

A NUMERICAL MODEL OF A 4K TWO-STAGE PULSE TUBE COOLER WITH MIXED EULERIAN-LAGRANGIAN METHOD

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ABSTRACT

In order to simulate and visualize the internal processes and performance of two-stage pulse tube coolers operating in the 4K-temperatures, a mixed Eulerian-Lagrangian numerical model is proposed to follow the exact tracks of gas particles as they move with pressure oscillation in the pulse tube and to simulate the detailed variations of dynamic parameters in the regenerators. A variety of physical factors, including real thermal properties of helium gas, multi-layered magnetic regenerative materials, pressure drop and heat transfer in the regenerator, are taken into account in this model. Predicted results of the time-variations of gas temperature, pressure, mass flow rate, enthalpy flow, in the 1st- and 2nd-stage regenerators are presented. Particular attention is given to the effects of different regenerative materials on the performance of 4K two-stage pulse tube coolers.

INTRODUCTION

Due to several advances in design and the development of magnetic materials, pulse tube coolers have developed rapidly in recent years. In contrast to the GM coolers pulse tube coolers operate without cold moving parts, thereby eliminating the potential problems (mechanical vibration, lifetime, reliability) inherent to the GM coolers.

Since 1994 when Matsubara and Gao first obtained a temperature of 3.6K using a three-stage pulse tube cooler [1], the lowest temperature of pulse tube coolers [2-4] is now well below the inversion temperature of ³He and ⁴He. As such, it can serve as a precooling stage for further reducing the temperatures to the millikelvin range using ³He-⁴He dilution refrigerators [5]. 4K-pulse tube coolers, which provide 0.5W cooling capacity at 4.2K and with COP up to 10⁻⁴ [6], are now commercially available. This cooling power is already sufficient for several applications, such as for the liquefaction of helium [7] and for cooling small-sized superconducting magnets [8].

However, the COP of 4K-pulse tube coolers is still much lower than that of GM coolers. Most of this is due to the absence of solid displacer in the pulse tube, the phase

shifter located at room temperature does not lead to an efficient phase shifting (by the so-called “gas piston”) of the moving liquid helium at the cold end. The other reasons are due to lack of fundamental knowledge of the effects of the salient characteristics of real helium properties, the unique features of thermal properties of magnetic material, and the existence of helium in the void space of the regenerator on the performance of the coolers.

Since the internal processes in 4K-pulse tube coolers are not well understood at present, a mixed Eulerian-Lagrangian model [9] recently developed by the present author is used to simulate and visualize the internal processes and performance of a 4K two-stage pulse tube cooler. Here, we briefly summarize the model and present some predicted results of the time-variations of gas temperature, pressure, mass flow rate, enthalpy flow, in the 1st- and 2nd-stage regenerators. Particular attention is given to the effects of different regenerative materials on the performance of the 4K two-stage pulse tube cooler.

FORMULATION

The schematic diagram of a 4K two-stage pulse tube cooler is shown in Fig. 1. The analysis assumes 1-D, compressible gas flow and neglects any geometric complexity in the regenerator, in order to reduce CPU time and avoid any difficulty involved with the two- or three-dimensional models. Other basic assumptions were as follows:

1. The gas flow is laminar, no turbulence
2. Constant wall temperature at the cold and hot end heat exchangers
3. The regenerative material in the regenerator is incompressible, uniform porosity
4. Boundary and variable permeability effects are neglected

Under these assumptions, the governing equations are given by:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0 \quad (1)$$

$$(1-f)\rho C_p \frac{\partial T}{\partial t} = -\rho C_p u \frac{\partial T}{\partial x} - u(1-T\alpha_v) \frac{\partial p}{\partial x} + (1-f)T\alpha_v \frac{\partial p}{\partial t} + \alpha F(T_r - T) + \frac{\partial}{\partial x} \left(\kappa \frac{\partial T}{\partial x} \right) \quad (2)$$

$$\frac{\partial p}{\partial x} = -\eta z_r u \quad (3)$$

$$\rho = f(T, p) \quad (4)$$

The energy equation for the solid regenerative materials in the regenerator is:

$$f\rho_r C_r \frac{\partial T_r}{\partial t} = \alpha F(T - T_r) + \frac{\partial}{\partial x} \left(\kappa_r \frac{\partial T_r}{\partial x} \right) \quad (5)$$

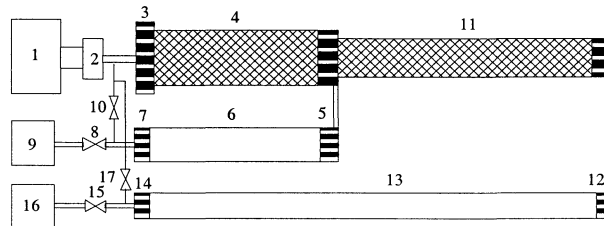


FIGURE 1. Schematic diagram of a 4K two-stage pulse tube cooler

Here $T, u, p, \rho, V_m, C_p, k, \eta$ and α_v are the temperature, velocity, pressure, density, specific volume, specific heat, thermal conductivity, viscosity and volumetric thermal expansion coefficient of helium gas respectively. $T_r, \rho_r, C_r,$ and κ_r are the temperature, density, heat capacity, thermal conductivity of regenerative materials respectively. $f, A,$ and F are the filling factor of regenerator, the cross sectional area of the regenerator and the heat exchange area per unit volume, respectively.

The heat transfer coefficient α and the flow impedance factor z_r of the regenerator are determined by the experimental data [10] and have been used in thermodynamic analysis of regenerator and pulse tube [11].

We assume constant (room) temperature in the compressor that is the left boundary. The right boundary in the simulation is the buffer, where the gas is regarded as isothermal. The initial pressure variation inside the pulse tube is assumed as harmonic variation and will be modified from the calculated pressure difference in the regenerator system. Both the orifice and double-inlet valves are considered as a resistance component. The mass flow rates through these valves are determined from the pressure differences [9].

The real thermal properties for helium gas, including density, heat capacity, thermal conductivity etc. are obtained from NIST TN-12 Helium Database. The density, heat capacity and thermal conductivity of the regenerator materials are obtained by curve fitting.

NUMERICAL METHODS

Equations (1)~(5) with boundary conditions make up the set of governing equations, which are used for determining 5 independent variables of the gas temperatures, velocity, pressure, density and the matrix temperature. They are non-linear and unsteady, and can only be solved by numerical integration.

To solve these equations a combination of the Eulerian and Lagrangian approaches has been developed [9]. We use the Eulerian method, a fixed computational grid, to simulate the time-variations of dynamic parameters in the regenerator. The Lagrangian approach, a moving grid, is used to follow the tracks of gas particles as they move with pressure oscillation in the pulse tube to avoid any numerical false diffusion.

The Eulerian method has been adopted widely and found to be useful for simulating this problem. In this method the grids are fixed. They solve the fluid variables on fixed grids and reconstruct the interface of gas particles based on the fluid fraction data in each node. In order to obtain the interfaces accurately, highly refined grids are required for sharp variations of thermal properties. The advantages of the Eulerian approach using fixed grids are simplicity and availability of well-tested, economical field solvers. However, it is not suitable when details of the gas particles are to be explicitly tracked since the interface is reconstructed from the distribution of certain computed field variables. The main drawback of the Eulerian method is that it may bring numerical false diffusion since additional field variables are introduced to model the presence of a moving discontinuity on the computational grid.

1. Eulerian Method

Numerical simulation of the governing equations by Eulerian method is based on the finite difference methodology. The regenerator is divided into many subsystems and each subsystem is considered as a uniform system, which can exchange heat and mass with the surroundings through its boundaries. The regenerator domain discretization is made using a

control volume approach by an inner nodal method as described by Patankar [12]. Equations (1)~(5) are solved by using the explicit scheme, thus the temperature, velocity and pressure are updated in an explicit fashion. Spatial derivatives are approximated using an upwind second-order difference formula [13], to ensure the transportive properties of the discretization equations. The interface of gas particles is advanced in time by the Lagrangian translation of the gas particles (as discussed below), while the normal mass flow rate is evaluated from the mass conservation equation (1). Additional details regarding the numerical simulation and procedures have been discussed in Ref. [13], to reduce CPU time and increase the calculation accuracy.

A solution is obtained from an initial guess through an iterative scheme using a line-by-line tridiagonal matrix algorithm (TDMA) method. The iteration results should be satisfied by the periodic stable conditions for all the time-dependent parameters and the energy balance condition for the regenerator matrix over one cycle. After a grid refinement study all calculations presented below are carried out using a 200 uniform grid size. Due to the explicit nature of the scheme, the time step is restricted by the convective Courant-Friedrichs-Lewy (CFL) limitations, which is set to $\sim 0.0055s$ (time period/ 180). With the space size and time step selected in this numerical study, our experiences indicated that the calculated results are essentially independent of the numerical discretization.

2. Lagrangian Method

In order to use the Lagrangian method we need to know the time-variations of the temperature of the gas element and the position of the gas element traveling inside the pulse tube with pressure oscillation. The detailed mathematical formulations have been discussed in Ref. [14]. Thus, only a brief summary is provided. For a real gas, the temperature of the gas element with the pressure oscillations inside the tube is given by

$$T = T^0 + \frac{1}{C_p} \alpha_v T V_m (p - p^0) \quad (6)$$

The superscript 0 means variables at the previous time and variables without superscript stand for the value at present time. The velocity of the gas element at the cold end of the tube, related to the gas velocity at the hot end of the tube, is given by

$$u_L = u_H + \frac{dp}{dt} \int_0^{L_t} \frac{C_v}{C_p} \kappa_T dl \quad (7)$$

here u_H is the gas velocity at the hot end of the tube and κ_T the isothermal compression coefficient of gas. The position of gas element traveling with the pressure inside the pulse tube, leaving the hot heat exchanger at $x = L_t$ on a time $t = t_0$, is given by solving Eq. (8)

$$l = L_t - (p - p^0) \cdot \int_l^{L_t} \frac{C_v}{C_p} \kappa_T dx \quad (8)$$

Knowing the temperature profile and the position of the gas element traveling with pressure oscillations inside the pulse tube using Eq. (6) and Eq. (8), we can directly obtain the time-variations of gas temperatures at the cold ends of 1st- and 2nd-stage regenerators and the heat exchange rate (cooling power) in them. Then we use Eulerian methods, to solve the variations of dynamic parameters in the regenerator through an iterative scheme.

RESULTS AND DISCUSSIONS

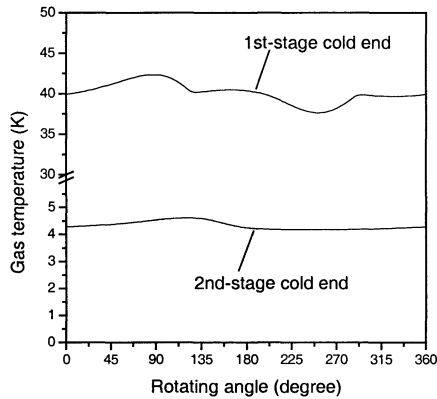
A typical calculation is made for a two-stage pulse tube cooler, as depicted in Fig. 1. The main structure parameters of the two-stage pulse tube cooler are given in Table 1. The operating parameters are listed as follows:

Temperature at hot end: 300K; Temperature at 1st-stage cold end: 40K; Temperature at 2nd-stage cold end: 4.2K; Operating frequency: 1Hz; Average pressure: 1.4MPa.

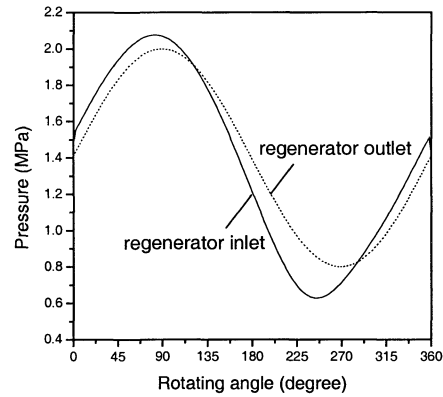
The 1st-stage regenerator is filled with 250-mesh stainless steel screens and, its filling factor is 0.4. The 2nd-stage regenerator is filled with lead spheres in 50% of the length and with Er₃Ni of 50% of the regenerative length and, its filling factor is 0.6.

Table 1. Main structure parameters of the two-stage pulse tube cooler

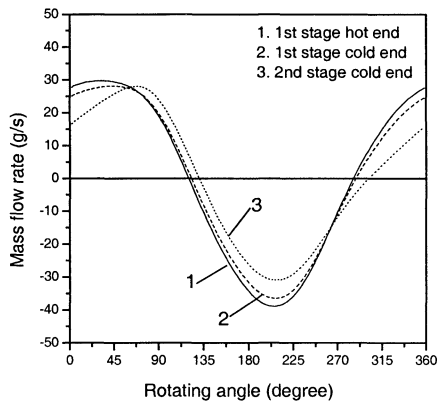
Components	1st-stage	2 nd -stage
Regenerator	∅ 60 mm × 140 mm	∅ 30 mm × 180 mm
Pulse tube	∅ 35 mm × 140 mm	∅ 18 mm × 330 mm
Buffer	1000cm ³	1000cm ³



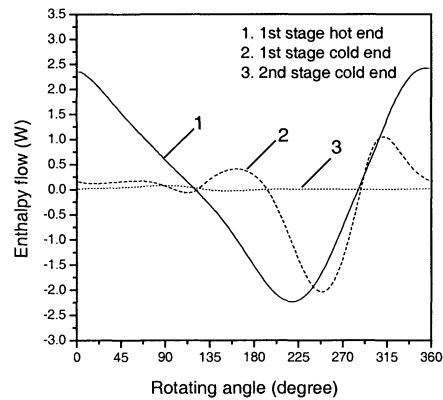
(a)



(b)



(c)



(d)

FIGURE 2. Variations of dynamic parameters in one cycle
(a) Gas temperature; (b) pressure; (c) mass flow rate; (d) enthalpy flow

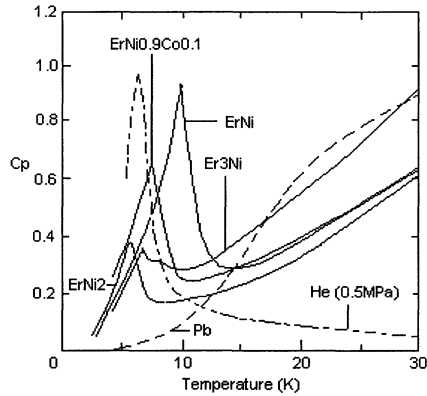


FIGURE 3. Specific heat capacity of regenerator materials and helium

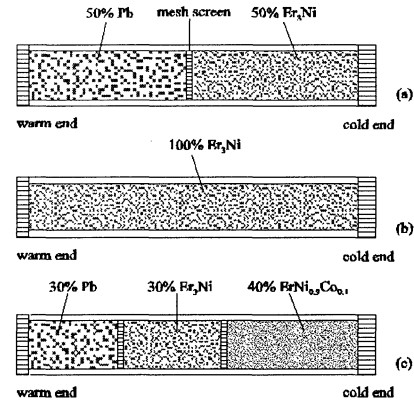


FIGURE 4. 2nd-stage regenerator arrangements

The time-variations of dynamic parameters, including the gas temperature, pressure, mass flow rate and enthalpy flow, in the 1st- and 2nd-stage regenerators during one cycle are shown in Fig. 2. The variations of the gas temperatures, at the cold ends of the 1st- and 2nd-stage regenerators during one cycle with the rotating valve angle are plotted in Fig. 2(a). The amplitudes of the gas temperature fluctuations at the cold ends of the 1st- and 2nd-stage regenerators are around 10K and 0.5K, respectively. Much smaller fluctuation at the cold end of the 2nd-stage regenerators is due to the reason that the isothermal compressibility of helium is very small around 4.2K. Fig. 2(b) gives the variations of the dynamic pressures at the inlet and outlet of the 1st-stage regenerator during one cycle. There is 2~3bar (nonlinear) pressure drop across the 1st-stage regenerator, but nearly no pressure drop across the 2nd-stage regenerator since the viscosity of the helium is much smaller in the temperature range of 40-4.2K than that of 300-40K. The dynamic mass flow rate and the enthalpy flow at the 1st-stage hot end, 1st-stage cold end and 2nd-stage cold end of the regenerators are plotted in Fig. 2(c) and (d), respectively.

The density and specific heat of helium gas increases greatly and has a sharp peak in the low temperature region (as shown in Fig. 3). The heat capacity of stainless steel and lead, materials usually used as regenerative materials, decrease rapidly in the low temperature region. Magnetic materials have larger specific heat capacity below 10K compared with common materials. We focus attention below to the optimal regenerative materials in the 2nd-stage regenerator to achieve larger cooling power for the cooler. We introduce a parameter ξ , which is defined as the ratio of volumetric specific heat capacity of the helium gas and the regenerative materials (taking into account the filling factor f) in the regenerator, to judge the efficiency of different multi-layered hybrid regenerators.

Several 2nd-stage regenerator arrangements and materials have been analyzed in the simulation. Fig. 4 shows three typical types of regenerator arrangements and materials studied in the numerical simulation.

Fig. 5(a) and (b) show the average specific heat of the regenerative materials along the 1st- and 2nd-stage regenerators during one cycle corresponding to the arrangements in Fig. 4(a) and (c), respectively. There are two peaks along the regenerator in Fig. 5(a), which are caused by the strong increase of specific heat of lead and Er₃Ni. Similar features can be found in Fig. 5(b).

Fig. 6(a) and (b) give the time-averaged ξ of specific heat of helium and regenerator materials along the 1st- and 2nd-stage regenerators during one cycle corresponding to the

arrangements in Fig. 4(a) and (c), respectively. Very large values of ξ can be seen in Fig. 6(a) in the 2nd-stage regenerator from 30% to 50% of the regenerator length (position from 0.6 to 0.72 in Fig. 6(a)). This region of the 2nd-stage regenerator is not efficient, demonstrating that lead in this section is not suitable as a regenerative material corresponding to the temperature profile inside the regenerator. In addition to this regenerative arrangement, predicted results of the case of Fig. 4(b) indicate that large values of ξ can be found in the section of the 2nd-stage regenerator from 65% to the cold end of the length. This means that the Er₃Ni grain in this section of the regenerator is not suitable as regenerative material. Therefore, the regenerative materials and arrangements showed in Fig. 4(a) and (b) are not suitable in the 4K two-stage pulse tube cooler.

According to Fig. 3, we see that the specific heat of ErNi_{0.9}Co_{0.1} below 10K is larger than that of the other materials, although it is still smaller than that of helium. The large specific heat is helpful to improve the regenerator efficiency and heat exchange between the working gas and regenerative materials, thereby increasing the cooling capacity and lowering the cooling temperature.

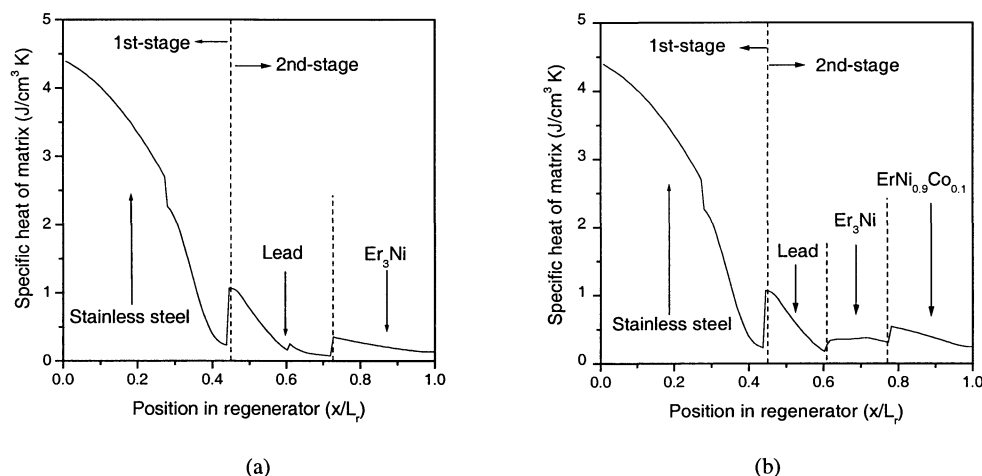


FIGURE 5. Specific heat of regenerative materials inside the regenerator

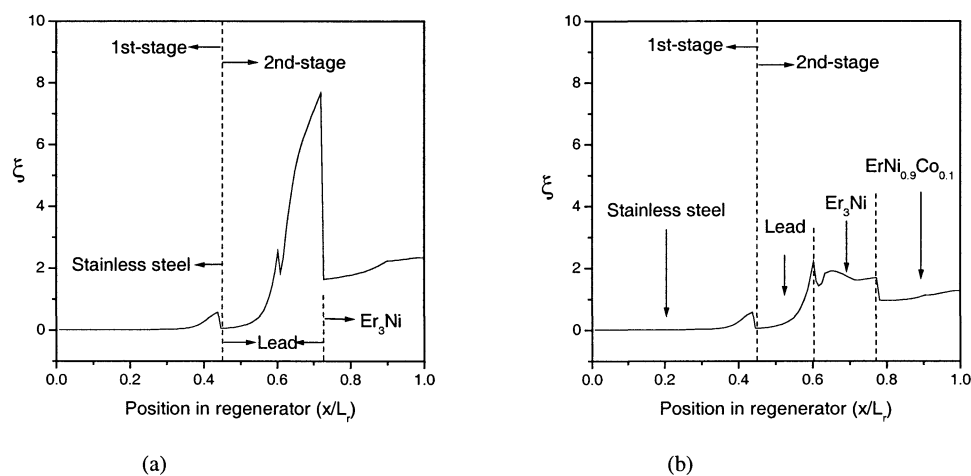


FIGURE 6. Ratio of specific heat of helium and regenerative materials in the regenerator

Comparing Fig. 6(b) with Fig. 6(a), as well as Fig. 5(b) with Fig. 5(a), we find that the regenerative materials and arrangements of Fig. 4(c) is more suitable for the 4K-pulse tube cooler than that of Fig. 4(a) and (b). The predicted results of cooling power of the cooler also demonstrate that the arrangements of Fig. 4(c) can achieve larger cooling powers than that of others. So far, the optimum combinations of materials studied in the present simulation in the 2nd-stage regenerator were lead, Er₃Ni and ErNi_{0.9}Co_{0.1}. The optimum volumetric ratios of those three materials from hot end to the cold end of the regenerator were around 30%, 30% and 40%, as shown in Fig. 4c. In this case, the predicted cooling capacity at 4.2K of the pulse tube cooler is larger by about 0.2W and 0.1W than the cases with regenerative arrangements of Fig. 4(a) and Fig. 4(b). It should be noted that the efficiency of multi-layered hybrid regenerative arrangements depends notably on the exact gas and matrix temperature profile along the regenerator.

The predicted results of the simulation were compared with the corresponding experimental data [3] and found in reasonable agreement between the two. The calculated cooling capacity is about 35% higher than the experimental value. The author reasons that the discrepancies can be attributed to the compressed heat generated inside the regenerator, additional heat losses due to radiation, DC gas flow through the double-inlet tube, transition to turbulence, which are not incorporated into the present numerical model.

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