Measurements of the flow resistance and inductance of inertance tubes at high acoustic amplitudes

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ABSTRACT

We recently measured the flow resistance and flow inductance of inertance tubes at high acoustic amplitudes for four different inner diameters of 0.6, 1.0, 1.5 and 2.0mm at various tube length ranging from 100 to 1500mm, at frequencies of 30, 40, 50, 60 and 70 Hz. The experimental data were compared with the explicit solution to the linear momentum equation for small acoustic amplitudes and were fitted with the modification coefficients in terms of the operating frequency and Reynolds numbers characterized by the amplitude of gas velocities.

INTRODUCTION

Recent studies [1-2] described a simple way of an inertance tube, to replace the orifice, at hot end of the pulse tube to generate a proper phase shift to improve the pulse tube performance. The inertance tube is a long, thin tube that provides a complex impedance at the warm end of the pulse tube rather than a simple resistive impedance provided by the orifice. The inertance tube adds a reactive impedance, analogous to inductance in a simple AC electrical circuit, that allows the phase difference between the pressure and mass flow rate in the pulse tube to be adjusted to an extent as efficiently as Stirling coolers. In brief, the inertance tube is an acoustic term connoting both inertia resistance and inductance of moving gas [2].

Studies show that use of the inertance tube is significantly beneficial for large-scale pulse tubes or at higher operating frequencies [1-3]. They indicate that the improvement in the performance by using the inertance tube comes from: (1) it can produce a desirable phase shift between the pressure and mass flow in the pulse tube, (2) it can balance the intrinsic tendency for DC gas flow in the double-inlet pulse tube cooler; and (3) it can increase work flow per mass flow within the pulse tube. However, little information is available on experimental measurements of the flow resistance and flow conductance of the inertance tube, particularly, at high acoustic amplitudes. We recently measured the flow resistance and flow inductance of inertance tubes at high acoustic amplitudes [4]. In this paper, we briefly summarize our previous work and present some useful results.

THEORETICAL CONSIDERATION AND FORMULATION

Considering an incompressible flow oscillating sinusoidally in an inertance tube of inner radius $r = a$, and of length L connecting two large reservoirs at room temperatures $T = T_0$. The outer sidewall of the tube is adiabatic. The tube is filled with pressurized ideal helium gas, which moves back and forth with the oscillatory pressure at the angular frequency. We assume the maximum Reynolds number associated with the oscillation is not too high (laminar flow) and one can neglect end effects. Gas velocity u is in the x direction only and varies only in the r-coordinate direction normal to the tube wall. Within these limitations, and considering the nonlinear inertia term can be neglected at low acoustic amplitudes, one can easily derive that the gas velocity in the tube is in the form

$$u_0 = \frac{1}{\rho_0 \omega} \frac{\partial p_0}{\partial x} \left[1 - \frac{J_0(\sqrt{2}r/\delta_v)}{J_0(\sqrt{2}a/\delta_v)} \right] \tag{1}$$

Here u_0 and p_0 are the amplitude of the gas velocity and pressure oscillation, respectively, J_0 is the Bessel function of the first kind of order zero, and $\delta_v = \sqrt{2\mu/\rho_0\omega}$ is the viscous penetration depth denoting the relevant boundary-layer thickness. Integrating and averaging Eq. (1) over the across-sectional area $S = \pi a^2$ one yields the cross-sectional mean velocity

$$\bar{u}_0 = \frac{1}{2a}\int_{-a}^{a} u_0 dr = \frac{1}{\rho_0 \omega} \frac{\partial p_0}{\partial x} \left[1 - \frac{2J_1(Ka)}{KaJ_0(Ka)} \right] \tag{2}$$

Where $K = \sqrt{2}/\delta_V$. The term $-\partial p_0/\partial x$ is the pressure drop per unit tube length and $-S\partial p_0/\partial x$ is the force on the tube per unit length, thus the force impendence per unit tube length can be expressed as

$$Z'_M = -S\frac{\partial p_0}{\partial x}/\bar{u}_0 \, . \tag{3}$$

Substituting Eq. (2) into Eq. (3), and integrating it over the total length L of the tube and assuming L is much smaller than the local sound wavelength, we find the force impendence along the tube

$$Z_A = -\frac{\rho_0 \omega L/S}{1 - 2J_1(Ka)/KaJ_0(Ka)} \tag{4}$$

The flow impendence is complex in general and can be simply approximated as

$$Z_A = \frac{8\mu L}{\pi a^4}\sqrt{1+|Ka|^2/32} + j\frac{\rho\omega L}{\pi a^2}\left(1+\frac{1}{\sqrt{3^2+|Ka|^2/2}}\right) \tag{5}$$

Its real part and imaginary part indicates the flow resistance and flow inductance, respectively,

$$R_A = \frac{8\mu L}{\pi a^4}\sqrt{1+|Ka|^2/32} \tag{6}$$

$$X_A = \frac{\rho_0\omega L}{\pi a^2}\left(1+\frac{1}{\sqrt{3^2+|Ka|^2/2}}\right) \tag{7}$$

The flow resistance coefficient for the inertance tube under oscillatory flow conditions can be rewritten as

$$f = \frac{64}{\mathrm{Re}_0}\sqrt{1+\frac{|Ka|^2}{32}} \tag{8}$$

Comparing Eq. (8) to the flow factor of the tube for the steady-state flow $f_{st} = 64/\mathrm{Re}_0$ one yields

$$\beta = f/f_{st} = \sqrt{1+|Ka|^2/32} > 1 \tag{9}$$

Eq. (9) indicates that at low acoustic amplitudes the flow resistance coefficient of the inertance tube in the oscillatory flow is larger by a factor of β than that of a steady-state flow at the same Reynolds numbers. Fig. 1

shows the factor of β of inertance tubes with different diameters of 0.6, 1.0, 1.5 and 2.0 mm as a function of frequency. It shows the factor is in the range of 1.04~1.1 for 0.6mm tube, and of 1.4~1.8 for 2mm tube, respectively. However, the correlation given above based on the assumption of the low acoustic amplitudes is usually not applicable for a pulse tube cooler operating at high acoustic amplitudes (large pressure amplitudes), in which the nonlinear inertia term cannot be neglected. Therefore, we introduce here two modification coefficients C_1 and C_2 for considering the nonlinear effects at high acoustic amplitudes.

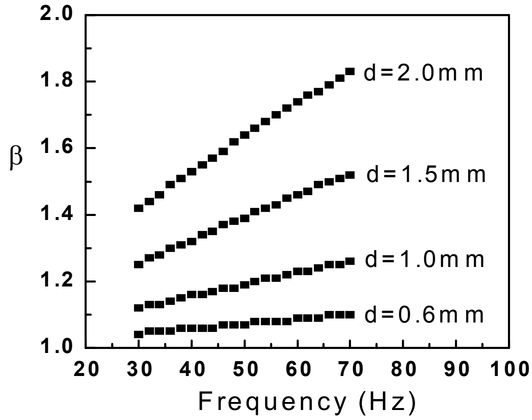


Figure 1 β as a function of frequency

$$R_A' = C_1 R_A \quad (10)$$

$$X_A' = C_2 X_A \quad (11)$$

Below we will describe the experimental procedures for the two parameters measurements to determine the modification coefficients C_1 and C_2.to determine the modification coefficients of C_1 and C_2.

EXPERIMENTS

The detail experimental set-up to determine the value of C_1 and C_2 has been shown in Ref. [4]. The oscillatory flow is generated by means of a self-made linear compressor with swept volume of 2cm^3, and its working frequency can be adjustable from 20 to 80Hz. The test section of different inertance tubes is made of thin wall stainless steel tube. At one end of the test section is placed a flow straightener. A reservoir is directly connected to the other end of the test section. Two small pressure transducers, connected to charge amplifiers are used to measure transient pressures at the inlet of the inertance tube, and inside the reservoir. Two thermocouples are placed on the surface of the flow straightener and in the reservoir to measure the temperatures.

The cross-sectional mean velocity of the oscillatory flow is determined by measuring the instantaneous pressure oscillations in the reservoir. Since this approach has already been described in the literatures [5,6], we show the results without describing the details. The transient pressure in the reservoir has usually small pressure amplitude; it is reasonable to assume the pressure oscillations are the steady and oscillatory part of the pressure in the reservoir, respectively. The amplitude of the gas velocity at the inlet of the reservoir can be expressed as

$$\overline{u} = \frac{p_1 V_{res} \omega}{A_t \rho c^2} e^{j\left(\omega t + \frac{\pi}{2}\right)} \quad (12)$$

Here A_t is the cross-sectional area of the flow straightener connecting to the reservoir and $c = \sqrt{\gamma RT}$ is the local sound speed. By measuring the transient pressure in the reservoir and using Eq. (12) one can determine the cross-sectional mean velocity of the oscillatory gas flow. We have used hot wire anemometer to measure the velocity of the oscillatory flow [7]. The average relative derivation between the velocities obtained by the hot wire anemometer and evaluated by Eq. (12) is about 3.35%, which is within experimental error.

Strictly speaking, Eq. (12) is valid only at: (1) the volume of inertance tube is much smaller than the volume of the reservoir, and (2) the tube length L is much smaller than the local sound wavelength of the working gas.

EXPERIMENTAL RESULTS AND DISCUSSES

Experiments were carried out for four different inner diameters (0.6, 1.0, 1.5 and 2.0mm) at various tube length ranging from 100 to 1500mm, at frequencies of 30, 40, 50, 60 and 70 Hz. The working gas was helium and the system mean pressure was 2.0MPa at room temperature. In order to determine the pressure amplitudes and their phase angle, the raw experimental data measured from the pressure transducers need data processing. The output signal of the pressure transducer was first amplified, pressure transformation and correction, and then collected with the spectrum analysis for high frequency harmonic filtration and Fourier analysis. The average uncertainty in the pressure amplitude and phase angle was evaluated to be less than 2%.

After obtained the transient pressures of p_t and p_{res} at the inlet of the inertance tube and in the reservoir and the amplitude of the gas velocity by using Eq. (12), one can calculate the flow impendence by using

$$Z_A = (p_t - p_{res}) / A_t \bar{u} = R_A' + jX_A' \tag{13}$$

Together with Eqs. (10) and (11) we can determine the coefficients of C_1 and C_2 in terms of Re_0 and f.

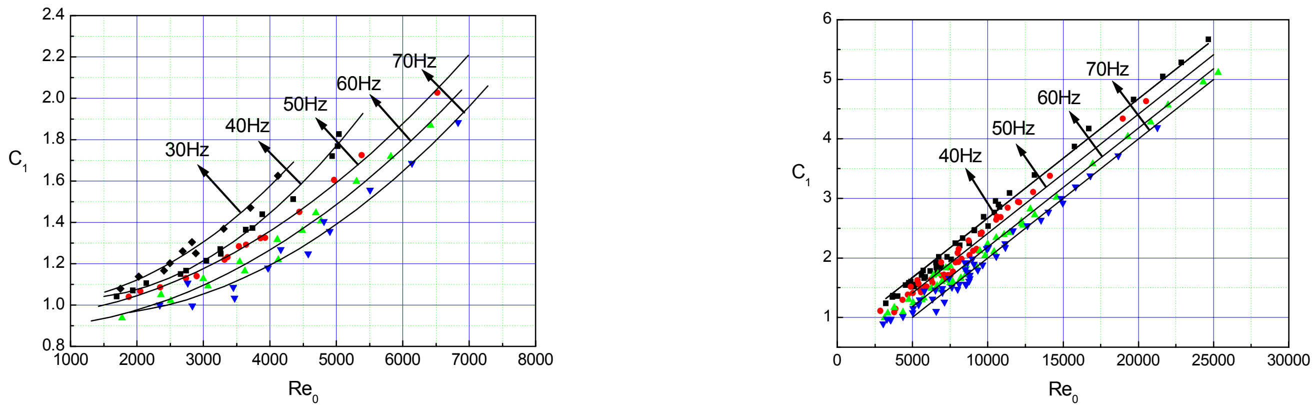
A series of experiments were carried out to determine the modification coefficients C_1 and C_2. Experimental results demonstrated that the flow resistance and inductance of inertance tubes at higher acoustic amplitudes strongly depend on the operating frequency and Reynolds numbers based on the amplitude of the cross-sectional mean velocity. The selected experimental results are summarized in Figs. 2 and 3.

Fig. 2(a) and (b) show the experimental data of the correlation coefficient C_1 of the flow resistance of inertance tubes for three different tube diameters of 0.6mm and 1.0mm, respectively, in terms of Re_0 and f. The correlation coefficient C_1 of the flow resistance (pure flow resistance) varies with different tube diameters, but is nearly independent of the length of the inertance tube. It is clearly shown that C_1 increases monotony with increasing Reynolds numbers and with decreasing operating frequencies. It means that the nonlinear effect becomes larger with the increasing of the amplitude of velocities. The correlation coefficient C_1 gradually tends to 1 when the Reynolds numbers reach to zero. Therefore, the explicit solution of Eq. (6) for small acoustic amplitudes is only applicable to small Reynolds numbers.

A comparison of the correlation coefficient C_1 for different tube diameters at the same Reynolds numbers shows that the larger the tube diameter, the smaller the C_1. The reason is obvious: the velocity is small for large inner diameter tube at the same Reynolds numbers thereby the nonlinear effects become relatively weak.

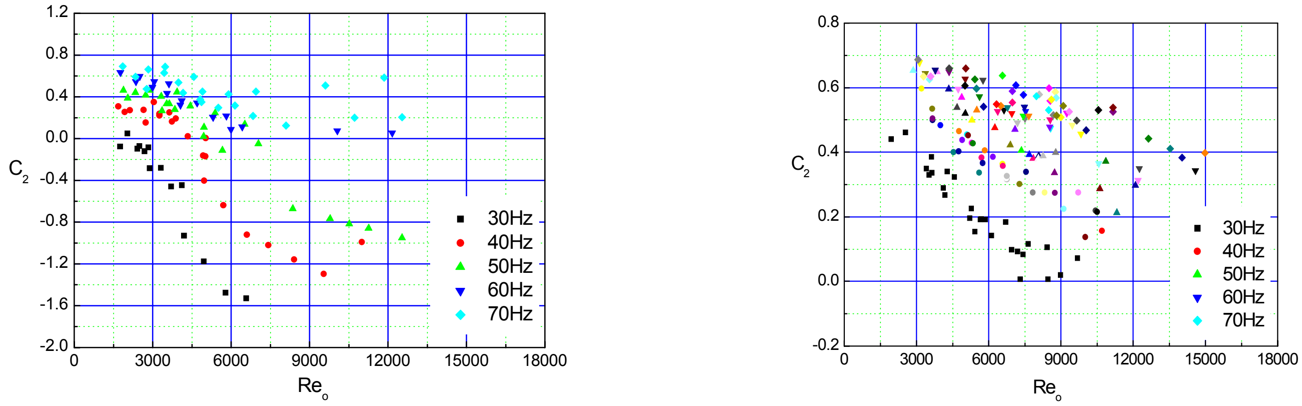
Fig. 3 (a) and (b) present the experimental data of C_2 of the flow inductance of inertance tubes for two different tube diameters of 0.6mm and 1.0mm, respectively, in terms of Re_0 and f. It is shown that C_2 are less than 1, which means that the flow inductance of the inertance tube at high acoustic amplitudes is smaller than that at low acoustic amplitudes. The C_2 decreases with increasing Reynolds numbers, and increases with increasing operating frequencies, which is contrary to the tendency of the correlation coefficient C_1 of the flow resistance. However, the influence of the operating frequency on C_2 gradually becomes smaller with the increasing of the tube diameter. The C_2 for the inertance tube with inner diameters of 0.6mm and 1.0mm are nearly independent of the length of the inertance tube, similar as that of C_1.

From these experimental results we can see evidently the difference of the flow resistance and flow inductance of inertance tubes at high and low acoustic amplitudes. It is helpful for understanding the physical mechanism of the inertance tube subjected to the oscillating gas flow in high frequency pulse tube operations and further conduct theoretical analysis.



(a) (b)

Figure 2 The coefficient C_1 in terms of Reynolds numbers and frequency for (a) 0.6mm and (b) 1.0mm



(a) (b)

Figure 3 The coefficient C_2 in terms of Reynolds numbers and frequency for (a) 0.6mm and (b) 1.0mm

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