27}$$

It can be rewritten as

$$p_c - p_0 = p_1\sqrt{(1+\psi)^2 + (\alpha\psi)^2}\cos(\omega t + \varphi). \tag{28}$$

Therefore, the pressure amplitude

$$p_A = |p_c - p_0| = p_1\sqrt{(1+\psi)^2 + (\alpha\psi)^2}.$$

Combining with Eqs. (21)–(23), the molar gas flow rate through the regenerator is found

$$\overset{*}{n}_r = \frac{p_0}{RT_L}\psi C_r p_1(\cos\omega t - \alpha\sin\omega t). \tag{29}$$

From Eq. (14), the molar gas flow rate through the orifice valve is

$$\overset{*}{n}_1 = \frac{p_0}{RT_H}C_1 p_1\cos\omega t. \tag{30}$$

From Eq. (15), the molar gas flow rate through the double-inlet valve is

$$\overset{*}{n}_2 = \frac{p_0}{RT_H}\psi C_2 p_1(\cos\omega t - \alpha\sin\omega t). \tag{31}$$

Combining with Eqs. (29) and (31), the molar flow rate through the compressor side is

$$\begin{aligned}\overset{*}{n}_c &= \overset{*}{n}_r + \overset{*}{n}_2\\ &= \psi\left(\frac{C_r}{T_L} + \frac{C_2}{T_H}\right)\frac{p_0 p_1}{R}\sqrt{1+\alpha^2}\cos(\omega t + \varphi).\end{aligned} \tag{32}$$

Integrating and averaging Eq. (32) over a half cycle gives

$$\overline{\overset{*}{n}}_c = \frac{\psi}{\pi}\left(\frac{C_r}{T_L} + \frac{C_2}{T_H}\right)\frac{p_0 p_1}{R}\sqrt{1+\alpha^2}. \tag{33}$$

From Eq. (7) the work needed to compress the gas from the low-pressure side to the high-pressure side is given by

$$\overline{\dot{W}} = \overline{\overset{*}{n}}_c RT_H\ln\frac{p_H}{p_L} \approx \overline{\overset{*}{n}}_c RT_H\frac{2p_A}{p_0}. \tag{34}$$

Comparing with Eqs. (28) and (33), Eq. (34) becomes

$$\begin{aligned}\overline{\dot{W}} &= \frac{4}{\pi}\frac{1}{2}C_1 p_1^2\frac{C_r}{C_2 + C_r}\left[\frac{C_2}{C_r} + \frac{T_H}{T_L}\right]\\ &\quad\times\sqrt{(1+\psi)^2 + (\alpha\psi)^2}\sqrt{1+\alpha^2}.\end{aligned} \tag{35}$$

The cooling power can be expressed as [15]

$$\overline{\dot{Q}}_L = \frac{C_r}{C_2 + C_r}\frac{1}{2}C_1 p_1^2. \tag{36}$$

Substituting Eq. (36) into Eq. (35) gives

$$\overline{\dot{W}} = \frac{4}{\pi}\left[\frac{C_2}{C_r} + \frac{T_H}{T_L}\right]\sqrt{(1+\psi)^2 + (\alpha\psi)^2}\sqrt{1+\alpha^2}\,\overline{\dot{Q}}_L. \tag{37}$$

Therefore the COP of GM-type PTC is

$$\begin{aligned}\mathrm{COP_{Sin}} &= \frac{\overline{\dot{Q}}_L}{\overline{\dot{W}}}\\ &= \frac{\pi}{4}\frac{1}{[C_2/C_r + T_H/T_L]\sqrt{(1+\psi)^2 + (\alpha\psi)^2}\sqrt{1+\alpha^2}}.\end{aligned} \tag{38}$$

Since we discuss it in the case of the sinusoidal wave, we use subscript Sin. Substituting Eq. (24) into Eq. (38) gives

$$\begin{aligned}&\mathrm{COP_{Sin}}\\ &= \frac{\pi}{4}\frac{1 + C_2/C_r}{[C_2/C_r + T_H/T_L]\sqrt{(1 + C_2/C_r + C_1/C_r)^2 + (\alpha C_1/C_r)^2}\sqrt{1+\alpha^2}}.\end{aligned} \tag{39}$$

Two ideal cases help us to interpret Eq. (39). The first is the case when there is no double-inlet valve (single-orifice PTC), this means $C_2 = 0$, then Eq. (39) becomes

$$\mathrm{COP_{Sin}} = \frac{T_L}{T_H}\frac{\pi}{4}\frac{1}{\sqrt{(1 + C_1/C_r)^2 + (\alpha C_1/C_r)^2}\sqrt{1+\alpha^2}}. \tag{40}$$

The second case is that if the flow resistance of the regenerator can be neglected, this means $C_r \to \infty$ ($Z_r \to 0$). In this limit

$$\mathrm{COP_{Sin}} = \frac{T_L}{T_H}\frac{\pi}{4}\frac{1}{\sqrt{1+\alpha^2}}. \tag{41}$$

For the Stirling-type PTC, we have (Eq. (79) in [15])

$$\mathrm{COP_{St}} = \frac{\overline{\dot{Q}}_\mathrm{L}}{\overline{\dot{W}}} = \frac{1 + C_2/C_\mathrm{r}}{[C_2/C_\mathrm{r} + T_\mathrm{H}/T_\mathrm{L}][1 + C_2/C_\mathrm{r} + C_1/C_\mathrm{r}(1+\alpha^2)]}. \quad (42)$$

If there is no double-inlet valve, $C_2 = 0$, then Eq. (42) becomes

$$\mathrm{COP_{St}} = \frac{T_\mathrm{L}}{T_\mathrm{H}} \frac{1}{[1 + (C_1/C_\mathrm{r})(1+\alpha^2)]}. \quad (43)$$

If neglecting the flow resistance of the regenerator, $C_\mathrm{r} \to \infty$, we obtain the well-known result for the COP of PTC

$$\mathrm{COP_{St}} = \frac{T_\mathrm{L}}{T_\mathrm{H}}. \quad (44)$$

Comparing Eq. (44) with Eq. (41), we have

$$\frac{\mathrm{COP_{Sin}}}{\mathrm{COP_{St}}} = \frac{\pi}{4} \frac{1}{\sqrt{1+\alpha^2}} < 1. \quad (45)$$

This is due to the dissipation of the rotary valve of GM-type PTC compared to that of Stirling-type PTC. Since the GM-type PTC only differs from the Stirling-type at the compressor side of the cooler, it uses a rotary valve, so that an additional compressor work is needed due to the irreversible entropy production in the rotary valve thereby decreasing the COP of GM-type PTC. From Eq. (25), we have

$$\alpha \propto 1/C_1. \quad (46)$$

Substituting Eq. (46) into Eq. (42), we can find the maximum COP for Stirling-type PTC with respect to $\alpha = 1$. From Eq. (26), this means that the phase angle θ between the gas molar flow rate (velocity) and the pressure at the cold end of the pulse tube is equal to 45°. This phase angle has been detected experimentally [18]. Substituting $\alpha = 1$ into Eq. (45) gives

$$\frac{\mathrm{COP_{Sin}}}{\mathrm{COP_{Si}}} = \frac{\pi}{4} \frac{1}{\sqrt{2}} = 0.55. \quad (47)$$

This means that in the ideal case of no flow resistance in the regenerator, the COP of ideal GM-type PTC is only 55% compared with that of ideal Stirling-type PTC. Substituting $\alpha = 1$ into Eq. (39) and Eq. (42) gives

$$\mathrm{COP_{Sin}} = \frac{0.55(1 + C_2/C_\mathrm{r})}{[C_2/C_\mathrm{r} + T_\mathrm{H}/T_\mathrm{L}]\sqrt{(1 + C_2/C_\mathrm{r} + C_1/C_\mathrm{r})^2 + (C_1/C_\mathrm{r})^2}}, \quad (48)$$

$$\mathrm{COP_{St}} = \frac{(1 + C_2/C_\mathrm{r})}{[C_2/C_\mathrm{r} + T_\mathrm{H}/T_\mathrm{L}][1 + C_2/C_\mathrm{r} + 2C_1/C_\mathrm{r}]}. \quad (49)$$

Figs. 3(a) and (b) show the variations of $\mathrm{COP_{Sin}}/(T_\mathrm{L}/T_\mathrm{H})$ and $\mathrm{COP_{St}}/(T_\mathrm{L}/T_\mathrm{H})$ as a function of the ratio C_2/C_r for the different refrigeration temperatures T_L at the ratio $C_1/C_\mathrm{r} = 0.468$ [15], which is a alue in our experimental set-up [17]. These two figures eliminate the influences of different refrigerator temperatures. Comparing Figs. 3(a) and (b), it is found that $\mathrm{COP_{Sin}}$ is always lower than that of $\mathrm{COP_{St}}$ for the same refrigeration temperature T_L. We also find that the effect of double-inlet on the COP of PTC has distinct improvement at the lower temperature region.

From these two figures, we find that if T_L is lower than a certain critical value T_LC, it is given by

$$T_\mathrm{LC} = \frac{(1+\alpha^2)T_\mathrm{H}}{(1+\alpha^2) + C_\mathrm{r}/C_1}. \quad (50)$$

In the case of $\alpha = 1$, $C_1/C_\mathrm{r} = 0.46$, $T_\mathrm{L} T_\mathrm{LC} \approx 145$ K, there is an optimum value of the flow conductance C_2 of the double-inlet for achieving a maximum COP

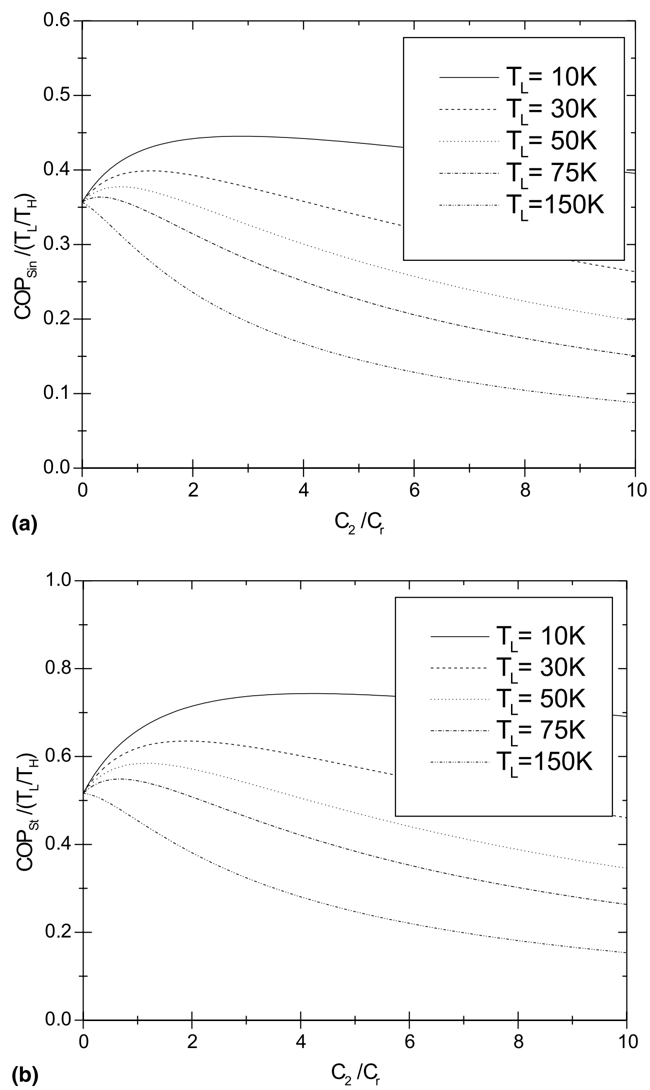

Fig. 3. Variations of $\mathrm{COP}/(T_\mathrm{L}/T_\mathrm{H})$ as the function of the ratio of C_2/C_r for different refrigeration temperatures T_L with $T_\mathrm{H} = 300$ K, $C_1/C_\mathrm{r} = 0.468$. (a) $\mathrm{COP_{Sin}}/(T_\mathrm{L}/T_\mathrm{H})$ and (b) $\mathrm{COP_{St}}/(T_\mathrm{L}/T_\mathrm{H})$.

corresponding to T_L, both for GM-type and Stirling-type PTC. Otherwise, if the refrigeration temperature T_L is above the critical value T_{LC}, the double-inlet is harmful for COP of PTCs.

4. COP of GM-type PTC (step function)

Next we will derive and discuss the COP of GM-type PTC based on the step function of the pressure profiles generated by a GM compressor and a rotary valve. For this kind of pressure profile, an ideal time-dependent pressure profile before the regenerator is shown in Fig. 4, which is divided into four steps. The pressure increases from low pressure p_L to high pressure p_H in step (a) and remains at high pressure during step (b), decreases to low pressure p_L in step (c) and remains constant at low pressure in step (d). It can be expressed as

$$p_c(t) = \begin{cases} p_0 - p_1 + (2p_1/\tau)t & (0 \leqslant t < \tau)\ (1a), \\ p_0 + p_1 & (\tau \leqslant t < \frac{1}{2}t_c)\ (1b), \\ p_0 + p_1 - (2p_1/\tau)(t - \frac{1}{2}t_c) & (\frac{1}{2}t_c \leqslant t < \frac{1}{2}t_c + \tau)\ (1c), \\ p_0 - p_1 & (\frac{1}{2}t_c + \tau \leqslant t < t_c)\ (1d), \end{cases} \tag{51}$$

where p_0 and p_1 are the mean pressure and the pressure amplitude, respectively. For convenience, we assume $\tau \ll t_c$ ($t_c = 2\pi/\omega$ the period), and only consider the period of the first half-cycle where the pressure profile is above the mean average pressure p_0 (similar to that of the part below p_0 for reasons of symmetry and continuity)

$$p_c = p_0 + p_1. \tag{52}$$

Similar to the derivation of Eq. (23), we can obtain the pressure drop across the regenerator

$$p_c - p_t = \psi\left[(p_t - p_0) + \frac{\alpha}{\omega}\frac{dp_t}{dt}\right]. \tag{53}$$

It can be rewritten as

$$\frac{\alpha\psi}{\omega}\frac{dp_t}{dt} + (1+\psi)p_t = p_c + \psi p_0. \tag{54}$$

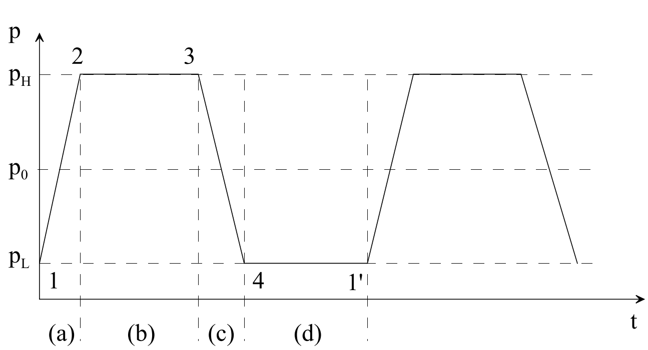

Fig. 4. An ideal step pressure wave at the compressor side generated by means of a GM compressor and a rotary valve.

This equation is exactly the same as Kirchoff's law of a resistance–inductance circuit, its solution is

$$p_t = p_0 + \frac{p_1}{1+\psi} + Ce^{-t/T} \tag{55}$$

with

$$T = \frac{\alpha\psi}{(1+\psi)\omega} = \frac{\alpha}{\omega}\frac{C_1}{C_1 + C_2 + C_r}. \tag{56}$$

Constant C can be determined by the boundary condition. For reasons of symmetry and continuity, the amplitude of the pressure oscillation inside the tube plus the pressure difference between the pressures in the compressor and the tube should be equal to the amplitude of the pressure oscillation at the compressor side, and this gives

$$C = -\frac{2p_1}{1+\psi}\frac{1}{1+e^{-t_c/2T}}. \tag{57}$$

Therefore, the pressure oscillation inside the tube can be expressed as

$$p_t = p_0 + \Phi p_1 \tag{58}$$

with parameter

$$\Phi = \frac{1}{1+\psi}\left(1 - \frac{2e^{-t/T}}{1+e^{-t_c/2T}}\right). \tag{59}$$

Fig. 5 shows a typical pressure profile inside the pulse tube given by Eq. (58).

Similar to the derivation of Eq. (33), a detailed analysis shows the average molar gas flow through the compressor side

$$\overline{\overset{*}{n}}_c = \frac{\psi}{2(1+\psi)}\left[\frac{C_r}{T_L} + \frac{C_2}{T_H}\right]\frac{p_0 p_1}{R}\left[1 + \frac{4T\Psi}{\psi t_c}\right]. \tag{60}$$

Here we introduced a dissipation parameter

$$\Psi = \frac{1 - e^{-t_c/2T}}{1 + e^{-t_c/2T}}. \tag{61}$$

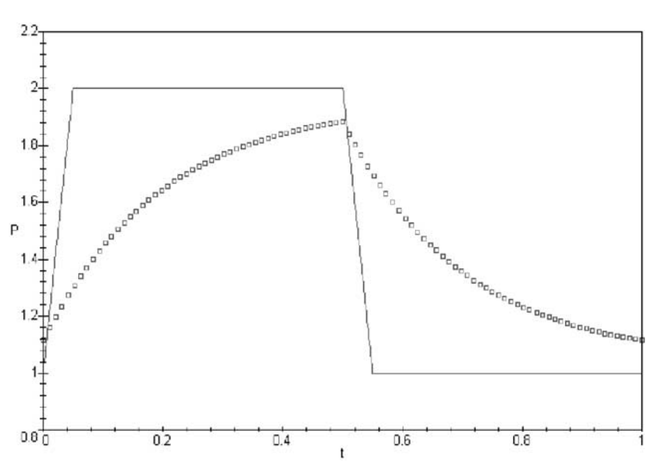

Fig. 5. A typical pressure profile inside the pulse tube (open box), the solid line is the pressure profile at the compressor side.

The maximum $\Psi = 1$ at $T = (\alpha/\omega)\{C_1/(C_1 + C_2 + C_r\} = 0$ means that there is no flow resistance ($C_r \to \infty$) in the regenerator. According to Eq. (34), the work needed from the compressor is given by

$$\overline{\dot{W}} = \frac{C_r}{C_r + C_2} C_1 p_1^2 \frac{1}{(1+\psi)} \left[\frac{T_H}{T_L} + \frac{C_2}{C_r}\right]\left[1 + \frac{4T\Psi}{\psi t_c}\right]. \quad (62)$$

The general entropy production in the orifice can be expressed as [14]

$$T_H \overline{\dot{S}}_{O1} = \frac{1}{t_c/2}\int_0^{t_c/2} V_1^*(p_t - p_0)\,dt = \frac{2}{t_c}\int_0^{t_c/2} C_1(p_t - p_0)^2\,dt. \quad (63)$$

Substituting Eq. (58) into Eq. (63) yields

$$T_H \overline{\dot{S}}_{O1} = \frac{2}{t_c}\int_0^{t_c/2} C_1 p_1^2 \Phi^2\,dt. \quad (64)$$

Integrating Eq. (64) gives

$$T_H \overline{\dot{S}}_{O1} = \frac{C_1 p_1^2}{(1+\psi)^2}\left[1 - \frac{4T\Psi}{t_c}\right]. \quad (65)$$

For simplicity, we first consider here the single-orifice pulse tube, which means that there is no double-inlet $C_2 = 0$, $T = (\alpha/\omega)\{C_1/(C_1 + C_r)\}$ with $\omega t_c = 2\pi$. Eq. (62) becomes

$$\overline{\dot{W}} = \frac{T_H}{T_L}\frac{C_r}{C_1 + C_r} C_1 p_1^2 \left[1 + \frac{C_r}{C_1 + C_r}\frac{2\alpha}{\pi}\Psi\right]. \quad (66)$$

In this case, the cooling power is equal to the entropy production in the orifice (Eq. (44) in [15]). From Eq. (66)

$$\overline{\dot{Q}}_L = \left(\frac{C_r}{C_1 + C_r}\right)^2 C_1 p_1^2 \left[1 - \frac{C_1}{C_1 + C_r}\frac{2\alpha}{\pi}\Psi\right]. \quad (67)$$

Therefore, the COP of GM-type PTC based on step function at the compressor side is

$$\mathrm{COP}_{\mathrm{Step}} = \frac{\overline{\dot{Q}}_L}{\overline{\dot{W}}} = \frac{T_L}{T_H}\frac{C_r}{C_1 + C_r} \times \left\{\left[1 - \frac{C_1}{C_1 + C_r}\frac{2\alpha}{\pi}\Psi\right] \Big/ \left[1 + \frac{C_r}{C_1 + C_r}\frac{2\alpha}{\pi}\Psi\right]\right\}. \quad (68)$$

If there is no resistance in the regenerator, $C_r \to \infty$, so $\Psi = 1$, Eq. (66) is deduced as

$$\mathrm{COP}_{\mathrm{Step}} = \frac{T_L}{T_H}\frac{\pi}{\pi + 2\alpha}. \quad (69)$$

Comparing Eq. (69) to Eq. (41) based on the sinusoidal function yields

$$\frac{\mathrm{COP}_{\mathrm{Sin}}}{\mathrm{COP}_{\mathrm{Step}}} = \frac{\pi + 2\alpha}{4\sqrt{1+\alpha^2}}. \quad (70)$$

For $\alpha = 1$, this is 0.908, which means that the sinusoidal function is slightly more inefficient than the step function. In the ideal case of no double-inlet and no flow resistance in the regenerator, the COP of GM-type PTC is independent of the shape of the pressure waves, which is the same as the result given in [16].

5. Double-inlet pulse tube

For the double-inlet pulse tube the cooling power is given by

$$\overline{\dot{Q}}_L = \frac{C_r}{C_2 + C_r} T_H \overline{\dot{S}}_{O1} = \frac{C_r}{C_2 + C_r}\frac{C_1 p_1^2}{(1+\psi)^2}\left[1 - \frac{4T}{t_c}\Psi\right]. \quad (71)$$

Similarly to Eq. (63), the entropy production in the double-inlet (second orifice) is

$$T_H \overline{\dot{S}}_{O2} = \frac{1}{t_c/2}\int_0^{t_c/2} \overset{*}{V}_2(p_c - p_t)\,dt = \frac{2}{t_c}\int_0^{t_c/2} C_2 p_1^2 (1-\Phi)^2\,dt. \quad (72)$$

Integrating Eq. (72) gives

$$T_H \overline{\dot{S}}_{O2} = \frac{C_1 p_1^2}{(1+\psi)^2}\frac{C_2}{C_1}\left[\psi^2 + \frac{(8\psi + 4)T\Psi}{t_c}\right]. \quad (73)$$

The entropy production in the regenerator is

$$T_H \overline{\dot{S}}_r = \frac{T_H}{T_L}\frac{1}{t_c/2}\int_0^{t_c/2} \overset{*}{V}_r(p_c - p_t)\,dt = \frac{T_H}{T_L}\frac{2}{t_c}\int_0^{t_c/2} C_r p_1^2 (1-\Phi)^2\,dt. \quad (74)$$

Integrating Eq. (74) yields

$$T_H \overline{\dot{S}}_r = \frac{C_1 p_1^2}{(1+\psi)^2}\frac{C_r}{C_1}\frac{T_H}{T_L}\left[\psi^2 + \frac{(8\psi + 4)T\Psi}{t_c}\right]. \quad (75)$$

Summing Eqs. (65), (73) and (75) gives the total entropy production

$$T_H \overline{\dot{S}}_t = \frac{C_1 p_1^2}{(1+\psi)^2}\left[1 - \frac{4T\Psi}{t_c} + \left(\frac{C_2 + C_r T_H/T_L}{C_1}\right) \times \left[\psi^2 + \frac{(8\psi + 4)T\Psi}{t_c}\right]\right]. \quad (76)$$

Eqs. (62), (71) and (76) satisfy the general relationship Eq. (10). Therefore the COP of GM-type PTC based on the step function at the compressor side is found by Eqs. (62) and (71)

$$\mathrm{COP}_{\mathrm{Step}} = \frac{C_r(C_2 + C_r)}{C_t(C_2 + C_r(T_H/T_L))} \times \left\{\left[1 - \frac{C_1}{C_t}\frac{2\alpha}{\pi}\Psi\right] \Big/ \left[1 + \frac{C_2 + C_r}{C_t}\frac{2\alpha}{\pi}\Psi\right]\right\}. \quad (77)$$

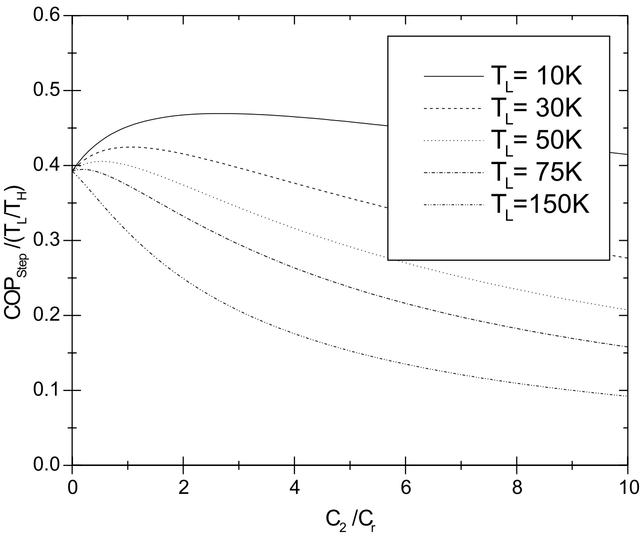

Fig. 6. Variations of $COP_{Step}/(T_L/T_H)$ as a function of the ratio of C_2/C_r for different refrigeration temperatures T_L with $T_H = 300$ K, $C_1/C_r = 0.468$.

With

$$C_t = C_1 + C_2 + C_r \quad (78)$$

is the total flow conduction in the PTC.

An example is given in Fig. 6, which shows the variations of $COP_{Step}/(T_L/T_H)$ as a function of the ratio C_2/C_r for the different refrigeration temperatures T_L at the ratio $C_1/C_r = 0.468$. It can be found that the variations of COP_{Step} are similar to those of COP_{Sin} in Fig. 4(a).

6. Conclusions

By using the first and second laws of thermodynamics, the thermodynamic loss of the rotary valve and the COP of GM-type PTC have been discussed and explained. General expressions of the COP for GM-type PTCs, based on two-type pressure profiles, the sinusoidal wave inside the pulse tube and the step wave at the compressor side have been derived and compared with those of Stirling-type PTC. Result shows that the additional compressor work is needed due to the irreversible entropy productions in the rotary valve thereby decreasing the COP of GM-type PTC. The effect of double-inlet mode on the COP of PTC has distinct improvement in lower temperature region. It is also shown that the COP of GM-type PTC is independent of the shape of the pressure waves in the ideal case of no double-inlet and no flow resistance in the regenerator.

Acknowledgements

The author would like to thank the members of Low Temperature Group, Eindhoven University of Technology, The Netherlands, especially Professor A.T.A.M. de Waele, for many stimulating discussions.

References

[1] Mikulin EI, Tarasov AA, Shrebyonock MP. Low-temperature expansion pulse tube. Adv Cryog Eng 1984;29:629.
[2] Zhu S, Wu P, Chen Z. Double inlet pulse tube refrigerator – an important improvement. Cryogenics 1990;30:514.
[3] Zhou Y, Han Y. Pulse tube refrigerator research. In: Proc. 7th Internat. Cryog. Conf., USA, vol. 7, 1992, p. 147.
[4] Ishizaki Y, Ishizaki E. Experimental study and medullization of pulse tube refrigerator below 80 K down to 23 K. In: Proc. 7th Internat. Cryog. Conf., USA, vol. 7, 1992, p. 140.
[5] Matsubara Y, et al. Four-valves pulse tube refrigerator, JSJS-4, Beijing, vol. 4, 1994, p. 54.
[6] Gardner DL, Jin C, et al. Characterization of 350 Hz thermoacoustic driven orifice pulse tube cryocooler with measurements of the phase of the mass flow and pressure. Adv Cryog Eng 1996;41:1411.
[7] Zhu SW, Kakimi Y, et al. Active-buffer pulse tube refrigerator. In: Proc. CEC16/ICMC, Japan, vol. 16, 1996, p. 291.
[8] Gao JL, Matsubara Y. An inter-phasing pulse tube refrigerator for high refrigeration efficiency. In: Proc. ICEC16/ICMC, Japan, vol. 16, 1996, p. 295.
[9] Chen GB, Qiu LM, et al. Experimental study on a double-orifice two-stage pulse tube refrigerator. Cryogenics 1997;37:271.
[10] Matsubara Y, Gao J. Novel configuration of three-stage pulse tube refrigerator for temperatures below 4 K. Cryogenics 1994;34:259.
[11] Wang C, Thummes G, Heiden C. A two-stage pulse tube cooler operating below 4 K. Cryogenics 1997;37:159.
[12] Wang C, Thummes G, et al. Use of a two-stage pulse tube refrigerator for cryogen free operation of a superconducting Niobium–Tin magnet. In: Proc. ICEC17, vol. 17, 1998, p. 69.
[13] Xu MY, de Waele ATAM, Ju YL. A pulse tube refrigerator below 2 K. Cryogenics 1999;39:865.
[14] de Waele ATAM, Steijaert PP, Gijzen J. Thermodynamical aspects of pulse tube. Cryogenics 1997;37:313.
[15] de Waele ATAM, Steijaert PP, Koning JJ. Thermodynamical aspects of pulse tube II. Cryogenics 1998;38:329.
[16] de Waele ATAM. Optimization of pulse tubes. Cryogenics 1999;39:13.
[17] Steijaert PP. Thermodynamical aspects of pulse-tube refrigerators. Doctoral Thesis, Eindhoven University of Technology, 1999, The Netherlands.
[18] Ju YL, Wang C, Zhou Y. Dynamic experimental investigation of a multi-bypass pulse tube refrigerator. Cryogenics 1997;37:357.