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Nonideal-gas effect in regenerators

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Abstract

Reformulating the theory of pulse-tube operation from the ideal-gas situation to a real gas, changes the properties drastically. The nonideal-gas properties have a profound effect on the energy balance in the regenerator and on the expression for the cooling power. The temperature profiles in the regenerator are strongly affected by the thermal properties of the fluid. We show that, for large flow rates, the dissipation in the regenerator is proportional to the flow rate and cannot be reduced by reducing the thermal conduction coefficient or the regenerator geometry. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Thermodynamics; Regenerators; Pulse tubes

Nomenclature

A	area (m^2)
C	heat capacity (J/K mol)
E	energy flow (J/s)
L	length (m)
H	enthalpy (J)
H_p	Eq. (3) (m^3/mol)
n	moles (mol)
n_p	Eq. (6) (Pa mol/s)
O	orifice
p	pressure (Pa)
Q	heat (J)
S	entropy (J/K)
t	time (s)
T	temperature (K)
v	velocity (m/s)
V	volume (m^3)

Greeks

α_V	volumetric expansion coefficient (1/K)
κ	thermal conductivity (W/Km)

Subscripts

c	conduction
H	high temperature
ir	irreversible
L	low temperature
m	molar
O	maximum value
p	pressure
r	regenerator
t	tube

1. Introduction

This paper is a continuation of our work on the thermodynamical aspects of pulse tubes [1–4]. A schematic diagram of a pulse tube refrigerator is given in Fig. 1. So far we have mainly assumed that the working fluid is an ideal gas. This is certainly invalid for the temperature ranges which can be reached nowadays [5,6]. Therefore, in this paper, we will drop the condition that the working fluid is an ideal gas. The operation of heat engines working with real fluids around the critical point has been treated by Allen et al. [7].

We will assume that the flow resistance of the regenerator is zero (so the pressure varies with time, but not with position), that the thermal contact between the fluid and the matrix is perfect, and that the heat capacity of the regenerator matrix is very large (so the gas temperature is constant in time but varies with position). Finally, we assume that the void volume in the regenerator is very small (so the molar flow rate, or the mass flow rate, is the same everywhere in the regenerator). Although these idealizations will limit the results of our treatment considerably they will help in forming a clear

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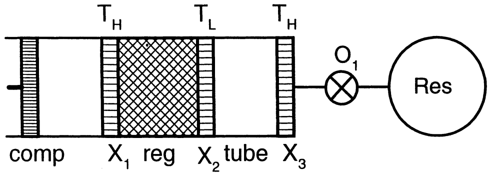

Fig. 1. Schematic diagram of a pulse-tube refrigerator consisting of a compressor (comp), three heat exchangers (X_1, X_2, X_3), a regenerator (reg), a tube, an orifice, and a buffer. In this paper we concentrate on the regenerator in which half of the cycle heat is stored from the gas and in the other half of the cycle heat is given of to the gas.

picture of the essentials of the phenomena, which might otherwise be obscured by complicated mathematics.

In a previous paper [3] we have shown that the thermal conduction plays an essential role in determining the temperature profile in the regenerator. First we discuss the consequences of the real-gas properties on the temperature profile of regenerators. Furthermore we derive expressions for the cooling power and the entropy production.

2. General expressions

The molar enthalpy H_m, treated as a function of the molar entropy S_m and the pressure p, satisfies

$$dH_m = T\, dS_m + V_m\, dp, \tag{1}$$

where T is the temperature and V_m the molar volume. If we consider H_m as a function of T and p, this can also be expressed as

$$dH_m = C_p\, dT + H_p\, dp \tag{2}$$

with

$$H_p = (1 - T\alpha_V)V_m, \tag{3}$$

and C_p the heat capacity at constant pressure, and α_V the volumetric thermal expansion coefficient. As an example, in Fig. 2, H_p of ^{4}He is given of a function of T for a pressure of 15 bar. If ^{4}He would be an ideal gas H_p would be zero. In reality H_p is not zero even at room temperature. The enthalpy flow, which plays a key role in the analysis of regenerators, is given by

$$\overset{*}{H} = \overset{*}{n}_r H_m, \tag{4}$$

where $\overset{*}{n}_r$ is the molar flow rate in the regenerator (positive from left to right in Fig. 1). We split H_m in a time average part and a varying part $H_m = \overline{H_m} + \delta H_m$ and use the fact that $\overline{\overset{*}{n}_r} = 0$. Furthermore, in our case, $\delta T = 0$ so with Eq. (2) we get for the average enthalpy flow

$$\overline{\overset{*}{H}} = H_p \overline{\overset{*}{n}_r \delta p}. \tag{5}$$

We introduce the quantity n_p, which is defined as

$$n_p \equiv \overline{\overset{*}{n}_r \delta p}. \tag{6}$$

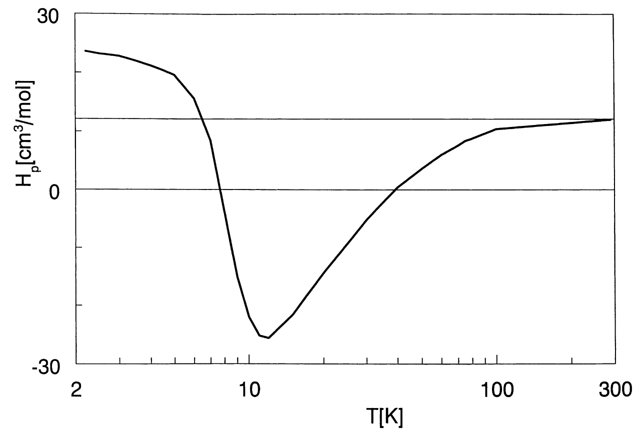

Fig. 2. $H_p - T$ dependence of ^{4}He at 15 bar. The horizontal line is drawn at $H_p = 12$ cm^3/mol. If $T < 6.4$ K, $H_p > 12$ cm^3/mol.

With this notation

$$\overline{\overset{*}{H}} = H_p n_p. \tag{7}$$

The value of n_p is determined by the molar flow and the pressure amplitude. In general n_p is a function of the length coordinate l which runs from the hot end of the regenerator ($l = 0$) to the cold end ($l = L_r$). However, when the void volume is very small, $\overset{*}{n}_r$ only is a function of time and not of l. If the flow resistance is zero also δp is only a function of time. In this case n_p is constant. So, for given flow rate and pressure amplitude, the enthalpy flow in the regenerator is proportional to H_p. [1] As H_p is a function of temperature this means that the enthalpy flow varies in the regenerator. This has important consequences.

3. T-profile in the regenerator

3.1. Energy conservation

The first law demands that in the steady state the total average energy flow

$$E_r = \dot{Q}_c + \overline{\overset{*}{H}} \tag{8}$$

is constant. The heat conduction is given by

$$\dot{Q}_c = -\kappa A_r \frac{dT}{dl}, \tag{9}$$

where κ is the coefficient of thermal conductivity and A_r is the area of the cross-section of the regenerator. So

$$-\kappa A_r \frac{dT}{dl} + n_p H_p = E_r. \tag{10}$$

[1] Stricktly speaking the equations given above only hold for small pressure amplitudes, but H_p is not a strong function of pressure and we will use the expressions also for higher pressure amplitudes.

This is a differential equation determining the temperature profile in the regenerator. For a given temperatures at the hot end (T_H) and at the cold end (T_L) the value of E_r is determined by the condition

$$\int_{T_L}^{T_H} \frac{\kappa A_r \, dT}{E_r(n_p) - n_p H_p} = L_r, \tag{11}$$

where L_r is the length of the regenerator. This expression is an implicit relation giving the energy flow E_r as a function of n_p. Once E_r is known the temperature profile $T(l)$ can be found from the relation

$$\int_{T}^{T_H} \frac{\kappa A_r \, dT}{E_r - n_p H_p} = l. \tag{12}$$

3.2. Constant κA_r

Even though the thermal conductivity and the cross-section usually are far from constant we will discuss the case of constant κA_r for didactical reasons.

If $n_p = 0$, Eq. (11) gives

$$\frac{\kappa A_r (T_H - T_L)}{L_r} = E_r(0). \tag{13}$$

The energy flow is just the heat conduction and Eq. (12) results in a linear temperature profile. The value of $E_r(0)$ is the characteristic energy-flow scale of this problem.

If $n_p > 0$ the profile is not linear even if κA_r is constant. In regions where H_p is small the heat flow is large so the temperature gradient is large and vice versa. This is illustrated in Fig. 3. At about 10 K H_p is negative and $\dot{Q}_c$ is about its maximum value. In the temperature ranges below 5 K or above 100 K H_p is relatively large, so $\dot{Q}_c$ is small.

If n_p is increased from zero both E_r and $n_p H_p$ increase (Fig. 4). However, $n_p H_p$ increases faster than E_r. For large flow rates E_r becomes approximately equal to the maximum value of $n_p H_p$ in the $[T_L, T_H]$ interval

$$E_r \approx \max(n_p H_p) \equiv n_p H_O. \tag{14}$$

If this is the case the conducted heat becomes very small and the temperature profile is flat in the region of the regenerator where H_p is a maximum.

In Fig. 1 it can be seen that for 2.2 K $\leqslant T_L \leqslant$ 6.4 K the maximum of H_p in the $[T_L, T_H]$ interval is at T_L, while for 6.4 K $\leqslant T_L \leqslant$ 300 K the maximum of H_p is at T_H. This gives rise to a distinctly different behavior of the solutions of Eqs. (11) and (12) if $T_L \leqslant 6.4$ K or $T_L > 6.4$ K (Figs. 4 and 5).

The thermal conductivity of regenerator material is typically an order of magnitude less then the bulk value [8]. An example is calculated taking $\kappa = 1$ W/K m, $A_r = 8$ cm^2, and $L_r = 40$ cm, giving $\dot{Q}_{c0} = \kappa A_r T_H / L_r = 0.6$ W, which is the characteristic energy flow of this system. The nonideal-gas effects are noticeable if the variations in $n_p H_p$ are comparable with $\dot{Q}_{c0}$. With variations in H_p of order 10 cm^3/mol this means that pronounced effects of the flow are to be expected if n_p is on the order of $0.6/(10 \times 10^{-6}) = 60$ kPa mol/s. With a pressure amplitude of 300 kPa the molar amplitude is 0.2 mol/s. For flows below this value the thermal conduction dominates, for flows above this value the flow phenomena dominate.

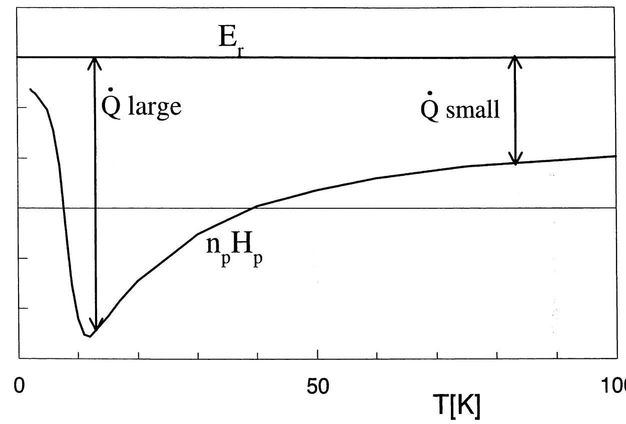

Fig. 3. Variation of the heat flow $\dot{Q}$ in the regenerator. In regions where H_p is small $\dot{Q}$ is large, and in regions where H_p is large (as around room temperature and at temperatures below 4 K) $\dot{Q}$ is small.

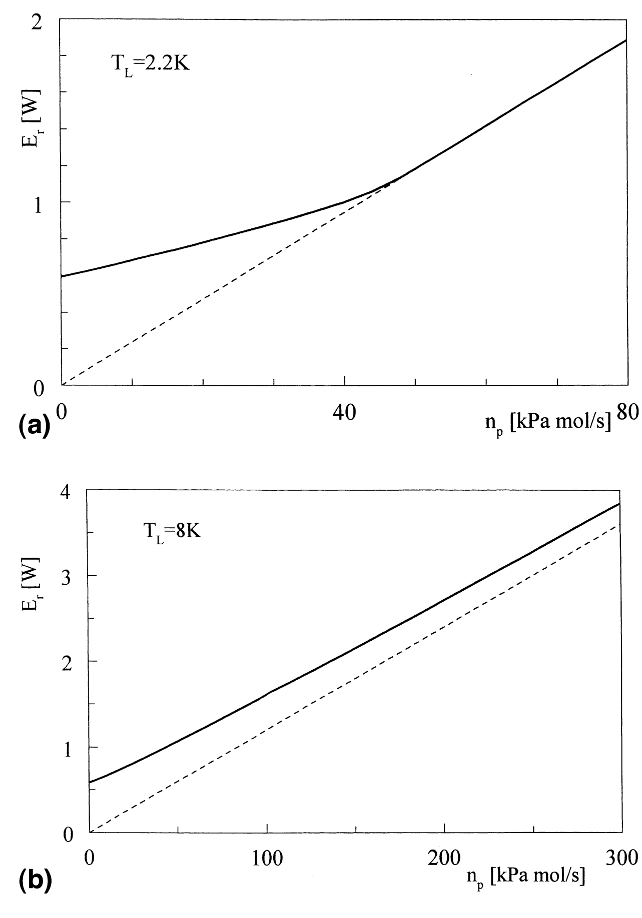

Fig. 4. Variation of E_r as a function of n_p for $T_L = 2.2$ K and $T_L = 8$ K. The dotted lines represent $E_r = n_p H_p(2.2\text{ K})$ and $E_r = n_p H_p(8\text{K})$, respectively.

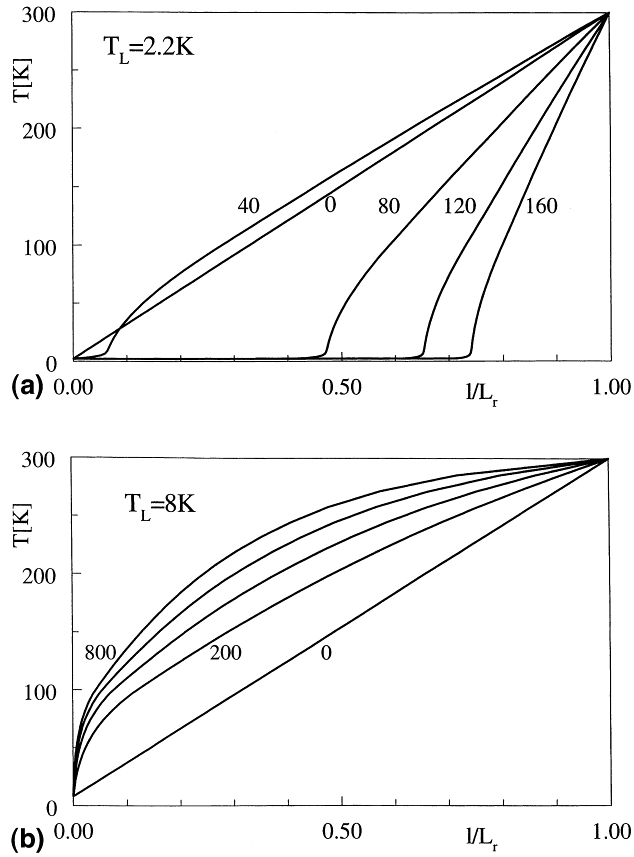

Fig. 5. Temperature profiles in the regenerator for $T_L = 2.2$ K and $T_L = 8$ K for various values of n_p (indicated in the graphs in kPa mol/K). In the case of $T_L = 2.2$ K the maximum of H_p is near the low temperature end and for $T_L = 8$ K the maximum is at the high-temperature end. Therefore the two T-profiles are completely different.

This is illustrated in the Figs. 4 and 5. Fig. 4 gives the dependance of E_r on n_p for $T_L = 2.2$ K and $T_L = 8$ K. The corresponding temperature profiles are given in Fig. 5. For $T_L < 6.4$ K and at larger values of n_p, we see $dT/dl \approx 0$ near the low-temperature side of the regenerator. The temperature profile is flat (Fig. 5) and the heat flow is small. The largest gradient occurs at 12 K where the difference between E_r and n_pH_p is a maximum. For $T_L > 6.4$ K and large n_p, $dT/dl \approx 0$ near the high-temperature side of the regenerator. The effect is less pronounced since the $H_p - T$ dependance at high temperatures is rather flat.

In general, for large flow rates, Eq. (14) holds, so the total energy flow is equal to the enthalpy flow corresponding with the maximum value of H_p, independent of the thermal conduction coefficient of the regenerator material.

3.3. Entropy production

The only irreversible process in the regenerator is heat conduction, so the entropy production in the regenerator is determined by $\dot{Q}_c$. With Eq. (8) [1]

$$\dot{S}_{ir} = \int_{T_L}^{T_H} \frac{\dot{Q}_c}{T^2}\, dT = \int_{T_L}^{T_H} \frac{E_r - n_pH_p}{T^2}\, dT. \tag{15}$$

If the flow rate is very high Eq. (14) holds so

$$\dot{S}_{ir} \approx n_p \int_{T_L}^{T_H} \frac{H_O - H_p(T)}{T^2}\, dT. \tag{16}$$

This expression has a rather unexpected consequence: usually, an ideal regenerator is assumed to have zero entropy production. If the fluid is an ideal gas this can be achieved by reducing the heat conduction to arbitrary low values using low-κ material or decreasing the A_r/L_r ratio. In the case of a real gas, however, there will come a point where the heat flow is reduced so far that $\dot{Q}_c$ is on that same order as the enthalpy flow n_pH_p. From that moment on one cannot further reduce the entropy production. In the limit the entropy production is not a function of the thermal conductivity. The only result of reducing the thermal conduction is that the temperature gradient increases in part of the regenerator. The limiting value of the entropy production is given by Eq. (16). So, the entropy production due to thermal conduction is the same even if the thermal conduction is very small. In engineering terms it means that it is not useful to reduce the thermal conduction below a certain value.

4. Cooling power

In the ideal case in the tube (Fig. 1) $\overline{\overset{*}{S}} = 0$ so the average energy flow in the tube, as derived with Eq. (1), is

$$E_t = V_m n_p. \tag{17}$$

Energy conservation at the low-temperature heat exchanger X_2 demands

$$E_r + \dot{Q}_L = E_t, \tag{18}$$

where $\dot{Q}_L$ is the applied heating power. So

$$\dot{Q}_c + \overline{\overset{*}{H}} + \dot{Q}_L = E_t. \tag{19}$$

Here $\dot{Q}_c$ and $\overline{\overset{*}{H}}$ in this, and in the following expressions, have to be evaluated at the cold end of the regenerator. With Eqs. (3) and (17) we find

$$\dot{Q}_L = T\alpha_V V_m n_p - \dot{Q}_c, \tag{20}$$

which is an expression for the net cooling power of the pulse tube. In the limit of high flow rates and $T_L < 6.4$ K we saw that $\dot{Q}_c \approx 0$, so

$$\dot{Q}_L \approx T\alpha_V V_m n_p. \tag{21}$$

The thermal expansion coefficient α_V satisfies

$$\alpha_V V_m = \left(\frac{\partial V_m}{\partial T}\right)_p = -\left(\frac{\partial S_m}{\partial p}\right)_T. \tag{22}$$

It must be zero at $T = 0$ according to the third law. However, in the case of ^4He, $\alpha_V = 0$ already just above the lambda point, so at about 2 K. It even is negative at lower temperatures. So at 2 K the cooling power is zero. For ^{3}He [9] the temperature where $\alpha_V = 0$ is varies from 0.5 K at low pressures to 1.2 K near the melting curve.

5. Velocities inside the tube

In general

$$T\mathrm{d}S_\mathrm{m} = C_\mathrm{V}\left(\frac{\partial T}{\partial p}\right)_{V_\mathrm{m}}\mathrm{d}p + C_\mathrm{p}\left(\frac{\partial T}{\partial V_\mathrm{m}}\right)_\mathrm{p}\mathrm{d}V_\mathrm{m} \quad (23)$$

from which follows

$$\left(\frac{\partial V_\mathrm{m}}{\partial p}\right)_{S_\mathrm{m}} = \frac{C_\mathrm{V}}{C_\mathrm{p}}\left(\frac{\partial V_\mathrm{m}}{\partial p}\right)_T. \quad (24)$$

The change of a volume V_t under adiabatic compressions is given by

$$\frac{\mathrm{d}V_\mathrm{t}}{\mathrm{d}t} = \int_0^{L_\mathrm{r}}\left(\frac{\partial V_\mathrm{m}}{\partial p}\right)_{S_\mathrm{m}}\frac{\mathrm{d}p}{\mathrm{d}t}\mathrm{d}n = \frac{\mathrm{d}p}{\mathrm{d}t}\int_0^{L_\mathrm{r}}\left(\frac{\partial V_\mathrm{m}}{\partial p}\right)_{S_\mathrm{m}}\frac{A_\mathrm{t}\mathrm{d}l}{V_\mathrm{m}}. \quad (25)$$

Using this expression and Eq. (23) gives the velocity v_L at the cold end, related to the velocity v_H at the hot end, by

$$v_\mathrm{L} = v_\mathrm{H} + \frac{\mathrm{d}p}{\mathrm{d}t}\int_0^{L_\mathrm{r}}\frac{C_\mathrm{V}}{C_\mathrm{p}}\kappa_\mathrm{T}\,\mathrm{d}l \quad (26)$$

with the isothermal compression coefficient

$$\kappa_\mathrm{T} = -\frac{1}{V_\mathrm{m}}\left(\frac{\partial V_\mathrm{m}}{\partial p}\right)_T. \quad (27)$$

Contrary to the ideal gas case the second term in Eq. (26) depends on the temperature distribution in the tube.

6. Discussion

We have shown that reformulating the theory from the ideal-gas situation to a real gas changes the properties of the system drastically. The nonideal gas properties have a profound effect on the energy balance in the regenerator and on the expression for the cooling power. The temperature profiles in the regenerator are strongly affected by the thermal properties of the fluid. The prediction of the theory, that the profile changes drastically if the low temperature passes the 6.4 K value has not been demonstrated experimentally yet. A fundamental difference between an ideal gas and a real gas is further that, for large flow rates, the dissipation in the regenerator is proportional to the flow and cannot be reduced by reducing the thermal conduction coefficient or the regenerator geometry.

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