

The Role of the Orifice and the Secondary Bypass in a Miniature Pulse Tube Cryocooler

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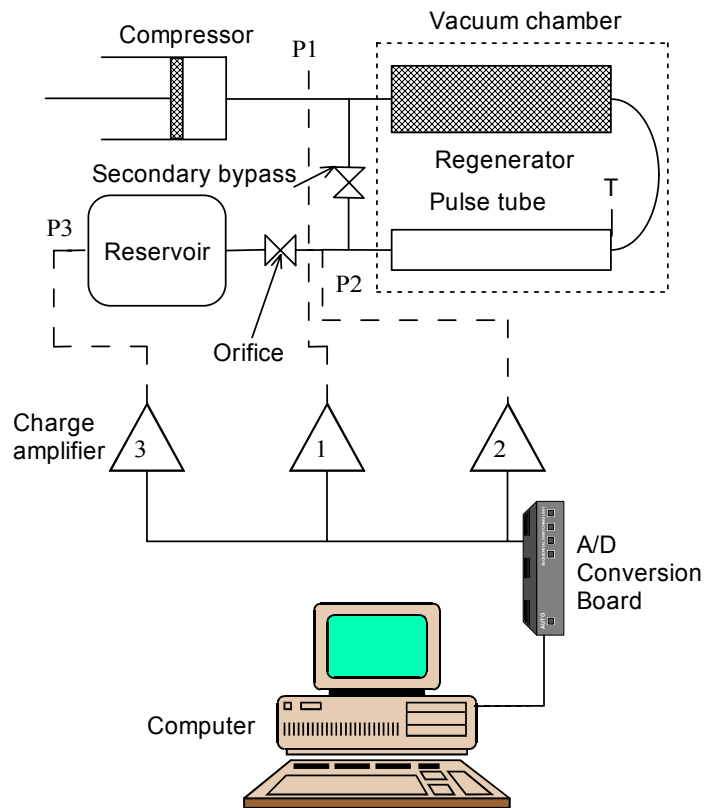
ABSTRACT

This paper is focus on the phase shifting role of the orifice and the secondary bypass in a miniature pulse tube cryocooler. Firstly, the equation of the mass flow rate through the valve is extended into the Fourier series to investigate the phase difference between dynamic pressure in the pulse tube and mass flow rate at the hot end of the pulse tube. The analytical results show that the orifice opening has weak effect on the phase difference between the pressure and the mass flow rate at hot end when the secondary bypass is closed. And the mass flow rate at the hot end of the pulse tube is almost in phase with the dynamic pressure in the pulse tube. The introduction of the secondary bypass will make the pressure in the pulse tube always lead the mass flow rate at the hot end. For the orifice pulse tube cryocooler, the experiments show that with the orifice opening increasing, the phase difference is increasing but less than 3 degree for the opening of the orifice within the range from 0.2 to 2 turns. Based on the optimum orifice opening, the phase difference between the pressure in the pulse tube and the mass flow rate at the hot end increases with the opening of the secondary bypass increasing. The experimental result agrees with the analytical result qualitatively.

INTRODUCTION

According to the enthalpy flow theory¹, the phase difference between the dynamic pressure in the pulse tube and the mass flow rate at the cold end is an important parameter for the performance of the cooler. It is considered at first that the phase difference close to zero will lead to better cooling performance. However, the optimum phase relationship is to have the mass flow rate lags the pressure wave at the cold end² when the losses in the regenerator are taken into account. For the pulse tube cryocoolers, this phase difference is mainly determined by the phase shift mechanisms such as orifice, reservoir and secondary bypass at the hot end of the pulse tube. Fully understanding of the phase relationship between the dynamic pressure in the pulse tube and the mass flow rate at the hot end is helpful.

Cai³ et al investigated the effect of the opening of the orifice and the secondary bypass on the phase difference between the pressure wave and mass flow rate at the hot end of the pulse tube early in 1993, they used the hot wire anemometer and the quartz pressure transducer to measure simultaneously the mass flow rate and the pressure, respectively. When the secondary bypass was closed, their experimental results showed that the dynamic pressure wave lagged the mass flow rate at the hot end from 85 degree to 48 degree with the opening of the orifice



T: type T thermocouple, P1-P3: pressure transducers

Figure 1. Experimental setup.

increasing. Inada⁴ et al also investigated the change of the phase difference between the pressure and mass flow rate at the hot end by using the hot wire anemometer and the pressure transducer in 1996. For the orifice pulse tube cryocooler, they found that the phase difference was almost independent of the opening of the orifice, and the mass flow rate was almost in phase with the pressure wave within the operating conditions discussed in their paper.

The difference of their experimental setups is that Cai placed the hot wire anemometer before the orifice and Inada placed it after the orifice, and the geometry of their cryocooler is also different.

In this paper, the analytical method similar to what Kuriyama⁵ applied is used to investigate the phase shifting role of the orifice and the secondary bypass in a high frequency miniature pulse tube cryocooler, and the experiments are carried out to confirm the analytical results.

EXPERIMENTAL SETUP

The experimental setup is shown in Fig. 1, which consists of a pulse tube cryocooler, a vacuum system and a measuring system. The pulse tube cryocooler is arranged in the “U” shape. The compressor is of a rotary type with a constant swept volume of 1.66cm^3 . The regenerator has an outer diameter of 8mm and a length of 60mm, the pulse tube has an outer diameter of 5mm and a length of 70mm. The reservoir has the volume of 55cm^3 . Two identical fine needle valves are used as the orifice and secondary bypass. They are placed outside of the vacuum chamber for convenience. The needle valve has a total of 10 turns, with each turn being graded in 50 divisions. The vacuum environment is maintained by a vacuum pump. The lowest vacuum of 1.0Pa can be reached. Temperature is measured at the cold end of the pulse tube with a copper-constantan thermocouple (Type T). Dynamic pressures are measured by small quartz pressure transducers (Kistler, type 601A) at the inlet of the regenerator, at the hot end of the pulse tube and in the reservoir. The pressure voltage signs are amplified by charge amplifiers (Kistler, type 5011) and collected by the computer. During the experiment, the constant room temperature is obtained by the air conditioning.

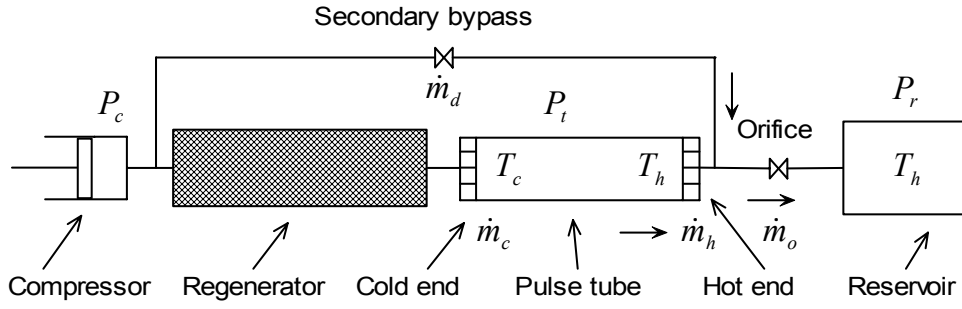


Figure 2. Schematic diagram of the double-inlet pulse tube cryocooler.

METHOD TO DETERMINE THE MASS FLOW RATE AT THE HOT END⁶

Fig. 2 is the schematic diagram of a double-inlet pulse tube cryocooler. It becomes the orifice pulse tube cryocooler when the secondary bypass is closed. Neglecting the void volume of the connecting tube and the hot end heat exchanger, the mass flow rate at the hot end of the pulse tube is given by

$$\dot{m}_h = \dot{m}_o - \dot{m}_d \quad (1)$$

where \dot{m}_h is the mass flow rate at the hot end; \dot{m}_o is the mass flow rate through the orifice and \dot{m}_d is the mass flow rate through the secondary bypass. Generally, needle valves are used as the orifice and the secondary bypass. If the inertial effect is neglected and the temperature is constant when gas flows through the valve, the mass flow rate through the valve can be expressed as⁵:

$$\begin{cases} \dot{m} = k\sqrt{|P_1^2 - P_2^2|} \dots\dots; P_1 \geq P_2 \\ \dot{m} = -k\sqrt{|P_1^2 - P_2^2|} \dots\dots; P_1 < P_2 \end{cases} \quad (2)$$

where k is the proportionality constant; P_1 and P_2 are the pressures before and after the valve, respectively.

In Eq. (2), the proportionality constant k is dependent on the opening of the valve and the

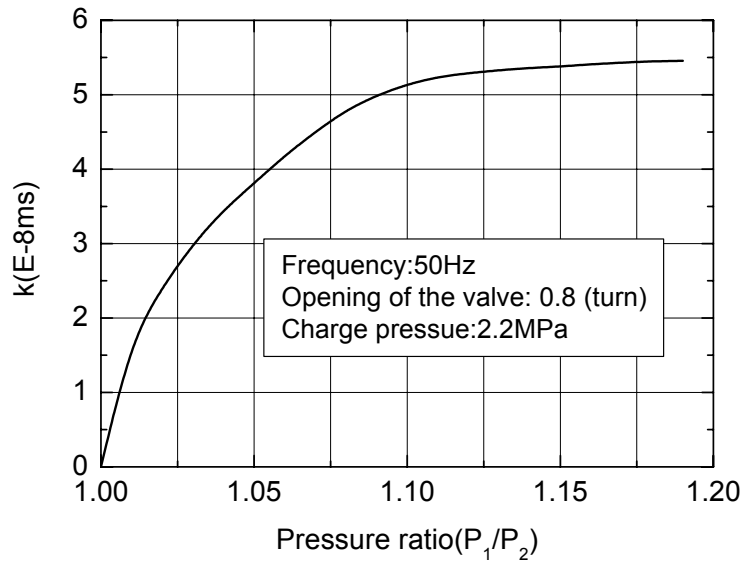


Figure 3. The proportionality constant k of the valve with opening of 0.8 turns.

pressure ratio, which is P_1/P_2 when P_1/P_2 is greater than or equal to 1 or P_2/P_1 when P_1/P_2 is less than 1. It will be determined based on the pressure measurements as follows.

According to mass conservation, the mass flow rate through the orifice valve equals to the change of mass in the reservoir, which is⁶

$$\dot{m}_o = \frac{dm_r}{dt} \quad (3)$$

where subscript “ r ” refers to the reservoir. Assuming that the gas oscillating in the reservoir is adiabatic, we have

$$\dot{m}_o = \frac{V_r}{\gamma RT_0} \frac{dP_r}{dt} \quad (4)$$

where V_r is the volume of the reservoir; P_r is the transient pressure in the reservoir; R is the ideal gas constant; γ is the ratio of specific heat capacities and T_0 is the room temperature. By measuring the transient pressure in the reservoir, the mass flow rate from Eq. (4) can be obtained. With this mass flow rate, the values of k can be determined at different openings from Eq. (2). Because the orifice and the secondary bypass are identical valves, the values k obtained by the orifice valve can be available for the secondary bypass valve. Fig. 3 shows the value of k for the valve with opening of 0.8 turns. The values of k for other valve openings are also obtained.

ROLE OF THE ORIFICE

For simplicity, some basic assumptions are made as follows:

- The pressure oscillation in the pulse tube cryocooler is sinusoidal.
- There is no pressure drop in the pulse tube.
- The initial phase angle of the pressure in the pulse tube is zero.

The pressure at the inlet of the regenerator (i.e. before the secondary bypass) can be expressed as

$$P_c = p_a + p_c \cos(\omega \cdot t - \theta_c) \quad (5)$$

where p_a is the average pressure; p_c is the amplitude of the dynamic part of the pressure; ω is the angular velocity and θ_c is the initial phase angle of the pressure.

The pressure in the pulse tube is:

$$P_t = p_a + p_t \cos(\omega \cdot t) \quad (6)$$

where p_t is the amplitude of the dynamic part of the pressure in the pulse tube.

The pressure in the reservoir is:

Table 1. Effect of the opening of the orifice and θ_r on the initial phase angle of the mass flow rate through the orifice.

θ_r (degree)	60	70	80	90
Initial phase angle for case 1 (degree)	≈ 0	≈ 0	≈ 0	≈ 0
Initial phase angle for case 2 (degree)	0.71	0.74	0.76	0.76
Initial phase angle for case 3 (degree)	1.5	1.55	1.55	1.54

Case 1 : Opening of the orifice is small (0.4 turns), $p_t/p_r = 129$.

Case 2 : Opening of the orifice is optimum (1 turns), $p_t/p_r = 44$.

Case 3 : Opening of the orifice is large (2 turns), $p_t/p_r = 21$.

$$P_r = p_a + p_r \cos(\omega \cdot t + \theta_r) \quad (7)$$

where p_r is the amplitude of the dynamic part of the pressure in the reservoir and θ_r is the initial phase angle of the pressure.

Substituting Eq. (6) and (7) into Eq. (2) and rearranging it gives the dimensionless mass flow rate through the orifice

$$\left. \begin{aligned} \dot{m}_o / kp_a &= \sqrt{\frac{1}{2} \left(\frac{p_t}{p_a} \right)^2 - \frac{1}{2} \left(\frac{p_r}{p_a} \right)^2 + 2 \left(\frac{p_t}{p_a} - \frac{p_r}{p_a} \cos \theta_r \right) \cos \omega \cdot t - 2 \frac{p_r}{p_a} \sin \theta_r \sin \omega \cdot t} \\ &\quad + \frac{1}{2} \left[\left(\frac{p_t}{p_a} \right)^2 - \left(\frac{p_r}{p_a} \right)^2 \cos 2\theta_r \right] \cos 2\omega \cdot t - \frac{1}{2} \left(\frac{p_r}{p_a} \right)^2 \sin 2\theta_r \sin 2\omega \cdot t} \end{aligned} \right\} \dots P_t \geq P_R \quad (8)$$

$$\left. \begin{aligned} \dot{m}_o / kp_a &= - \sqrt{\frac{1}{2} \left(\frac{p_t}{p_a} \right)^2 - \frac{1}{2} \left(\frac{p_r}{p_a} \right)^2 + 2 \left(\frac{p_t}{p_a} - \frac{p_r}{p_a} \cos \theta_r \right) \cos \omega \cdot t - 2 \frac{p_r}{p_a} \sin \theta_r \sin \omega \cdot t} \\ &\quad + \frac{1}{2} \left[\left(\frac{p_t}{p_a} \right)^2 - \left(\frac{p_r}{p_a} \right)^2 \cos 2\theta_r \right] \cos 2\omega \cdot t - \frac{1}{2} \left(\frac{p_r}{p_a} \right)^2 \sin 2\theta_r \sin 2\omega \cdot t} \end{aligned} \right\} \dots P_t < P_R$$

where the subscript “o” refers to the orifice.

Eq. (8) shows that the amplitude and the initial phase angle of the mass flow rate through the orifice are the functions of $\frac{P_t}{p_a}$, $\frac{P_r}{p_a}$ and θ_r . Eq. (8) is a period function which can be extended to the Fourier series⁵. It is reasonable to consider that the amplitude and the initial phase angle of the mass flow rate through the orifice are mainly determined by the main term of the fundamental oscillation. Therefore the mass flow rate through the orifice can be expressed as

$$\dot{m}_o / kp_a \approx a0_o + \sqrt{a1_o^2 + b1_o^2} \cos(\omega \cdot t - \phi_o) \quad (9)$$

where $a0$ is the constant in the Fourier series; $a1$ and $b1$ are the coefficients of the first term in the Fourier series and ϕ_o is the initial phase angle, which is

$$\phi_o = \arctan\left(\frac{b1_o}{a1_o}\right) \quad (10)$$

The positive value of ϕ_o means that the mass flow rate through the orifice leads the dynamic pressure in the pulse tube. The Fourier coefficients can be calculated by numerical integral.

When the secondary bypass is closed, it becomes the orifice pulse tube cryocooler. According to Eq. (1), the mass flow rate at the hot end of the pulse tube is equal to the mass flow rate through the orifice for the orifice pulse tube cryocooler. Referring to Eq. (6) and (10), it is known that the initial phase angle of the mass flow rate through the orifice represents the phase difference between the mass flow rate at the hot end and the dynamic pressure in the pulse tube. In the experiment, it is measured that the amplitude of the dynamic pressure in the reservoir is far less than that in the pulse tube because the reservoir is large enough. Table 1 shows the relationship between the initial phase angle and the ratio of $\frac{P_t}{p_r}$ for three cases. Case 1, 2, 3

represent the small, optimum and large opening of the orifice, respectively. The values of $\frac{P_t}{p_r}$ for every case are taken from the experimental data. The calculations are performed under the condition of average pressure of 2.2MPa and frequency of 50Hz. Within the orifice opening

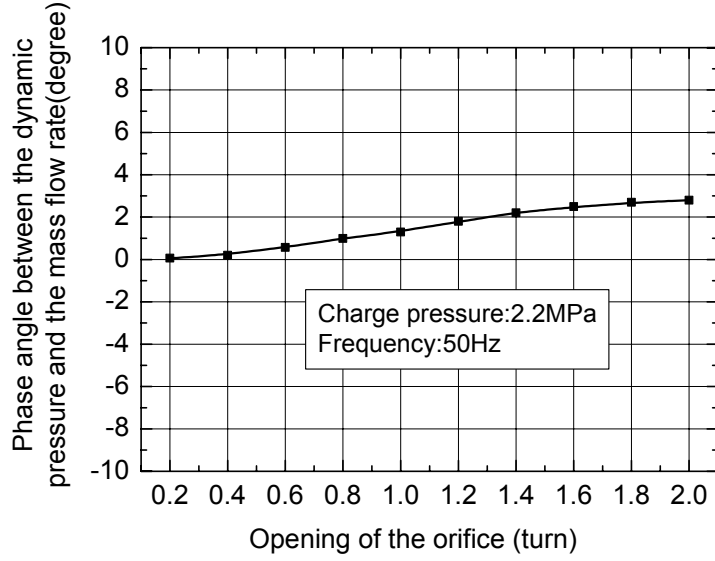


Figure 4. The phase angle (the dynamic pressure lagging the mass flow rate) V.S. the opening of the orifice.

range (less than or equal to 2 turns) considered in the experiment, the phase difference is less than 2 degree no matter how much the initial phase angle of θ_r is.

The mass flow rate at the hot end of the orifice pulse tube cryocooler can also be obtained by the measured oscillating pressure in the reservoir. Fig. 4 is the experimental result by Eq. (4). In the experiment, the maximum opening of the orifice is 2 turns. With the opening of the orifice increasing, the phase difference is increasing, but is less than 3 degree. The theoretical result and the experimental result all indicate that opening of the orifice has weak effect on the phase difference, and the mass flow rate at the hot end of the pulse tube is almost in phase with the dynamic pressure in the pulse tube.

The increasing of the opening of the orifice can reduce the phase difference between the mass flow rate at cold end and the pressure in the pulse tube. The mass flow rate at the cold end can be expressed as

$$\dot{m}_c = \frac{T_h}{T_c} \dot{m}_h + \left(\frac{V_t}{\gamma T_c} \right) \frac{\dot{P}_t}{R} \quad (11)$$

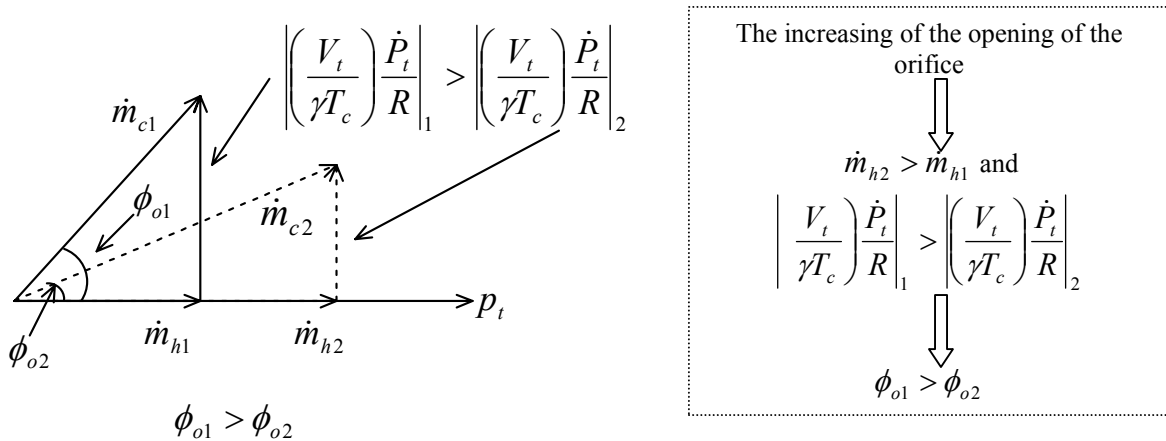


Figure 5. The effect of the opening of the orifice increasing on the phase angle at the cold end.

Table 2. Effect of the opening of the secondary bypass and θ_c on the initial phase angle of the mass flow rate through the secondary bypass

θ_c (degree)	10	20	30	40
Initial phase angle for case 1 (degree)	24.3	45.2	62.0	71.5
Initial phase angle for case 2 (degree)	32.7	56.6	70.1	82.4
Initial phase angle for case 3 (degree)	46.5	67.2	81.5	89.7

Case 1 : Opening of the secondary bypass is small, $p_c/p_t = 1.9$.

Case 2 : Opening of the secondary bypass is optimum, $p_c/p_t = 1.4$.

Case 3 : Opening of the secondary bypass is large, $p_c/p_t = 1.07$.

where T_c is the cold end temperature; T_h is the hot end temperature and V_t is the volume of the pulse tube. Fig. 5 schematically shows the phase angle reducing along with the opening of the orifice increasing. With the opening of the orifice increasing, the first term in the right hand of Eq. (11) increases and the second term decreases, thereby the phase angle reduces.

ROLE OF THE SECONDARY BYPASS

Similar to the analysis of the orifice, substituting Eq. (5) and (6) into Eq. (2) and rearranging it, the dimensionless mass flow rate through the secondary bypass is

$$\left. \begin{aligned} \dot{m}_d/kp_a &= \sqrt{\frac{1}{2}\left(\frac{p_c}{p_a}\right)^2 - \frac{1}{2}\left(\frac{p_t}{p_a}\right)^2 + 2\left(\frac{p_c}{p_a}\cos\theta_c - \frac{p_t}{p_a}\right)\cos\omega\cdot t - 2\frac{p_c}{p_a}\sin\theta_c\sin\omega\cdot t}{+ \frac{1}{2}\left[\left(\frac{p_c}{p_a}\right)^2\cos 2\theta_c - \left(\frac{p_t}{p_a}\right)^2\right]\cos 2\omega\cdot t - \frac{1}{2}\left(\frac{p_c}{p_a}\right)^2\sin 2\theta_c\sin 2\omega\cdot t} \end{aligned} \right\} \dots P_c \geq P_t \quad (12)$$

$$\left. \begin{aligned} \dot{m}_d/kp_a &= -\sqrt{\frac{1}{2}\left(\frac{p_c}{p_a}\right)^2 - \frac{1}{2}\left(\frac{p_t}{p_a}\right)^2 + 2\left(\frac{p_c}{p_a}\cos\theta_c - \frac{p_t}{p_a}\right)\cos\omega\cdot t - 2\frac{p_c}{p_a}\sin\theta_c\sin\omega\cdot t}{+ \frac{1}{2}\left[\left(\frac{p_c}{p_a}\right)^2\cos 2\theta_c - \left(\frac{p_t}{p_a}\right)^2\right]\cos 2\omega\cdot t - \frac{1}{2}\left(\frac{p_c}{p_a}\right)^2\sin 2\theta_c\sin 2\omega\cdot t} \end{aligned} \right\} \dots P_c < P_t$$

where the subscript “d” refers to the secondary bypass. Writing Eq. (12) with a Fourier series and taking the fundamental term as approximation gives

$$\dot{m}_d/kp_a \approx a0_d + \sqrt{a1_d^2 + b1_d^2} \cos(\omega\cdot t - \phi_d) \quad (13)$$

where ϕ_d is the initial phase angle, which is

$$\phi_d = \arctan\left(\frac{b1_d}{a1_d}\right) \quad (14)$$

The positive value of ϕ_d means that the mass flow rate through the secondary bypass leads the dynamic pressure in the pulse tube.

Referring to the calculation in the prior section, the initial phase angle of the mass flow rate through the secondary bypass is evaluated. Table 2 shows the calculated results for three cases.

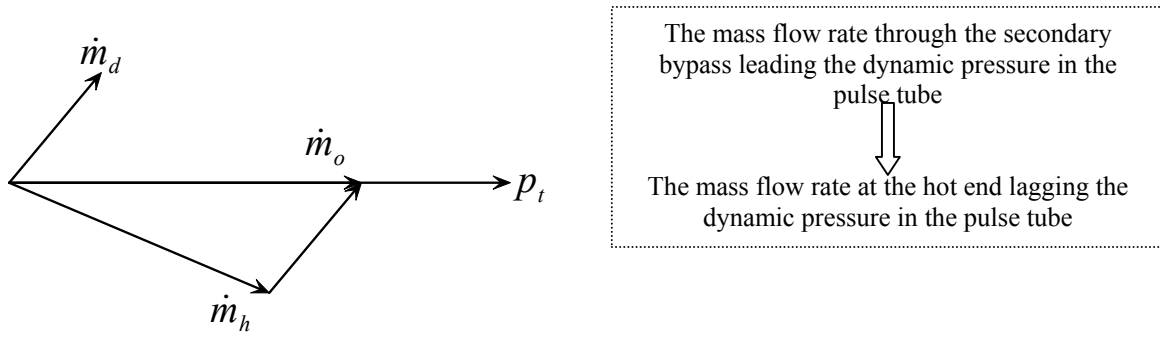


Figure 6. Schematic diagram of Eq. (1).

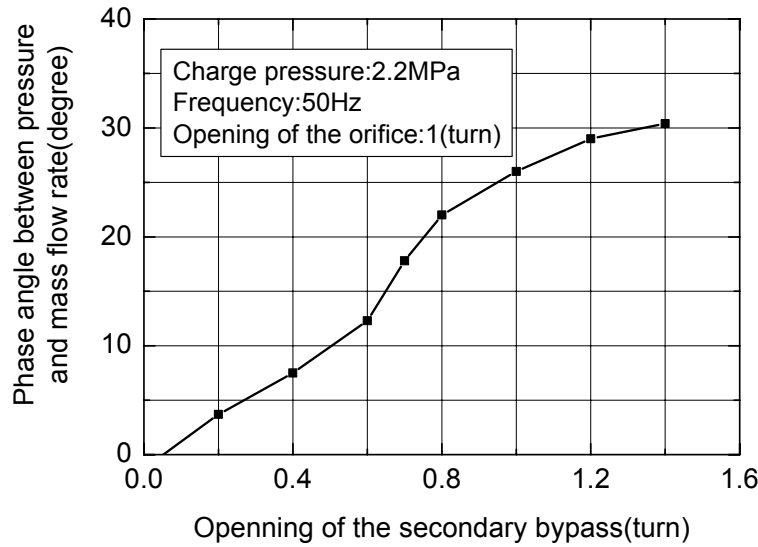


Figure 7. Phase angle (dynamic pressure leading the mass flow rate) VS. the opening of the secondary bypass.

Case 1, 2, 3 represent the small, optimum and large opening of the secondary bypass, respectively. The calculations are performed on the basis of the orifice set to its optimum opening. The calculations show that the initial phase angle of the mass flow rate through the secondary bypass strongly depends on the value of P_c/P_t and θ_c . For the miniature pulse tube cryocooler considered, the experiments show that the value of θ_c is less than 30 degree; the increasing of the secondary bypass reduces the value of θ_c ; but increases the value of P_c/P_t .

Table 2 also shows that the initial phase angle of the mass flow rate through the secondary bypass is always positive and can be as large as about 90 degree along with the opening of the secondary bypass increasing.

From Eq. (1), the mass flow rate at the hot end of the pulse tube is determined not only by the phase angle of \dot{m}_o and \dot{m}_d but also by their amplitude. As shown in Fig. 6, the positive value of the initial phase angle of the mass flow rate through the secondary bypass makes the pressure in the pulse tube lead the mass flow rate at the hot end.

With the determined proportionality constant k and the measured dynamic pressures at the inlet of the regenerator, in the pulse tube and in the reservoir, the mass flow rate at the hot end of the double-inlet pulse tube cryocooler can be obtained by Eq. (2). The experiment is carried out under the conditions of average pressure of 2.2MPa, frequency of 50Hz and the orifice being set

to its optimum opening. Fig. 7 shows the relationship between the phase difference (pressure leads the mass flow rate) and the opening of the secondary bypass. With the opening of the secondary bypass increasing, the phase difference between the pressure in the pulse tube and the mass flow rate at the hot end increases. When the opening is larger than 1.2 turns, the slope of the curve is gentle.

CONCLUSION

The role of the orifice and the secondary bypass is theoretical and experimental investigated. The mass flow rate through the orifice and the secondary bypass are approximately expressed with the fundamental term of the Fourier series. For the orifice pulse tube cryocooler, calculations and experiments show that the mass flow rate at the hot end of the pulse tube is almost in phase with the dynamic pressure in the pulse tube, and the opening of the orifice has weak effect on the phase angle under the condition that the opening of the orifice is less than 2 turns. Theoretical calculation shows that the initial phase angle of the mass flow rate through the secondary bypass is always positive and can be as large as about 90 degree along with the opening of the secondary bypass increasing, which makes the pressure in the pulse tube always leads the mass flow rate at the hot end for the double-inlet pulse tube cryocooler. With the opening of the secondary bypass increasing, experimental result shows that the phase difference between the pressure in the pulse tube and the mass flow rate at the hot end increases.

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