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On the numerical design of a new type of 4 K GM/PT hybrid refrigerators

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Abstract

In this paper we developed and designed, based on theoretical considerations, a new type of 4 K GM/PT hybrid refrigerators. The upper warm stage of the hybrid refrigerator is a typical GM refrigeration cycle, and the cold stage is a pulse tube refrigerator (PTR), on which is thermodynamically coupled the upper warm stage. Four different types of phase shifting assembly: (1) a cold auxiliary piston that is connected to the displacer of the upper GM refrigerator stage, (2) an orifice with reservoir encircled the cold head of the upper stage, (3) an orifice with double-inlet, and (4) in combination with the cold auxiliary piston, orifice and double-inlet, has been proposed and analyzed for the adjustment of the phase shift between the gas mass flow and pressure in the pulse tube. Numerical simulation is performed to understand the unique thermo-physical features, to reveal the time-dependent dynamic parameters and to quantify the overall cooling performance of the hybrid refrigerator. We rely on a one-dimensional, unsteady compressible flow numerical model that is based on a mixed Eulerian–Lagrangian method developed by the present author. The model will be first applied to analyze the cooling performance of the hybrid refrigerator with different types of phase shifting assembly. In what following, it is used to simulate the dynamic parameters in the cold stage of the pulse tube cycle. Next, optimization of the structure parameters and geometrical configuration of the new refrigerator will be presented. Finally, the influences of different hybrid regenerative materials on the cooling capacity of the new hybrid GM/PTR will be also discussed.

Keywords: GM/PT hybrid refrigerator; Numerical study; Liquid helium temperature

1. Introduction

There are a growing number of superconducting devices that require cryogenic cooling at liquid helium temperatures. Due to the introduction and developments of magnetic regenerative materials, the low temperature performance of GM and pulse tube refrigerators (PTRs) has been significantly improved, particularly at 4.2 K-temperature range [1,2]. GM refrigerators are compact and reliable machines and have been used for commercial application in the last decade. Up till now, two-stage GM refrigerators, which provided more than 3.6 W cooling power at 4.2 K with a compressor power of 8 kW [3] and a maximum coefficient of performance (COP) of 6×10^{-4} [4], have been reported. However, the displacer and associated multiple seal rings operated in the low temperature region cause inevitably serious mechanical vibrations, magnetic noise and temperature fluctuations, and possible limitation of lifetime. Furthermore the structure of its cold stage is complex.

The absence of the displacer in the PTRs gives them many potential advantages over GM refrigerators in many applications. The development and improvement in recent years enables PTRs to be an ideal alternative of GM refrigerators in terms of design fabrication, efficiency, reliability, and lifetime for space applications. Since 1994 Matsubara and Gao first obtained a temperature of 3.6 K using a three-stage PTR [5], the lowest temperature of PTRs is now well below the inversion temperature of ^3He and ^4He. By using ^3He as the working fluid the lowest temperature 1.78 K has been achieved [6]. Such temperatures have not yet been realized with conventional GM refrigerators. Up till now, the PTR with 600 mW cooling capacity at 4.2 K on the second stage is now commercially available with an

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Nomenclature

A	area (m^2)
C_P	specific heat of helium gas (J/Kkg)
C_r	specific heat of buck regenerator material (J/Kkg)
D_h	hydraulic diameter of regenerator (m)
f	filling factor
F	heat exchange area per unit volume (m^2/m^3)
H	enthalpy (J)
L	length (m)
$\dot{m}$	mass flow rate (g/s)
p	pressure (Pa)
Pr	Prandtl number
R	ideal gas constant (J/Kkg)
Re	Reynolds number
S_0	stroke of the cold auxiliary piston (mm)
t	time (s)
T	temperature (K)
u	velocity (m/s)
V	volume (m^3)
V_m	specific volume of gas (m^3/kg)
z_r	flow impedance factor ($1/m^2$)
Z_r	flow impedance ($1/m^3$)

Greek letters

α	heat transfer coefficient (W/Km^3)
α_V	volumetric thermal expansion coefficient (1/K)
ρ	gas density (kg/m^3)
η	viscosity (sPa)
θ	Crank angle (degr)
κ	thermal conductivity (W/Km)

Subscripts

0	average
b	buffer
c	cold end
g	gas
h	warm end
H	high pressure
L	low pressure
m	molar quantity
o	orifice
r	regenerator matrix materials
t	pulse tube

additional heat load of 30 W at 65 K on the first-stage with an input power of 4.9 kW [7]. This cooling capacity is already sufficient for several applications, such as for the liquefaction of helium [8] and for cooling small-sized superconducting magnets [9].

However, the COP of 4 K-PTRs is only in the range of 1.5×10^{-4} and is still much lower than that of GM refrigerator, which are being achieved upto 6×10^{-4}. Most of this is due to the absence of solid displacer in the pulse tube, the phase shifter located at room temperature does not lead to an efficient phase shifting (by the so-called "gas piston") of the moving helium at the cold end. The thermodynamic efficiency is reduced due to the work losses within the rotary valve [10]. The other reasons are due to lack of fundamental knowledge of the effects of the salient characteristics of real helium properties, the unique features of thermal properties of magnetic materials, and the existence of helium in the void space of the regenerator on the cooling performance.

In addition to a reasonable cooling power at liquid helium temperatures it is often of interest to provide a high cooling power by the precooling stage. This can be achieved by using a single-stage PTR with a precooling GM refrigerator. The first configuration of such hybrid cooler was realized by Gao and Matsubara [11] in 1994. They achieved 3.5 K with a combination of a two-stage GM and a pulse tube stage using a regenerative tube as a connection between the warm end of the pulse tube and room temperature. Later Gao et al. [12], Tanida et al. [13] studied a hybrid GM/PTR, in which they connected the warm end of the pulse tube to the cold end of the precooling single-stage GM refrigerator. This configuration needs only one gas circuit since the gas for the pulse tube stage is directly taken out of the GM refrigerator. They obtained 370 mW at 4.2 K with no heat load and 200 mW cooling power with an additional heat load of 20 W at 48 K on the first GM stage. Just recently, Liu et al. [14] numerically analyzed the cooling performance of this type of hybrid refrigerator. However, in their model they neglected the axial heat conduction of the gas and regenerative materials, which has some unique effects on the pronounced temperature gradient [15]; furthermore, the pressure drop in the regenerator was also neglected. So that it was impossible to obtain satisfactorily quantitative results.

Recently, von Schneidemesser et al. [16] experimentally studied a hybrid GM/PTR, in which a single-stage GM cooler coupled by thermal connection at a suitable location of the regenerator to a single-stage pulse tube cooler. The advantage of their configuration was the fact that both stages used the same 6 kW compressor but has separate rotary valves by which allow for independent adjustment of the individual frequencies of the pressure wave. This provided a better decoupling of both stages during the optimization procedure for cooling performance. They obtained a minimum temperature of 2.2 K and a cooling power of 0.26 W at 4.2 K at the pulse tube stage with an additional heat load of 50 W at the GM stage at 56 K.

In order to overcome the shortcomings of GM refrigerators and to increase the cooling performance of PTRs, a new type of 4 K GM/PT hybrid refrigerators has been developed and designed, based on theoretical considerations. The upper warm stage of the new hybrid refrigerator is a typical GM refrigeration cycle, the cold stage is a PTR, on which is thermodynamically coupled the upper warm stage. Four different types of phase shifting assembly have been proposed and analyzed for the adjustment of the phase shift between the gas mass flow and pressure oscillations in the pulse tube.

The objective of this paper is to perform numerical simulation to understand the unique thermo-physical features, to reveal the transient parameters and to quantify the overall cooling performance of the hybrid refrigerator. Particular attention is focused on the thermodynamically couple between the warm upper stage of the GM refrigeration cycle and the cold stage of the pulse tube cycle and its phase shifter. As discussed in the subsequent section, we rely on a one dimensional (1-D), unsteady compressible flow numerical model that is based on a mixed Eulerian–Lagrangian method [17], which has been developed from the well-used Eulerian methods [18–21]. The model described in the paper will be first applied to analyze the performance of the hybrid refrigerator with four different types of phase shifting assembly introduced in the last paragraph. In what following, it is used to simulate the dynamic parameters in the cold stage of the pulse tube cycle. Next, optimization of the structure parameters and geometrical configuration of the hybrid refrigerator will be presented. The effects of different hybrid regenerative materials on the cooling capacity of the new hybrid GM/PTR will be discussed in the last section.

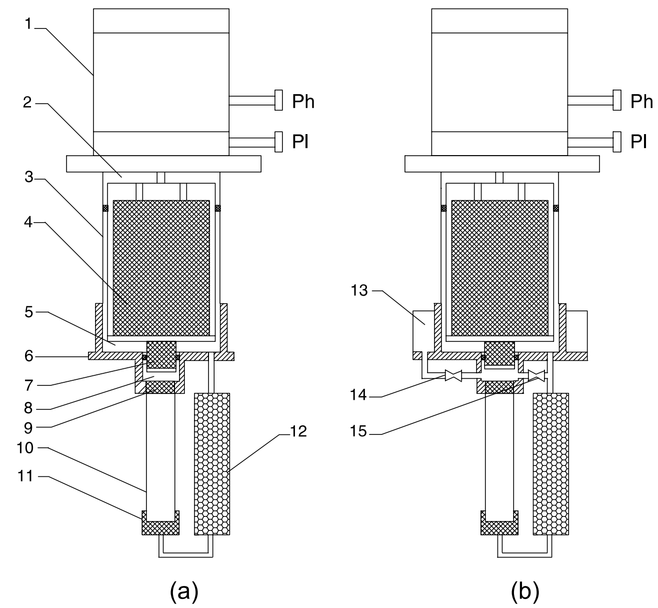

Fig. 1. Schematic diagram of the new type of the hybrid GM/PTR (a) type 1: a cold auxiliary piston connected to the displacer of the upper stage; (b) type 4: in combination with the cold auxiliary piston, orifice and double-inlet (1) rotary valve; (2) hot space; (3) cylinder; (4) main displacer; (5) main cold space; (6) warm stage cold head; (7) cold auxiliary piston; (8) auxiliary cold space; (9) warm end heat exchanger, (10) pulse tube; (11) cold stage cold head; (12) regenerator; (13) reservoir; (14) orifice valve; (15) double-inlet valve.

2. Structure of the new type of refrigerator

Fig. 1 shows the configuration of the new type of the hybrid GM/PTR using one rotary valve connected to one helium compressor. The hybrid GM/PTR has two stages. The upper warm stage of the refrigerator is a typical GM refrigeration cycle, on which the cold stage of a pulse tube cycle is thermodynamically coupled. The upper stage of the GM refrigeration cycle consists of a hot space, a cylinder, a main displacer, a main cold space, a warm stage cold head, a cold auxiliary piston and an auxiliary cold space. The cold stage of the pulse tube cycle consists of a regenerator, a cold stage cold head, a pulse tube, a warm end heat exchanger, which is inserted into the warm stage cold end in an oversized hole along the axis of the cylinder, and a set of phase shifting assembly.

One can see from Fig. 1 that the new hybrid GM/PTR is not just a pulse tube cooler precooled by a GM refrigerator, differs from the configuration described in Ref. [16], where both GM stage and pulse tube stage were operated in parallel driven by one compressor and independent rotary valves for each stage. In this new design, only one rotary valve is integrated in the GM refrigerator and both stages are operated at a fixed frequency. Thus all gas passages are linked, and the pressure, temperature, and mass flow oscillations share the same frequency. The design construction is also a little different from the hybrid refrigerator described in Refs. [12,13], in which the pulse tube warm end was operated at the first stage cold end temperature by use of a thermal bridge. In the present design, we introduced a cold auxiliary piston, which is linkage of the main displacer, and the pulse tube warm end is directly connected to the auxiliary cold space. Therefore, analyses of either GM refrigerators or PTRs are not capable of explaining the work mechanisms of the new hybrid refrigerator. Particular attention will be focused on the thermodynamically couple between the warm stage of the GM refrigeration cycle and the cold stage of the pulse tube cycle and its phase shifter.

Four different types of phase shifting assembly: (type 1) a cold auxiliary piston that is connected to the displacer of the upper warm stage, (type 2) an orifice with reservoir encircled the cold head of the upper stage, (type 3) an orifice with double-inlet, and (type 4) in combination with the cold auxiliary piston, orifice and double-inlet, has been proposed and analyzed of the phase shift between the gas mass flow and pressure oscillation in the pulse tube.

The phase shifting assembly of type 1 is accomplished by means of a cold auxiliary piston that is connected to the displacer of the upper warm stage of the GM refrigeration cycle, as shown in Fig. 1a. The main displacer and cold auxiliary piston are driven by the same rotary valve and their motions have the same phase. Since the warm end of the pulse tube and the regenerator are connected respectively, to the cold auxiliary piston and the main displacer, the relation between the phases of the cold auxiliary piston motion and the pressure wave inside the pulse tube is reversed (180°) in the type 1. The similar controllable system mounted on the hot end of a single-stage PTR was first introduced by Kasuya et al. [22] in 1992, and further modified by Wang et al. [23]. The pulse tube cycle of the hybrid refrigerator can also operate in the orifice model (type 2) or the double-inlet model (type 3) located at cold head of the upper warm stage of the GM refrigeration cycle, as shown in Fig. 1b. In the present configuration, the PTR can be operated in a conventional orifice model if the double-inlet valve 14 closed or be operated in a basic model when both the orifice valve 13 and double-inlet valve 14 are closed. With the goal of enhancing the ability to adjust the phase shifter to generate better cooling performance of the pulse tube cycle, the above phase shifting concepts can be used either independently, or in combination with the cold auxiliary piston through skillful design. As a result, other new methods of the phase shifter, like inertance tube, can be also introduced for further improving the performance of the hybrid refrigerator.

3. Formulation

Fig. 2 illustrates the physical model of the new hybrid GM/PTR that is used for the simulation. As can be seen we focus on the low temperature stage. The junction between the cold space of the upper stage and the low temperature regenerator is taken as the left boundary in

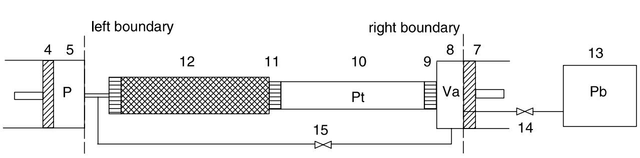

Fig. 2. Physical model of the new type of the hybrid GM/PTR.

the simulation model. The right boundary is the inner surface of the phase shifter (an auxiliary piston or an orifice and reservoir) of the pulse tube. Each method of phase shifters can be embodied by properly dealing with the right boundary condition. For example, when both the auxiliary piston and the orifice are used, the right boundary is movable, and the width of the grid to the left of it is varied accordingly. In addition, the gas velocity through the boundary is determined by the orifice and reservoir. Otherwise, the length of the grid left to the right boundary is constant if the auxiliary piston is not employed, and the gas velocity is zero if the orifice is not applied.

The numerical model used in the present study is an extended version of that discussed in Ref. [17]. Thus, only a brief outline is provided. The formation is based on 1-D, unsteady compressible gas flow and neglect any geometric complexity in regenerator, in order to reduce CPU time and avoid any difficulty involved with the two- or three-dimensional models. Other basic assumptions are as follows:

1. The gas flow is laminar, no turbulence
2. Constant wall temperature at the cold and hot end heat exchangers
3. The regenerative material in the regenerator is incompressible, uniform porosity
4. Boundary and variable permeability effects are neglected
5. The pressure drop inside the pulse tube is neglected.

Under these assumptions, the equations governing the motion of the gas can be reduced to [17].

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0 \tag{1}$$

$$\begin{aligned}(1-f)\rho C_P \frac{\partial T}{\partial t} = & -\rho C_P u \frac{\partial T}{\partial x} - u(1 - T\alpha_V)\frac{\partial p}{\partial x} \\ & + (1-f)T\alpha_V \frac{\partial p}{\partial t} + \alpha F(T_{\mathrm{r}} - T) \\ & + \frac{\partial}{\partial x}\left(\kappa \frac{\partial T}{\partial x}\right)\end{aligned} \tag{2}$$

$$\frac{\partial p}{\partial x} = -\eta z_{\mathrm{r}} u \tag{3}$$

$$\rho = f(T, p) \tag{4}$$

Meanwhile, energy equation of the matrix in the regenerator

$$f\rho_{\mathrm{r}} C_{\mathrm{r}} \frac{\partial T_{\mathrm{r}}}{\partial t} = \alpha F(T - T_{\mathrm{r}}) + \frac{\partial}{\partial x}\left(\kappa_{\mathrm{r}} \frac{\partial T_{\mathrm{r}}}{\partial x}\right) \tag{5}$$

The thermal properties of real helium gas, including density, heat capacity, thermal conductivity, viscosity, and volumetric thermal expansion coefficient are obtained from the wide-ranging real thermal properties of helium of NIST TN-12 Helium Database based on the 32 term Jacobsen equation of state that is a modification of Benedict–Webb–Rubin equation of state [24]. The NIST Helium Database is incorporated within the simulation program. The density, heat capacity and thermal conductivity of the regenerator materials were obtained by curve fitting.

The heat transfer coefficient α of the regenerator is calculated by the correlation equations given in Ref. [20]:

$$\alpha = 0.023\frac{\kappa}{D_h}Re^{0.8}Pr^{0.4} \tag{6}$$

with Reynolds number

$$Re = \rho u D_h/\eta \tag{7}$$

The flow impedance factor z_r of the regenerator has been determined by the experimental data [25]. Here we take $z_r = 31 \times 10^9\ \mathrm{m}^{-2}$.

The gas velocity (or the mass flow rate) is defined as positive if the gas flows from the left side to the right side and as negative for the opposite flow. The gas temperature at the left boundary is given as follows

$$T_1 = T_{in} = \mathrm{const} \quad u \geqslant 0 \tag{8}$$

$$T_1 = T_{gas} \quad u \leqslant 0 \tag{9}$$

The inlet gas temperature T_{in} is maintained constant (the temperature at the cold head of the upper GM refrigerator) and the out temperature depends on the efficiency of the regenerator.

The pressure oscillation at the left boundary is taken as input data. The pressure variations with the crank angle θ of the rotary valve for the present simulation are assumed as harmonic variations

$$p = p_L + \frac{(p_H - p_L)}{2}(1 + \sin\theta) \tag{10}$$

The input pressure oscillation can be also obtained from experimental measurements.

The right boundary in the simulation is either the surface of the cold auxiliary piston or the gas buffer or in combination of them. The gas temperature at the right boundary is as follows

$$T_N = T_{gas} \quad u \geqslant 0 \tag{11}$$

$$T_N = T_b = \mathrm{const} \quad u \leqslant 0 \tag{12}$$

It is reasonable to assume constant wall temperature at the warm and cold ends of heat exchangers of the pulse tube

$$T_{hi} = T_H = \mathrm{const} \tag{13}$$

$$T_{ci} = T_c = \mathrm{const} \tag{14}$$

The process in the gas buffer is regards as isothermal and adiabatic. The gas mass flow rate through the right boundary (the mass flow rate is zero if the orifice is not employed) is determined from the pressure differences [26].

$$\dot{m}_o = \frac{p_t - p_b}{\eta Z_o}\frac{p_b}{RT_b} \tag{15}$$

where Z_o is the flow impedance of the orifice valve and has been determined by the experimental data [25].

The motion of the cold auxiliary piston for phase shift is given by

$$S = S_d + \frac{S_0}{2}[1 + \cos(\theta - \pi + \theta_0)] \tag{16}$$

where S_0 is the stroke of the cold auxiliary piston, S_d is the dead gap, which is generally taken as 0.5 mm, θ_0 is the initial phase angle between the cold auxiliary piston and the main displacer and is assumed as zero without special note in the paper.

4. Numerical methods and procedure

The above set of equations is non-linear and unsteady, and can only be solved by numerical integration. To solve these equations a combination of the Eulerian and Lagrangian method has been developed by the present author. Additional details regarding the numerical constructions and the implementations are given in Ref. [17].

As discussed in Ref. [17], one of the key features of the present model is that it enables us to follow the tracks of gas particles in the pulse tube and obtain the gas temperature profiles and the position of the gas elements traveling with pressure oscillations inside the pulse tube through explicitly analytical expressions. At the interface between the regenerator 11 and the pulse tube 9 the temperature and velocity distributions within the cold end heat exchanger 10 are assumed to match smoothly with the tracks of the gas particles entering and leaving the pulse tube, which is advanced in time by the Lagrangian translation. This enables us to avoid any numerical iteration to reduce CPU time, and any numerical false diffusion to increase the calculation accuracy.

Numerical simulation of the governing equations by Eulerian method is based on the finite difference methodology. The regenerator is divided into many subsystems and each subsystem is considered as a uniform system, which can exchange heat and mass with the surroundings through its boundaries. The regenerator domain discretization is made using a control volume approach by internal node method as described by Patankar [27]. The governing equations are solved by using the explicit scheme, thus the temperature, velocity and pressure are updated in an explicit fashion. Spatial derivatives are approximated using an upwind second-order difference formula [20,21], to ensure the transportive properties of the discretization equations. The code has been tested and validated by comparing numerical predictions with experimental data.

5. Predicted results and discussions

It is well known that the right conditions for cooling to occur in the pulse tube cooler are that the amplitude of the gas mass flow and pressure oscillations must be large enough and the phase shift between the mass flow rate and pressure must be appropriate to carry the heat away (by enthalpy flow) from the cold point (cold end heat exchanger) to the hot point (hot end heat exchanger). The size and geometry of each component of the pulse tube cooler and the phase shifting assembly work together to control the amplitude and phase shifts between the gas mass flow and pressure inside the pulse tube to achieve the maximum cooling power.

Therefore, the model described in the previous section is first applied to analyze the cooling performance of the new type of hybrid GM/PTR with four different types of phase shifting assembly: the cold auxiliary piston (type 1), the orifice (type 2), the orifice and the double-inlet (type 3), and in combination with the cold auxiliary piston and the orifice and the double-inlet (type 4). Next it is used to optimize the geometrical configuration of the new type refrigerator. The major operating conditions are as follows: the first stage temperature $T_H = 40$ K, the second stage is $T_c = 4.2$ K, the high and low pressures are $p_H = 2.0$ MPa and $p_L = 0.6$ MPa, respectively, and the operating frequency is $f = 1$ Hz. Without special note the main structure parameters are given in Table 1. The regenerator is filled with Er_3Ni grain, and its filling factor is assumed as 0.6.

The cooling power as a function of the lowest temperature with the four types of phase shifters is shown in

Table 1
Main structure parameters of the two-stage pulse tube cooler

Components	
Regenerator	
Inner diameter	29 mm
Length	160 mm
Materials	Er_3Ni grain
Cold heat exchanger	
Diameter	20 mm
Length	15 mm
Pulse tube	
Inner diameter	15 mm
Length	160 mm
Cold auxiliary piston	
Diameter	22 mm
Stroke	25 mm
Gas buffer volume	1000 cm^3

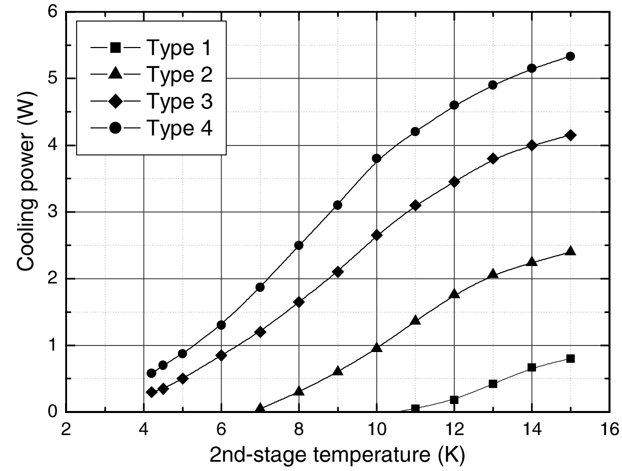

Fig. 3. Predicted cooling power versus the cold end temperature with the four types of phase shifter.

Fig. 3. It is obvious that the cooling performance of the refrigerator with only the cold auxiliary piston (type 1) or the orifice as its phase shifter is not as good as expected. The cooling temperatures of the refrigerator with the phase shifting assembly of type 1 and type 2 cannot reach 4.2 K temperature. Nevertheless, the cooling performance can be greatly improved in the case when the orifice is combined with the double-inlet (type 3) or the cold auxiliary piston is combined with the orifice and the double-inlet (type 4) as its phase shifter. The best phase shifting assembly in our simulation is that the cold auxiliary piston combines with the orifice and the double-inlet (type 4). In this case the new refrigerator can provide a net cooling power of 0.58 W at 4.2 K.

The effects of the different phase shifting assembly on the cooling performance of the hybrid refrigerator can be better understood from the transient behavior of the dynamic parameters of the pressure, mass flow rate and temperature at warm and cold ends of the pulse tube.

Fig. 4a shows the dynamic pressures inside the pulse tube and the time-variations of the volume of the auxiliary piston in one cycle with the phase shifter of the type 4. The phase difference between the motion of the cold auxiliary piston and the pressure wave inside the pulse tube is about 90°. The time-variations of the mass flow rates and the gas temperatures at the warm and cold ends of the pulse tube with time in one cycle with the type 4 phase shifter are plotted in Fig. 4b and c, respectively. The half-cycle averaged mass flow rate at the cold end is larger by about 60% than that at warm end of the pulse tube, which is a unique feature for 4 K refrigerators and has been discussed in Ref. [19]. The amplitudes of the gas temperature fluctuation at the warm and cold ends of the pulse tube are around 5 and 0.5 K, respectively. Much smaller fluctuation at the cold end of the pulse tube originates from the fact that the

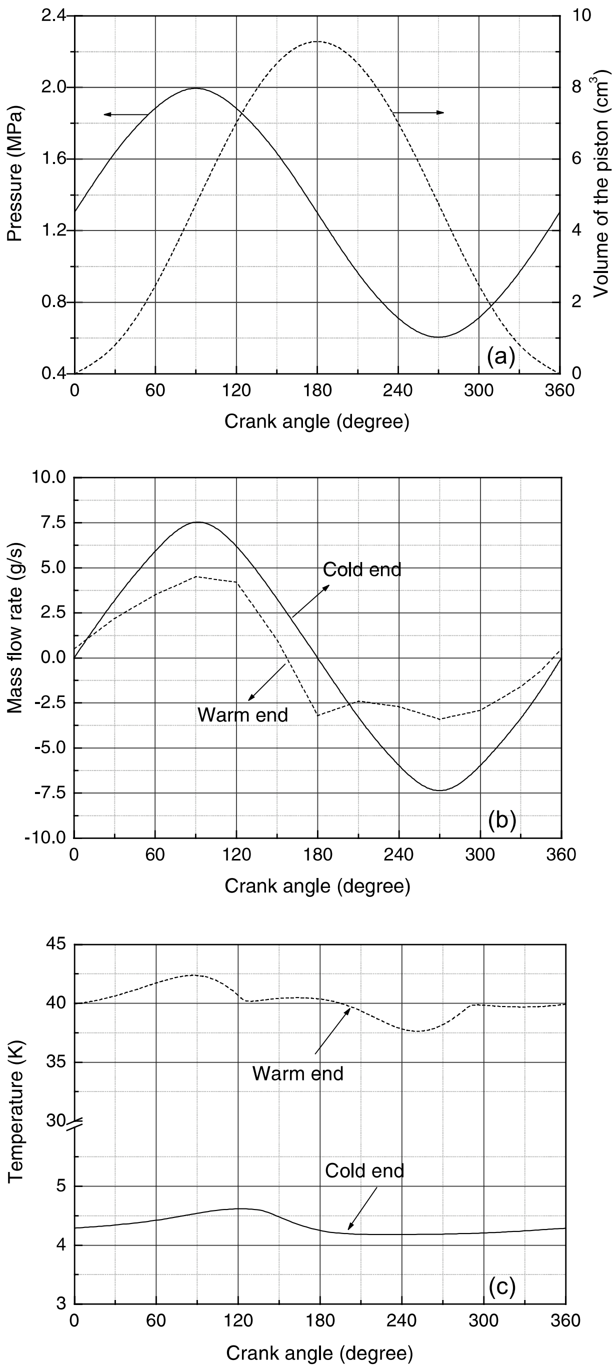

Fig. 4. Variations of dynamic parameters with time in one cycle (a) pressure waves and volume of the auxiliary piston; (b) mass flow rate; (c) gas temperature.

isothermal compressibility of helium is very small around 4.2 K. This also implies that the different types of phase shifters located at warm end of the pulse tube do not lead to an efficient phase shifting of the moving helium at the cold end.

The transient gas temperature distributions along the regenerator length at three different crank angles θ of the rotary valve are illustrated in Fig. 5. It shows that a

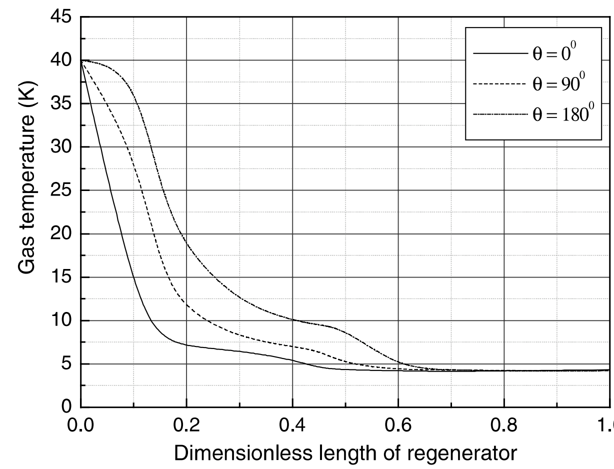

Fig. 5. Variations of gas temperature profile along the regenerator length.

region of almost constant temperature appears near the cold end of the pulse tube. The temperature profiles are rather flat in the 40% of the regenerator length near the cold end. About 80–90% of the temperature drop is concentrated in the 40% of the regenerator length near the hot end. The author reasons that this is caused by the larger heat capacity of helium than that of regenerator materials in the low-temperature range.

Next we will draw our attention to optimize the geometrical configuration of the new type refrigerator. Figs. 6–9 show the effects of the regenerator length, the volume ratio of the regenerator to the pulse tube, the volume ratio of the cold auxiliary piston to the pulse tube, and the initial phase angle of the cold auxiliary piston, respectively, on the cooling power at 4.2 K of the new hybrid refrigerator.

One can see from Fig. 6 that the cooling power increases rapidly with increasing length of the regenerator and then remains almost constant after reaching a

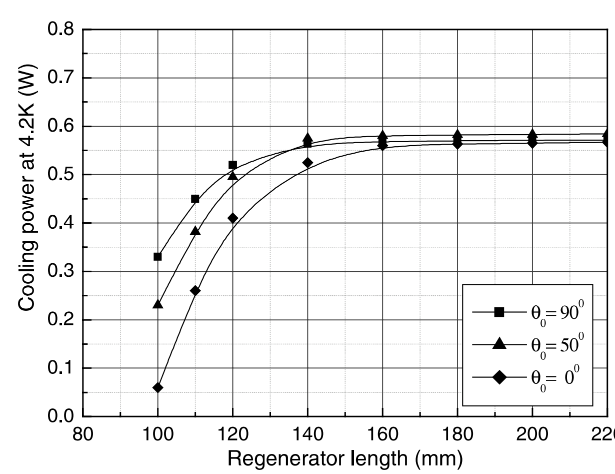

Fig. 6. Predicted cooling power as a function of regenerator length with three initial angles of the cold auxiliary piston.

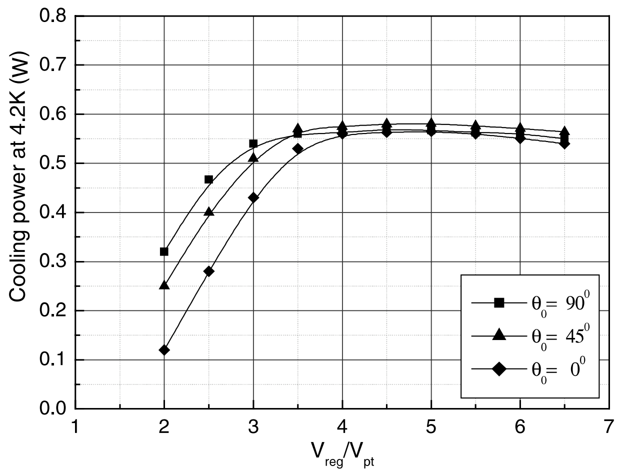

Fig. 7. Predicted cooling power versus the volume ratio of regenerator to pulse tube.

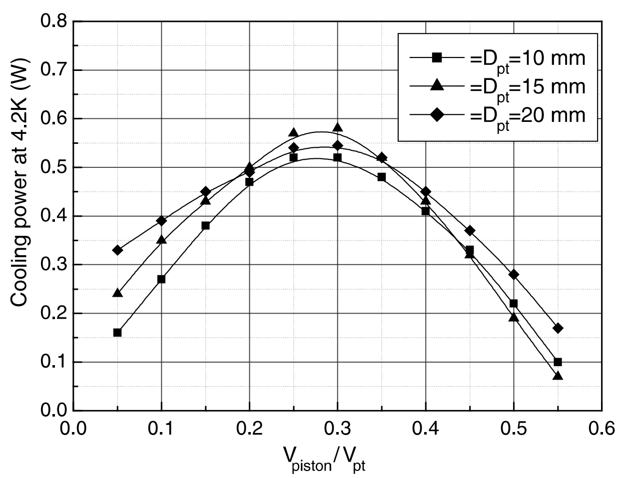

Fig. 8. Predicted cooling power versus the volume ratio of cold auxiliary piston to pulse tube.

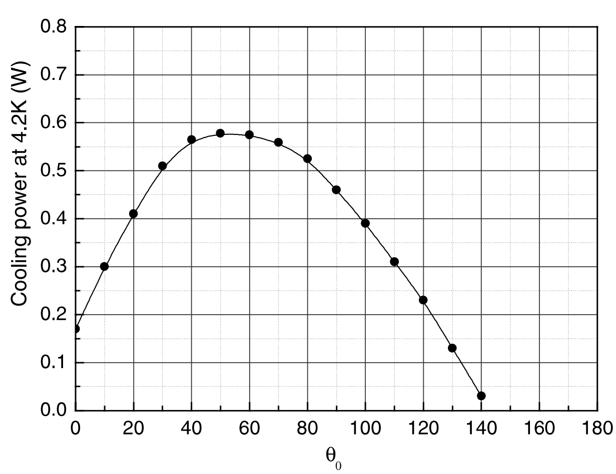

Fig. 9. Predicted cooling power in terms of the initial angle of the cold auxiliary piston.

maximum value. The most suitable length for the regenerator is about 160 mm in the present configuration. The initial phase angle θ_0 between the cold auxiliary piston and the main displacer on the cooling power has considerable influence at short regenerator length where the cooling power increases with increasing θ_0. When the regenerator length become longer than 160 mm the effect of the θ_0 on the cooling power is negligible.

Fig. 7 shows the influences of the volume ratio of the regenerator to the pulse tube (keeping the length constant and altering the diameter of the regenerator) on the cooling power at 4.2 K at three different initial phase angles. The tendency is similar with that in Fig. 6, while the cooling power decreases slightly with increasing the specific volume.

The effects on the cooling power at 4.2 K of the volume ratio of the cold auxiliary piston to the pulse tube (keeping the stroke constant and altering the diameter of the piston) at three different diameters of the pulse tube are illustrated in Fig. 8. The refrigerator with the pulse tube diameter of 15 mm can achieve a slightly larger refrigeration power than that of others. The larger the pulse tube diameter, the smoother the influence of the specific volume on the cooling power of the hybrid refrigerator.

From Figs. 7 and 8, one can determine the optimum dimension for the new hybrid refrigerator. The most suitable volume ratio of the regenerator to the pulse tube is around 4–5 and the optimal volume ratio of the cold auxiliary piston to the pulse tube is about 0.3–0.35.

The relation between the initial phase angle θ_0 of the cold auxiliary piston and the cooling power at 4.2 K of the hybrid refrigerator is shown in Fig. 9. It shows that there is an optimum initial phase angle of 45°–50° for obtaining the maximum cooling power at 4.2 K. It also demonstrates that the effect of θ_0 is slight in the range of 40°–70°. It should be pointed out that the initial phase angle between the cold auxiliary piston and the main displacer with the phase shifting assembly of type 1 (only by the cold auxiliary piston) is reversed, as given by Eq. (16).

From above figures, one can find that the volume ratio of the cold auxiliary piston to pulse tube and the initial phase angle θ_0 between the cold auxiliary piston and the main displacer are of two most important parameters which need to be well designed for achieving the better performance of the hybrid refrigerator. It is worth noting that the optimum values of the two parameters depend on the structure form and the size of the refrigerator.

6. Effects of multi-layered regenerator materials

Because of the unique features of the low temperature regenerator at 4.2 K [16,18,19], the specific heat of regenerator materials in the cold stage is the key factor to further improve the cooling performance. Although many magnetic materials (like Er_3Ni, $ErNi_2$, ErNi, $ErCo_2$, $HoCu_2$, $ErNi_{0.9}$ $Co_{0.1}$, etc) have larger specific heat at their magnetic phase transition, the width of the specific heat peak is often narrow and it is impossible, at present, to cover the specific heat peak of helium by a single type of magnetic material. Fortunately, the different magnetic phase transitions of these magnetic materials with different temperature dependence of the specific heat allow the choice of optimal regenerative materials in form of multi-layered hybrid regenerator. The concept of multi-layer regenerator was developed to compensate the above weak point of single magnetic material, thereby to increase regenerator efficiency. Many theoretical and experimental results have shown that larger cooling capacity could be achieved by using multi-layered hybrid regenerator materials at 4.2 K.

In order to further improve the refrigeration performance of the new hybrid GM/PTR, we focus below on the effects of different multi-layered hybrid regenerative materials in the regenerator of the pulse tube cycle to achieve larger cooling power. The main structural parameters considered here are the same as shown in Table 1. Eight cases (4 cases of two components and 4 cases of three components) of regenerator arrangements and materials are studied in the present work. The volumetric ratios of two components from the warm end to the cold end in the regenerator of the pulse tube cycle are 50% and 50%, and those of three components are 30%, 30% and 40% in sequence. Table 2 and Table 3 indicate the effects of four cases of two components and four cases of three components on the cooling performance, respectively.

So far, the optimum combinations of two components for the regenerator are Er_3Ni and $ErNi_{0.9}Co_{0.1}$. The optimum combinations of three components studied in the present simulation in regenerator are lead, Er_3Ni and $ErNi_{0.9}Co_{0.1}$. The optimum volumetric ratios of those three materials from warm end to the cold end of the regenerator are around 30%, 30% and 40% in sequence. In this case, the predicted cooling power at 4.2 K of the refrigerator is about 0.88 W, which is larger by 0.3 W than that of the single regenerative materials of Er_3Ni grain. It should be noted that the accurate gas temperature profile inside the regenerator is a necessary condition for analyzing the multi-layered structure of the regenerative materials.

Table 2
Effects of two hybrid regenerator materials on cooling performance

Components	Q_c (W)	T_r/T_c (K)	Regenerator efficiency (%)
$Pb + Er_3Ni$	0.575	4.131/4.082	96.34
$Pb + ErNi_2$	0.801	4.075/4.043	97.21
$Er_3Ni + ErNi_2$	0.821	4.073/4.039	98.35
$Er_3Ni + ErNi_{0.9}Co_{0.1}$	0.835	4.071/4.038	98.62

T_r is the averaged gas temperature entering and leaving the regenerator during one cycle. T_c is the averaged gas temperature inside the cold heat exchanger during one cycle.

Table 3
Effects of three hybrid regenerator materials on cooling performance

Components	Q_c (W)	T_r/T_c (K)	Regenerator efficiency (%)
$Pb + Er_3Ni + ErNi$	0.694	4.105/4.062	97.02
$Pb + Er_3Ni + ErNi_2$	0.835	4.074/4.039	97.88
$Pb + Er_3Ni + ErNi_{0.9}Co_{0.1}$	0.871	4.063/4.034	98.40
$Er_3Ni + ErNi + ErNi_{0.9}Co_{0.1}$	0.880	4.061/4.032	98.86

T_r is the averaged gas temperature entering and leaving the regenerator during one cycle. T_c is the averaged gas temperature inside the cold heat exchanger during one cycle.

7. Conclusions

A new type of 4 K GM/PT hybrid refrigerator was developed and designed based on numerical simulation. Four different types of phase shifting assembly was proposed and analyzed for adjustment of the phase shift between the gas mass flow and pressure in the pulse tube cycle. Numerical simulation was performed to understand the unique thermo-physical features, to reveal the dynamic parameters and to quantify the overall cooling performance of the hybrid refrigerator. Some useful results was obtained as follows:

(1) The cooling performance of the refrigerator with only the cold auxiliary piston or the orifice as its phase shifter is not as good as expected. The reason is that these structures do not lead to an efficient phase shifting. However, the performance is greatly improved in the case when the orifice is combined with the double-inlet or the cold auxiliary piston is combined with the orifice and the double-inlet as its phase shifter.

(2) The initial phase angle between the cold auxiliary piston and the main displacer and the volume ratio of the cold auxiliary piston to pulse tube are of two most important parameters for achieving the better performance of the refrigerator. The optimum initial phase angle is about 45°–50°. The optimal volume ratio of the regenerator to the pulse tube and of the cold auxiliary piston to the pulse tube is around 4–5 and 0.3–0.35, respectively.

(3) The optimum combinations of three components for the regenerator are lead, Er_3Ni and $ErNi_{0.9}Co_{0.1}$. The optimum volumetric ratios of those three materials from warm end to the cold end of the regenerator are around 30%, 30% and 40% in sequence.

(4) More work should be carried out to analyze the attribution on the performance of the refrigerator of the compressed heat generated inside the regenerator, DC gas flow, additional heat losses due to transition to

turbulence, which are not incorporated into the present paper. Experimental evaluation is now underway.

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