

ON THE HEAT CONDUCTION LOSSES OF PULSE TUBE AND REGENERATOR AT TEMPERATURE RANGE OF 300-4K

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ABSTRACT

Performance predictions of pulse tube coolers involve estimating pressure drop and heat transfer to determine regenerator efficiency under oscillating flow and oscillating pressure conditions. This paper discusses the enhanced heat conduction of helium gas in the pulse tube and regenerator by the oscillations. It shows that the helium gas confined in the pulse tube as well as in the void volume of the regenerator can produce enhanced heat conduction at a rate an order of magnitude greater than by pure gas conduction, provided the gas is oscillated sinusoidally. Thereby, the effective coefficient of thermal conductivity of the gas in the regenerator is typically an order of magnitude larger than the pure gas conductivity; while the effective coefficient of thermal conductivity of the regenerator material is typically an order of magnitude less than the bulk value. In practical pulse tube coolers, the effective heat conduction losses of the helium gas, solid wall, and regenerator materials are typically of the same order of magnitude.

INTRODUCTION

The pulse tube and regenerator, two of the main components of a pulse tube cooler, are subjected to oscillating flow conditions (or periodically reversing flow). The pulse tube is usually made of a thin stainless steel tube, in which there is no moving displacer like in Stirling and G-M machines, except the working gas, which moves back and forth in the compression and expansion processes with the oscillating pressure in a cycle. The regenerator consists essentially of a cylindrical tube filled with a porous matrix of stacked metallic wire screen (stainless steel, phosphor bronze, etc) or powder (sphere lead, magnetic materials, etc), and acts as a thermal sponge, which absorbs heat from the high-temperature gas during its compression period and gives off the heat to the low-temperature gas during its expansion. The right condition for cooling to occur requires that the regenerator does not only have small pores and fine wires to give a large surface area for heat exchange, but little resistance for the gas flow as well. It is well known that there is a phase difference not only between the mass flow and the pressure [1-3], but also between the gas temperature and the heat flux [4-5].

An understanding of pressure drops and heat transfer rates in the pulse tube and the regenerator under oscillating flow conditions is essential to accurately predict the performance of pulse tube coolers. Performance predictions are fundamental to design, optimization and evaluation. One-dimensional numerical models are widely used and have been developed extensively in recent years for pulse tube coolers, thanks to a wealth of steady-flow literature which correlates heat transfer and pressure drop in terms of cross-section averaged flow parameters. In a one-dimensional model, fluid parameters are averaged in the cross section normal to the principle flow direction and the governing equations are expressed in terms of these mean parameters, and the heat transfer and pressure drop correlations need to be inserted into such a model.

There are important differences between the steady and the oscillating flow. It is apparent that the popular correlations given by Tong and London [6] (reviewed by Kays and London [7]) based on steady flow would not be able to predict accurately the pressure drop and heat transfer through stacked screens in the regenerator under oscillating flow conditions. Many researchers [2-5, 8-11] found higher friction factors in the oscillating flow regenerator than that in the steady flow. For example, our experiments [2] showed that the value of the cycle-averaged pressure drop of the oscillating flow is 2 to 3 times higher than that of the steady flow calculated by the correlations given by Kays and London [7] at the same Reynolds numbers based on the cross-sectional mean velocity.

Besides the pressure drops, experimental and theoretical studies have predicted the heat conduction losses through the porous matrix inside the regenerator [12-14]. In an actual regenerator, the helium gas in the void volume can transport a large fraction of the heat through the matrix. Most experimental and analytical results for heat conduction losses are not available for practical use in the design of the cryocooler regenerators. Kuriyama et al. [13,14] measured the heat conduction loss from room temperature to cryogenic temperatures through stacked screens and showed that the helium gas between each screen plays an important role in transporting the heat. The heat conduction through the stacked screens was enhanced by helium gas by several orders of magnitude compared to that in vacuum. While the ratio of actual heat conduction to the heat conduction where the regenerator material is assumed to be a bulk, was about 0.1.

It has been found that the superposition of oscillations on a gas fluid confined in a single tube or a series of parallel tubes with thermally insulating side walls and connection two different temperature reservoirs at its ends produces a considerable increase in axial heat transfer between the reservoirs without a net transfer of mass [15,16]. The physical mechanism of this enhancement is a large time-dependent radial temperature gradient produced by the oscillations. In most of the oscillating cycle, the gas flow near the wall region will have a temperature difference from the core, resulting in large quantities of heat being transferred radially and hence axially. These enhancements, which are essentially due to the interaction of the oscillating gas flow and the boundary layer, have been discussed thoroughly by Swift in his noteworthy paper [17].

The paper discusses the enhanced heat conduction of the helium gas in the pulse tube and regenerator subjected to the oscillating flow conditions. We draw attention to the heat conduction loss in the pulse tube and regenerator in the temperature range from 300K to 4K. We propose using the effective coefficient of thermal conductivity of the gas, instead of the pure gas thermal conductivity, and of the matrix, instead of the bulk conductivity, to evaluate the heat conduction losses in the pulse tube and regenerator by the oscillations. We will show that the value of the effective heat conduction losses of the helium gas, solid wall, and regenerator materials in a pulse tube cooler are typically of the same order of magnitude.

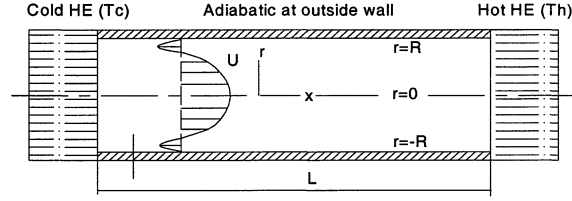


FIGURE 1. Schematic illustration of a circular tube with two heat exchangers at different temperatures at its ends. Temperature boundary conditions and velocity profile are also shown.

MATHEMATICAL FORMULATION

To simplify the analysis and to avoid the complicated mathematics as much as possible, we assume that the regenerator is made of a series of parallel circular tubes and neglect any geometric complexity in it. Therefore, we treat the regenerator and the pulse tube as the same configuration and restrict our attention to an incompressible laminar flow oscillating sinusoidally in a single or a series of parallel tubes of inner radius $r = R$, of thickness δ and of length L connecting two heat exchangers at cold temperature $T = T_C$ (left side) and at high temperature $T = T_H$ (right side), as illustrated in Fig. 1. The outer sidewall of the tube is thermally insulating (adiabatic). The tube is filled with pressurized helium gas (we restrict attention to ideal gases), which moves back and forth with the oscillatory pressure P at the angular frequency $\omega = 2\pi f$. We assume the maximum Reynolds number associated with the oscillations is not too high (laminar flow) and one can neglect entrance effects. Gas velocity U is in the x direction only and varies only in the r -coordinate direction normal to the tube wall. Within these limitations, the momentum equation becomes:

$$\rho \frac{\partial U}{\partial t} = -\frac{\partial P}{\partial x} + \mu \frac{1}{\eta} \frac{\partial}{\partial \eta} \left(\eta \frac{\partial U}{\partial \eta} \right) \quad (1)$$

Here $\eta = r/R$ is the dimensionless radial distance and μ is the dynamic viscosity.

One can easily derive [18] that the gas velocity of the one-dimensional flow in the tube is in the time-dependent form:

$$U(\eta, t) = U_0 \left[\frac{J_0(\sqrt{-i}\sqrt{2l/\delta_v}) - J_0(\eta\sqrt{-i}\sqrt{2l/\delta_v})}{J_0\sqrt{-i}\sqrt{2l/\delta_v} - 1} \right] \exp(i\omega t) = U_0 F(\eta) \exp(i\omega t) \quad (2)$$

where U_0 is the amplitude of the gas velocity on the centerline, J_0 the Bessel function of the first kind of order zero, and $F(\eta)$ the complex-value function for the gas velocity profile. We introduce here the relevant boundary-layer thickness [17]: the viscous penetration depth $\delta_v = \sqrt{2\mu/\rho\omega}$ and the thermal penetration depth $\delta_k = \sqrt{2k/\rho\omega C_p}$. Note that the Prandtl number $\text{Pr} = (\delta_v/\delta_k)^2$. For the sinusoidal oscillating flow considered here, the amplitude of the gas velocity U_0 is related to the amplitude of the fluid displacement x_0 by

$$U_0 = X_0 \omega / 2 \quad (3)$$

Having determined the gas velocity distribution, the gas energy equation can be used to solve for the corresponding temperature distribution in the tube, which is given by

$$\rho C_p \frac{\partial T}{\partial t} + \rho C_p U \frac{\partial T}{\partial x} = \frac{\partial P}{\partial t} + U \frac{\partial P}{\partial x} + k \left[\frac{\partial}{\partial x} \left(\frac{\partial T}{\partial x} \right) + \frac{1}{\eta} \frac{\partial}{\partial \eta} \left(\eta \frac{\partial T}{\partial \eta} \right) \right] + \mu \left(\frac{\partial U}{\partial \eta} \right)^2 \quad (4)$$

Equation (4) can only be solved numerically. Fortunately, in many cases the general solution to equation (4) is not needed and it can be approximated by simple expressions, which can be solved analytically by the method of successive approximations. The viscous dissipation term $\mu(\partial U / \partial \eta)^2$ in equation (4) is often neglected since it is typically a few orders of magnitude less than the other terms as long as one does not deal with high Prandtl number fluids.

Merkli and Thomann [19] worked out the acoustic solutions to first and second-order at the mean fluid velocity $U_0 = 0$ (the basic state is at rest) in terms of a uniform wall temperature T_0 for circular cylinder geometry (in cylindrical coordinates). Swift [17] derived the expressions for the first-order temperature $T_1(\eta)$ and the corresponding time-averaged second-order energy flux $\dot{H}_2(x)$ at the mean fluid velocity $U_0 = 0$ in terms of the mean temperature distribution $T_m(x)$ for parallel plate geometry.

A restricted solution, which applies to linear time-averaged longitudinal temperature distributions, was given by Kurzweg [15] using the appropriate boundary condition. He recognized that the viscous sinusoidal oscillatory flow, within parallel plate channels with thermally insulating side walls and connecting two different temperature reservoirs at its ends can conduct heat at rates orders of magnitude greater than by pure gas conduction. The enhanced heat conduction by the oscillation was found to be proportional to the square of the oscillation amplitude and a function of the Prandtl number, the frequency, and the tube radius. The prediction was also verified by experimental measurements [16].

The result on the enhanced heat transfer in parallel plates suggests that a similar phenomenon should occur in the present geometry. In view of the mathematical manipulations for the parallel plate geometry with two large reservoirs maintained at different temperatures given by Kurzweg [15], we find that the effective enhanced thermal conductivity k_e can be expressed as a function of the Prandtl number Pr , the kinetic Reynolds number Re_ω , and the dimensionless oscillation amplitudes of the gas flow A_0 :

$$\frac{k_e}{k} = 1 + \left(\frac{Pr A_0}{2} \right)^2 f(Re_\omega) \quad (5)$$

Where $A_0 = X_0 / D_h = 2U_0 / \omega D_h$ is the dimensionless oscillation amplitude of the gas flow and $Re_\omega = \rho \omega D_h^2 / \mu$ is the kinetic Reynolds number with $D_h = 2R$ being the hydraulic diameter of the circular tube. These two parameters were introduced by Zhao and Cheng [11] in experimental studies on oscillatory pressure drops through a woven-screen packed column subjected to a cyclic flow. Note that $A_0 = 2Re_0 / Re_\omega$, in which $Re_0 = \rho U_0 D_h / \mu$ is the Reynolds number for a steady flow based on the hydraulic diameter and the maximum axial velocity of the tube, provides an approach for correlation of fluid flow characteristics in a steady flow and in a sinusoidally oscillatory flow. It also indicates that the fluid

displacement (or gas velocity) and the frequency of oscillation cannot be isolated for the oscillating flow. The term $f(\text{Re}_\omega)$, which is a function of kinetic Reynolds number, represents a measure of the ratio of the enhanced thermal conductivity k_e to the thermal conductivity k of pure gas. In the case of monatomic gases with the Prandtl number $\text{Pr} \sim 1$, $f(\text{Re}_\omega) = 1 \sim 10$, for liquid metals with $\text{Pr} \sim 0.01$, $f(\text{Re}_\omega) = 10^2 \sim 10^3$.

Knowing the effective coefficient of thermal conductivity k_e (de Waele, et al. [20], first introduced it for considering the geometry of the regenerator matrix), the effective axial heat flow per tube is given by the revised Fourier's law of heat conduction

$$q = -\pi R^2 k_e \frac{\partial T}{\partial x} \quad (6)$$

The fundamental difference between equation (6) and the basic Fourier's law of heat conduction for the steady state condition is that we use here the effective coefficient of thermal conductivity k_e instead of the thermal conductivity k of pure gas. Substituting equation (5) into equation (6) and with the temperature gradient $\partial T / \partial x = -(T_H - T_C) / L$, one can obtain

$$q = \pi R^2 k \left[1 + f(\text{Re}_\omega) \left(\frac{\text{Pr} A_0}{2} \right)^2 \right] \frac{T_H - T_C}{L} \quad (7)$$

The term within the square bracket in equation (7) can be considered as a measure of the ratio of axial heat flow through the tube under oscillating flow conditions to that under unidirectional steady flow. An example is calculated taking values for helium as the working gas: for $\text{Pr} = 0.7$, $U_0 = 5 \text{ m/s}$, $f = 10 \text{ Hz}$ in a 10mm diameter tube, one finds $A_0 \sim 10$, then $(\text{Pr} A_0 / 2)^2 \sim 10$, with $f(\text{Re}_\omega) = 1 \sim 10$. According to equation (7), the increase in conduction heat flow, will be at least 10 times higher than that in the steady flow for this case. It suggests that a thermally conducting helium gas confined in a single circular pulse tube connecting two heat exchangers at different temperatures at its ends can conduct heat at rates an order of magnitude greater than by pure gas conduction, provided the gas flow is oscillated sinusoidally in the tube. Similarly enhanced heat conduction should occur in the void volume (between stacked screens) of the regenerator, which consists essentially of a series of parallel circular tubes, via sinusoidal oscillatory flow.

EXPERIMENTAL VERIFICATION

Compared to steady flow, oscillating flow can produce enhanced heat conduction, which is essentially due to the interaction of the oscillating gas flow and the boundary layer. This has been verified by experimental measurements [16]. Recently, Kuriyama et al. [13,14] designed an experimental apparatus for measurement of the heat conduction through stacked screens from room temperature to cryogenic temperature. The experimental results showed that the helium gas between each screen enhanced the heat conduction through the stacked screens by several orders of magnitude compared to that in vacuum. They also found that the conduction degradation factor, defined as the ratio of the actual heat conduction to the heat conduction where the regenerator material is assumed to be a bulk, was about 0.1, and the factor was almost constant for the temperature range between 40K and 80K at the cold end. We will use this conclusion in next section.

REALISTIC CASES

In this section we will apply our results to realistic cases. The dimensions of two pulse tube coolers are presented in Table 1. The solid walls of the pulse tube and regenerator are made of thin stainless steel tubes. Some typical data for helium at 1.5MPa and in the temperature range from 300K to 4K are listed in Table 2. The effective coefficient of thermal conductivity of the gas in the regenerator matrix is assumed to be ten times larger than the pure gas thermal conductivity. Table 3 is a selection of typical data for stacked stainless steel screens as the regenerator matrix. The effective coefficient of thermal conductivity of the matrix is assumed to be ten times smaller than the bulk conductivity.

We use the correlation of arithmetic mean to calculate the temperature-dependent averaged thermal conductivities for the helium gas, regenerator matrix, and solid wall, yielding 1.08W/mK for helium gas, 1.125W/mK for regenerator matrix, and 11.25W/mK for solid wall at temperature range of 300-70K and 0.421W/mK for helium gas, 0.389W/mK for regenerator matrix, and 3.89W/mK for solid wall at temperature range of 70-4K, respectively. Then equation (7) is adapted to calculate the effective heat conduction losses for the helium gas, the solid wall, and the regenerator matrix.

TABLE 1. A selection of dimensions for two typical pulse tube coolers

Components	O. D. (mm)	I. D. (mm)	Length (mm)	Volume (cm ³)	Cross-section area (cm ²)	Tube wall area (cm ²)
Regenerator 1	60	59	200	382.56	27.32	0.934
Pulse tube 1	35	34	200	127.04	9.07	0.542
Regenerator 2	30	29	200	118.83	6.60	0.463
Pulse tube 2	18	17	200	40.83	2.26	0.275

TABLE 2. Some typical data for helium gas (As an example pressure P=1.5 MPa)

Temperature (K)	300	150	100	70	40	20	10	4
Thermal conductivity for pure gas (W/m K)	0.157	0.0982	0.0752	0.0599	0.0432	0.0311	0.0267	0.0243
Effective thermal conductivity (W/m K)	1.57	0.982	0.752	0.599	0.432	0.311	0.267	0.243

TABLE 3. A selection of typical data for stainless steel screens as the regenerator matrix

Temperature (K)	300	150	100	70	40	20	10	4
Thermal conductivity (W/m K)	15	12.5	9.5	7.5	4.8	1.6	0.72	0.28
Effective thermal conductivity (W/m K)	1.5	12.5	0.95	0.75	0.48	0.16	0.072	0.028

TABLE 4. Typical effective heat conduction losses in the temperature range of 300K-70K
(Effective coefficient of thermal conductivity: 1.08 W/m K for helium gas, 1.125 W/m K for regenerator matrix, and 11.25 W/m K for stainless steel wall)

Heat conduction loss (W)	Gas heat conduction	Wall heat conduction	Matrix heat conduction	Ratio
Regenerator 1 (porosity= 0.7)	3.39	1.73	1.52	2.2 : 1.1 : 1
Pulse tube 1	1.61	1.00		1.6 : 1
Regenerator 2 (porosity= 0.7)	0.637	0.665	0.285	1 : 1.4 : 1
Pulse tube 2	0.319	0.395		1 : 1.2

TABLE 5. Typical effective heat conduction losses in the temperature range of 70K-4K (Effective coefficient of thermal conductivity: 0.421 W/m K for helium gas, 0.389 W/m K for regenerator matrix, and 3.89 W/m K for stainless steel wall)

Heat conduction loss (W)	Gas heat conduction	Wall heat conduction	Matrix heat conduction	Ratio
Regenerator 1 (porosity= 0.5)	0.271	0.171	0.251	1.6 : 1 : 1.5
Pulse tube 1	0.180	0.0994		2 : 1
Regenerator 2 (porosity=0.5)	0.0509	0.0660	0.0471	1 : 1.3 : 1
Pulse tube 2	0.0349	0.0392		1 : 1.1

Table 4 and Table 5 present the typical data of the effective heat conduction losses for the helium gas, the solid wall, and the regenerator matrix for the two pulse tube coolers operating at temperature range of 300K-70K and of 70K-4K, respectively. One finds that the ratios of the effective heat conduction losses of the three are about 1~2. This means that the effective heat conduction losses of the helium gas, the solid wall, and the regenerator matrix in a practical pulse tube cooler are typically of the same order of magnitude. As a result, the heat conduction losses of the solid wall and the gas inside the pulse tube and the regenerative matrix should be included as well, provided the heat conduction loss of the regenerator matrix is considered.

If, one does not consider the enhanced heat conduction of the gas in the pulse tube and the regenerator matrix, the ratios of the heat conduction losses of the helium gas, solid wall, and regenerative matrix in a pulse tube cooler are of orders of magnitude from 1:5:50 to 1:10:100. This means that the heat conduction losses of the solid wall and regenerative matrix are at least 5 and 50 times larger than that of the gas inside the regenerator matrix. In conclusion, the heat conduction losses of the gas in the pulse tube and the regenerator matrix can be neglected only in the steady flow or very low frequency conditions.

It should be pointed out that in the present paper only a very crude approximation is made for estimating the enhanced heat conduction of the gas inside the pulse tube and the regenerator matrix with adiabatic sidewalls, under oscillating flow conditions. In order to predict heat transfer rates and pressure drops accurately in a cryocooler regenerator, extensive work should be carried out to build and formulate generalized Nusselt number and generalized friction factor correlations, for oscillating flow that is in tubes or channels or a porous matrix, for which is turbulent, or for which entrance effects are significant or the longitudinal temperature gradients is nonlinear. This is arduous and is likely to come only from a combined theoretical analyses and experimental measurements.

CONCLUSIONS

The paper discussed the enhanced heat conduction characteristics of the helium gas in the pulse tube and regenerator subjected to the oscillating flow conditions. It has been shown that the enhanced heat conduction of the helium gas confined in the pulse tube or in the void volume of the regenerator by the oscillations is typically of an order of magnitude larger than the heat conduction of pure gas. As a result, the effective coefficient of thermal conductivity of the oscillating gas flow in the pulse tube and the regenerator is typically of an order of magnitude larger than the pure gas thermal conductivity. While the effective coefficient of thermal conductivity of the regenerator matrix is typically of an order of magnitude less than the bulk value. In practical cases, the value of the effective heat conduction losses of the helium gas, the solid wall, and the regenerator matrix in a pulse tube cooler are of the same order of magnitude. The effective coefficient of thermal conductivity introduced here is useful to perform and develop one-dimensional numerical

model, in which the heat transfer correlation needs to be inserted into the model, for the pulse tube cooler under oscillating flow conditions.

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