REAL GAS FEATURES ON THE PERFORMANCE OF PULSE TUBE CRYOCOOLERS

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ABSTRACT

The working helium gas in a pulse tube cryocooler operating at temperatures down to 80K is mainly assumed to be an ideal gas. Therefore, the time-variations of the temperature profiles and the position of the gas element traveling with pressure oscillations inside the pulse tube can be readily determined by the law of Poisson function. However, this is certainly invalid for the pulse tube cryocooler operating at temperature range of liquid helium, in which the thermal properties of the helium gas change drastically. The temperature profiles in the regenerator and the pulse tube are strongly affected by the real thermal properties of the helium gas. We derive in this paper, the respective expressions to follow the tracks of the gas elements as they move in the pulse tube, and to reveal the time dependence of the temperature profiles and the position of gas elements traveling with the pressure oscillations inside the pulse tube. The approach is based on the thermodynamic equations for the real gas. We will show that contrary to the ideal gas case there is another term which determines the dynamic behaviors of the temperature distributions and the position of the gas elements. A typical calculation is presented for visualizing the time dependence of the cooling-down processes of the temperature profiles in the pulse tube of a 4K two-stage pulse tube cryocooler from room temperature down to low temperature.

INTRODUCTION

Pulse tube cryocooler, first introduced in 1963 [1], has been extensively studied in recent years due to its inherent simplicity, high reliability and low vibration owing to the absence of mechanical moving components in the low temperature region. Two important improvements called orifice pulse tube and double-inlet pulse tube were made by Mikulin et al. [2] in 1984 and Zhu et al. [3] in 1990, respectively. Together with other innovations [4-9] pulse tube coolers have achieved comparable refrigeration capabilities to Stirling and G-M coolers [10-12].

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The internal processes of pulse tube cryocoolers are very complex due to the unsteady, oscillating compressible gas flow, the regenerator and the orifice and double-inlet valves, etc. The understanding of the internal physical processes and time-variations of dynamic parameters associated with the pulse tube cryocooler is of crucial importance in optimal design. Unfortunately, besides experimental measurements, quantitative evaluation usually requires a full solution of the Navier-Stokes equations and the complexities of these equations make an analytical solution essentially impossible. Thus, numerical simulations [13-16] are widely used and have been extensively developed in recent years to study the performance of pulse tube cryocoolers. Due to the complexity of partial differential equations for the mass, momentum, energy conservation, and equation of state, accurate evaluation of these compressible equations for the gas velocity, pressure and temperature is complicated and needs highly refined computational grids and numerical iteration for sharp variations of thermal properties.

In order to avoid complicated mathematic calculation and computational iteration of the numerical simulation, the method of characteristics was used by Wu and Chen [17] to analyze the gas elements in the pulse tube entering from the cold end to the hot end. In such method along a characteristic direction, the integration of a partial differential equation reduces to the integration of an equation involving total differentials only. Therefore, Wu and Chen [17] directly obtained the expressions for the velocity of gas flow along the pulse tube, and the position of gas elements traveling with pressure oscillations inside the pulse tube without any numerical iteration. More recently, de Waele et al. [18] and Xu et al. [19] used the method of characteristics in the analysis of the dynamic behavior of the temperature profiles of the gas near the hot and cold ends of the pulse tube, and the behavior of various gas elements entering the cold end in an orifice pulse tube cryocoolers. To our knowledge, the method of characteristics was first used by Beson et al. [20-21] in 1960s in the analysis of diesel engines. In 1984 Taylor [22] used this method to model Stirling engines.

However, all of above researchers in using the method of characteristics, assumed the working gas to be an ideal gas thereby, the time-variations of the temperature profiles and the position of the gas elements traveling with the pressure oscillation inside the pulse tube can be readily determined by the law of Poisson function. This is certainly invalid for pulse tube cryocoolers operating at liquid helium temperature, in which the thermal properties of the working gas change drastically. The temperature profiles in the pulse tube and in the regenerator are strongly affected by the thermal properties of the real helium gas, which have a profound effect on the energy balance and the performance of the cryocooler. For example, de Waele et al. showed that the non-ideal gas properties have a profound effect on the energy balance in the regenerator and on the expression for the cooling power [23]. But up till now, the contributions of the real gas effects to the method of characteristics has not been analyzed.

We will derive in this paper, the respective expressions to follow the tracks of the gas elements as they move inside the pulse tube, and to reveal the time-variations of the temperature profiles and the position of the various gas elements traveling with the pressure oscillations inside the pulse tube. The approach is based on the thermodynamic equations for the real gas. We will show that contrary to the ideal gas case there is another term, by which determines the dynamic behavior of the temperature distributions and the position of the gas elements. A typical calculation is given for visualizing the time dependence of the cooling-down processes of the gas temperature profiles in the regenerator and the pulse tube of a 4K two-stage pulse tube cryocooler from the room temperature down to the lowest temperature.

GENERAL EQUATIONS

In order to derive the general expressions for the time-variations of the temperature and the position of the gas element traveling inside the pulse tube with pressure oscillations, we begin from the first and second laws of thermodynamics. For the gas molar internal energy U_m, molar volume V_m, molar enthalpy H_m, molar entropy S_m with gas pressure p and temperature T, the general thermodynamic equations for a real gas are given by

$$dU_m = TdS_m - pdV_m \tag{1}$$

$$dS_m = \frac{C_P}{T}dT - \left(\frac{\partial V_m}{\partial T}\right)_P dp \tag{2}$$

$$dH_m = TdS_m + V_m dp \tag{3}$$

$$dH_m = C_P dT + \left\{V_m - T\left(\frac{\partial V_m}{\partial T}\right)_P\right\}dp = C_P dT + (1 - T\alpha_V)V_m dp \tag{4}$$

here α_V is the volumetric thermal expansion coefficient

$$\alpha_V = \frac{1}{V_m}\left(\frac{\partial V_m}{\partial T}\right)_P \tag{5}$$

In the ideal gas case $\alpha_V = 1/T$.

GAS TEMPERATURE PROFILES INSIDE THE TUBE

Combining equations (3) and (4) yields

$$TdS_m = C_P dT - T\alpha_V V_m dp \tag{6}$$

So we have

$$\left(\frac{\partial T}{\partial p}\right)_{S_m} = \frac{1}{C_P}\alpha_V T V_m \tag{7}$$

In the adiabatic compression process, the change of gas temperature with the change of gas pressure can be expressed as

$$\frac{dT}{dt} = \left(\frac{\partial T}{\partial p}\right)_{S_m}\frac{dp}{dt} \tag{8}$$

Substituting equation (7) into equation (8) gives

$$dT = \frac{1}{C_P}\alpha_V T V_m dp \tag{9}$$

Therefore, the temperature of any one of gas elements inside the pulse tube with the gas pressure oscillations is given by solving equation (9)

$$T = T^0 + \frac{1}{C_P}\alpha_V T V_m (p - p^0) \qquad (10)$$

The superscript *0* means variables at last time, variables without superscript stand for the value at present time. In the ideal gas case, equation (10) becomes

$$T = T^0 + \frac{V_m}{C_P}(p - p^0) \qquad (11)$$

GAS VELOCITY INSIDE THE TUBE

In general, we have (equation (23) in Ref [23])

$$T dS_m = C_V \left(\frac{\partial T}{\partial p}\right)_{V_m} dp + C_P \left(\frac{\partial T}{\partial V_m}\right)_P dV_m \qquad (12)$$

from which follows

$$\left(\frac{\partial V_m}{\partial p}\right)_{S_m} = \frac{C_V}{C_P}\left(\frac{\partial V_m}{\partial p}\right)_T \qquad (13)$$

Equation (13) can be rewritten as

$$\left(\frac{\partial V_m}{\partial p}\right)_{S_m} = -\frac{C_V}{C_P}\kappa_T V_m \qquad (14)$$

with the isothermal compression coefficient

$$\kappa_T = -\frac{1}{V_m}\left(\frac{\partial V_m}{\partial p}\right)_T \qquad (15)$$

In ideal gas case $\kappa_T = -1/p$. The change of a subsystem volume V_i that contains N number of moles of gas elements inside the pulse tube under adiabatic compression is

$$\frac{dV_i}{dt} = \int_x^{L_t} \left(\frac{\partial V_m}{\partial p}\right)_{S_m} \frac{dp}{dt} dN = \frac{dp}{dt}\int_x^{L_t} \left(\frac{\partial V_m}{\partial p}\right)_{S_m} \frac{A_t}{V_m} dl \qquad (16)$$

where l is the axial length coordinate, L_t the length, and A_t the cross-section area of the pulse tube. Substituting equation (15) into equation (16) gives

$$\frac{dV_i}{dt} = -\frac{dp}{dt}\int_x^{L_t} \frac{C_V}{C_P}\kappa_T A_t dl \qquad (17)$$

Therefore, the velocity of any one of gas elements at axial position x of the pulse tube, related to the velocity at the hot end of the pulse tube is given by

$$u_i = u_H + \frac{dp}{dt}\int_x^{L_t} \frac{C_V}{C_P}\kappa_T dl \tag{18}$$

Contrary to the ideal gas case the second term in equation (18) depends on the pressure oscillation and the temperature distribution inside the pulse tube. The gas velocity at the hot end of the pulse tube, can be calculated from the molar flow rate

$$u_H = \frac{n_H R T_H}{p_0 A_t} \tag{19}$$

From equation (18), the distance of any one of gas elements at axial position x of the pulse tube, traveling inside the pulse tube can be expressed as

$$x_i = x_H + dp\int_x^{L_t} \frac{C_V}{C_P}\kappa_T dl \tag{20}$$

The position of gas element traveling with the pressure inside the pulse tube, leaving the hot heat exchanger at $x = L_t$, at time $t = t_0$, is given by solving equation (20)

$$l = L_t - (p - p^0)\cdot\int_l^{L_t} \frac{C_V}{C_P}\kappa_T dx \tag{21}$$

By using expressions (10) and (21) we can readily obtain the gas temperature profile and the position of the gas element traveling with pressure oscillations inside the pulse tube. Therefore, we can follow the tracks of any one of gas particles in the pulse tube.

Upon knowing the temperature profile and the position of the gas elements traveling with pressure oscillations inside the pulse tube of a pulse tube cryocooler, we can directly obtain the time-dependent temperature variations of the gas and wall at the cold ends of the pulse tube and the heat exchange rate (cooling power) in them. Next we can solve the time-variations of dynamic parameters in the regenerator by using numerical calculation from an initial guess through an iterative scheme. Such works will be presented elsewhere.

RESULTS AND DISCUSSIONS

A typical calculation is made for a two-stage pulse tube cryocooler, as illustrated in Figure 1. The structure and operating parameters are listed as follows:

The 1st-stage regenerator is 59mm in inner diameter and 150mm in length, filled with 250-mesh stainless steel screens and, its filling factor is assumed as 0.3. The 2nd-stage regenerator is 29mm in inner diameter, and 150mm in length, filled with lead spheres in 50% of the length and with Er_3Ni in 50 % of the length, and its filling factor is 0.6. The 1st-stage pulse tube has 35mm inner diameter and is 150mm long, and the 2nd-stage pulse tube has 18mm inner diameter and is 300 mm long. The volume of the two gas buffers is 1000cm^3. The temperature is 300K at hot end, 40K at 1st-stage cold end, and 4K at 2nd-stage cold end. The operating frequency is 1Hz and the average pressure is 1.6MPa.

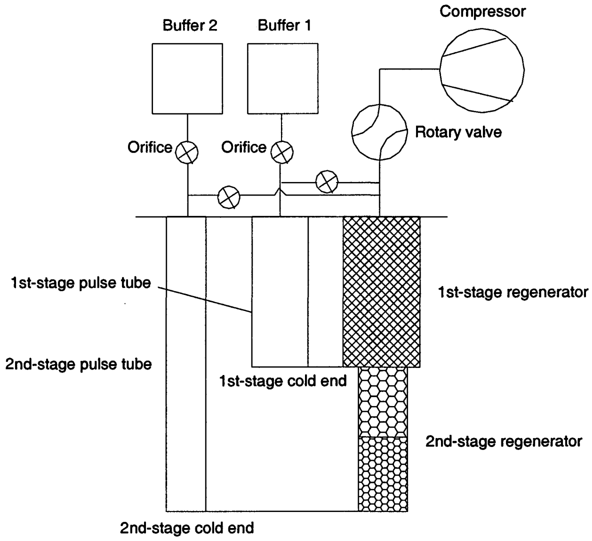

FIGURE 1. Schematic diagram of a two-stage pulse tube cryocooler, the total length of regenerator and the 2^{nd}-stage pulse tube are the same. The 1^{st}-stage pulse tube is connected at middle point of the regenerator.

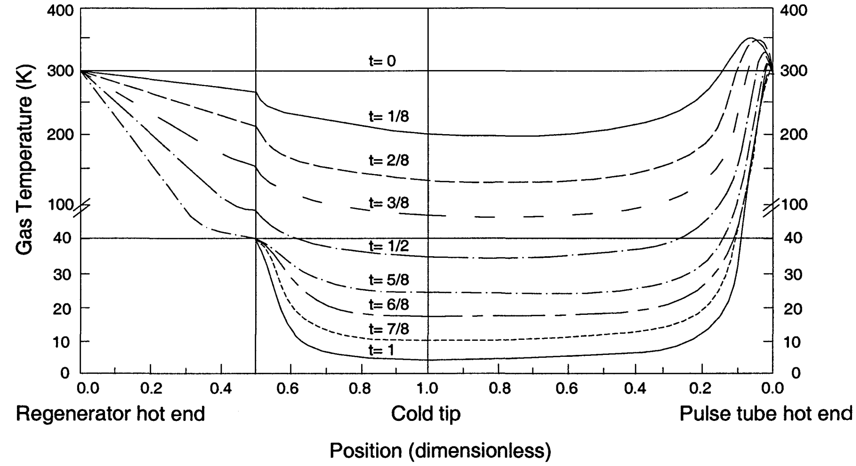

FIGURE 2. Time dependence of the cooling-down processes of the gas temperature profiles in the regenerator and the pulse tube

Figure 2 shows the time dependence of the cooling-down process of the gas temperature profiles from room temperature $T = 300K$ at initial dimensionless time $t = 0$ down to the lowest temperature $T = 4K$ at dimensionless time $t = 1$ along the total length of the regenerator and along the length of the 2^{nd}-stage pulse tube. Eight time interval is presented in Figure 2. At initial time (dimensionless time $t = 0$), the temperatures of all gas elements in the regenerator and the pulse tube are set at room temperature $T = 300K$, then the gas temperatures of gas elements drop with time. After many cycles (dimensionless time $t = 1$), the pulse tube cryocooler reaches its minimum temperature of $T = 4K$.

Figure 2 shows a monotonous, almost linear temperature profile in the 1^{st}-stage regenerator at time intervals of $t = 1/8$, $t = 2/8$, and $t = 3/8$. Non-linear temperature profile at other time intervals and in the 2^{nd}-stage regenerator are indicated. It also shows that about 80~90% of the temperature drop is concentrated in the 40% of the pulse tube length near the hot end. The temperature gradients of the gas elements are very large in this region. While almost constant temperature appears in the low-temperature side of the pulse tube. The temperature profiles are rather flat in the 40% of the pulse tube length near the cold end. Similar features can be found inside the regenerator.

It has been shown that the behavior of the cooling-down processes and the temperature profiles in the low temperature regenerator and the pulse tube, especially in the liquid helium range, is completely different from that of the high temperatures in which the working helium gas is assumed to be ideal. The author reasons that the unique feature originates from the thermal properties of the real helium gas that change drastically in the low temperatures. The time-dependent temperature profiles in the regenerator and the pulse tube are strongly affected by the real helium thermal properties.

CONCLUSIONS

We have derived in this paper, the respective expressions to follow the tracks of the gas elements as they move in the pulse tube, and to reveal the time-dependence of the temperature profiles and the position of gas elements traveling with the gas pressure oscillations inside the pulse tube. The approach is based on the thermodynamic equations for the real gas. We have showed that contrary to the ideal gas case there is another term, by which determines the dynamic behaviors of the temperature distributions and the position of the gas elements. A typical calculation and results have been presented for visualizing the time dependence of the cooling-down processes of the gas temperature profiles in the regenerator and the pulse tube of a 4K two-stage pulse tube cryocooler from room temperature down to the lowest temperature. Some unique features and phenomena of the real helium gas on the temperature profiles of a 4K two-stage pulse tube cryocooler have been revealed and discussed.

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