A method for estimating the parameters of electrodynamic drivers in thermoacoustic coolers

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The electroacoustic efficiency of high-power actuators used in thermoacoustic coolers may be estimated using a linear model involving a combination of six parameters. A method to identify these equivalent driver parameters from measured total electrical impedance and velocity-voltage transfer function data was developed. A commercially available, moving-magnet driver coupled to a functional thermoacoustic cooler was used to demonstrate the procedure experimentally. The method, based on linear electrical circuit theory, allowed for the possible frequency and amplitude dependence of the driver parameters to be estimated. The results demonstrated that driver parameters measured in vacuo using this method can be used to predict the driver efficiency and performance for operating conditions which may be encountered under load. © 2005 Acoustical Society of America.

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I. INTRODUCTION

Electrodynamic drivers are used in a class of electrically driven thermoacoustic refrigeration systems. The mechanical and electrical characteristics of the driver, in conjunction with the acoustic load impedance at the driver piston, determine the electroacoustic efficiency of the actuator. The electroacoustic efficiency is, of course, a key factor in the overall efficiency of the cooling system. For this reason, it is useful to develop models that allow the efficiency of any such driver to be predicted for varying operating conditions and loads.

A detailed description of linear models of loudspeakers using equivalent electrical circuits is readily available (for example, Ref. 4 or 5). Several methods based on such linear models have been proposed in order to determine the model parameters experimentally. Measurement methods may be categorized as static or dynamic. The electrical resistance can be measured, say, using a four-wire technique, a dc ammeter, and a dc power source. The mechanical stiffness can be measured statically using a force gauge, a dc current source, and a position sensor. Dynamic methods use free decay curves of the driver response with known weights added to the piston. Alternately, one may measure the moving mass and characterize the force constant statically using similar procedures as for the stiffness determination. Dynamic methods use free decay curves of the driver response. These measurement methods are not always convenient because they require dedicated experimental setups and instrumentation, with often the need to remove the driver from the cooling system.

LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>B_l</td>
<td>force constant</td>
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<tr>
<td>f</td>
<td>frequency</td>
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<tr>
<td>f_d</td>
<td>damped natural frequency</td>
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<tr>
<td>f_o</td>
<td>undamped natural frequency</td>
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<td>F</td>
<td>force</td>
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<tr>
<td>H_u/V</td>
<td>velocity-voltage transfer function</td>
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<td>I</td>
<td>current</td>
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<tr>
<td>k_l</td>
<td>Helmholtz number</td>
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<tr>
<td>K_gas</td>
<td>gas spring stiffness</td>
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<tr>
<td>K_m</td>
<td>mechanical stiffness</td>
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<tr>
<td>L_e</td>
<td>coil inductance</td>
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<tr>
<td>M_m</td>
<td>mechanical moving mass</td>
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<tr>
<td>P</td>
<td>pressure</td>
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<tr>
<td>P_o</td>
<td>mean pressure</td>
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<tr>
<td>R_a</td>
<td>real part of ( S^2Z_a )</td>
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<tr>
<td>R_e</td>
<td>electrical resistance</td>
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<tr>
<td>R_m</td>
<td>mechanical resistance</td>
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<tr>
<td>S</td>
<td>piston cross-sectional area</td>
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<tr>
<td>S_p</td>
<td>cross-sectional area of cavity</td>
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<tr>
<td>u</td>
<td>velocity</td>
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<td>U</td>
<td>volume velocity</td>
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<td>V</td>
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<td>V</td>
<td>volume</td>
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<tr>
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<td>initial piston displacement</td>
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<tr>
<td>x_p</td>
<td>piston peak displacement</td>
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<tr>
<td>X</td>
<td>imaginary part of ( Z_{ma} )</td>
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<tr>
<td>Z_a</td>
<td>acoustic impedance</td>
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<tr>
<td>Z_b</td>
<td>blow-by impedance</td>
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<tr>
<td>Z_e,a</td>
<td>equivalent impedance due to acoustic loading in the electrical domain</td>
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<tr>
<td>Z_e,m</td>
<td>equivalent mechanical impedance with no current in the electrical domain</td>
</tr>
<tr>
<td>Z_i</td>
<td>total electrical impedance</td>
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</tbody>
</table>

a)Electronic mail: paek@ecn.purdue.edu
Methods for parameter identification without direct measurements have been developed. Most of them have been applied to loudspeakers, which require a uniform frequency response and modest electroacoustic efficiencies over a large frequency range.\textsuperscript{[9–11]} The possible influence of piston displacement amplitude on the parameters has not typically been considered in these studies. Jacobsen\textsuperscript{et al.}\textsuperscript{11} obtained driver parameters using the measured voltage response for a step current excitation. The decay curves were found to be only approximately exponential, with unevenly spaced zero crossings. This was attributed to nonlinear effects, which result in amplitude and time-dependent equivalent driver parameters. The parameters identified using this method were accurate within 10\% compared with parameter values measured directly on a test bench. Wright\textsuperscript{10} suggested an empirical impedance model for the measured electrical impedance. He modeled the electrical resistance and the inductance as exponential functions of frequency, and found the model parameters using a curve fitting technique. A significant increase of the electrical resistance with frequency was observed, reportedly due to eddy current effects. Knudsen \textit{et al.}\textsuperscript{11} studied the low-frequency behavior of a loudspeaker using a time-domain system identification method featuring a low-pass filtered square-wave input. This method yielded estimates within 4\% compared with directly measured values for all driver parameters, except the coil inductance. It was suggested that the coil inductance could be neglected for drivers having an inductance much smaller than the ratio of the electrical resistance and the angular frequency, due to its relatively small contribution to the overall input impedance.\textsuperscript{11} This simplification is not appropriate for power actuators in thermoacoustic systems because the coil inductance is often comparable to the ratio of electrical resistance and angular frequency.

Other efforts to model nonlinear effects have been made to overcome the limitations of linear driver models. Olson\textsuperscript{12} used a third-order polynomial to model the nonlinear suspension stiffness with respect to the cone displacement, and solved the associated differential equations of motion. Birt\textsuperscript{13} obtained the nonlinear force constant profile with respect to the voice-coil displacement, and the voice-coil current for an assembled loudspeaker driver. He proposed a method to eliminate nonlinearity using a programmable read-only memory in conjunction with a compensating coil and center pole extension. Kaizer\textsuperscript{14} used a truncated Volterra series expansion to solve a governing nonlinear differential equation for enclosed loudspeakers having nonlinearities in the electric inductance, the suspension stiffness and the force constant. Reasonable agreement was obtained between the calculated and directly measured responses. Klippel\textsuperscript{15} further extended Kaiser’s work by considering bass reflex loudspeakers using the same Volterra series expansion. A fair agreement between measured and calculated responses was obtained. More recently, Scott \textit{et al.}\textsuperscript{16} proposed a way to determine nonlinear driver parameters using an impedance sweep for a fixed coil displacement, assuming the linear equivalent circuits are accurate in this condition. The measured data was used to obtain seventh-order regression curves for the amplitude-dependent parameters. This method is simpler than Kaiser’s and Klippel’s methods, yet requires additional measurements of the cone’s displacement for a DC current excitation and the force constant at zero coil displacement.

In contrast with hi-fi loudspeakers, electrodynamically driven thermoacoustic cooling systems require drivers that can provide much greater acoustic power at one single target frequency, with good efficiency. This problem is well described in a recent investigation of the optimization of electrodynamic drivers for high electroacoustic efficiency using linear models based on equivalent electrical circuits.\textsuperscript{17} Drivers in thermoacoustic coolers need to operate only in the vicinity of the target frequency. Therefore it is not necessary to characterize the driver over a wide frequency range.

Methods specific for thermoacoustic coolers have been proposed before. Ballister and McKelvey\textsuperscript{18} found the force constant using a three-parameter regression of the impedance measured using an impedance analyzer. Either the moving mass or the mechanical stiffness was required. The frequency and amplitude dependence of the linear parameters was not considered. Smith\textsuperscript{1} assumed that the electrical resistance is a linear function of frequency at low frequencies, based on measured total electrical impedance data. This model was found to supply accurate estimates of the measured total electrical impedance of a 2-kW model C-2A CFIC moving magnet driver.

The objective of the present study was to develop a simple method to identify the frequency- and amplitude-dependent driver parameters without time-consuming static or dynamic measurements or elaborate test facilities. The goal was to improve upon Scott \textit{et al.}’s method using a simple model derived from linear electrical equivalent circuit theory. Only electrical impedance and velocity-voltage transfer function data are required for two distinct but similar frequencies, and a fixed piston amplitude. A direct analytical formulation derived from the linear electrical equivalent circuit theory was used to identify all the driver parameters. The parameters obtained \textit{in situ} using this method were compared with the values measured on a dedicated test bench, and values provided by the manufacturer.

\section*{II. THEORETICAL BACKGROUND}

A schematic of the linear electrical equivalent circuit model for an electroacoustic driver is shown in Fig. 1. The model parameters are $R_e$, the electrical resistance; $L_e$, the coil inductance; $M_m$, the mechanical moving mass; $K_m$, the mechanical stiffness; $R_m$, the mechanical resistance; $BI$, the force constant; $S$, the piston cross-sectional area; and $Z_a$, the complex acoustic impedance at the piston. All parameters except $Z_a$ are assumed to be real. The electrical circuit is connected to the mechanical circuit through a gyrator and the

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Linear electrical equivalent circuit of an electroacoustic driver.}
\end{figure}

\begin{tabular}{|c|c|c|}
\hline
$R_e$ & $L_e$ & $M_m$ \\
\hline
$K_m$ & $R_m$ & $BI$ \\
\hline
$S$ & $Z_a$ & \\
\hline
\end{tabular}
mechanical circuit is connected to the acoustical circuit through a transformer. The electrical, mechanical, and acoustical circuits can be brought into the electrical domain and represented as shown in Fig. 2. In Fig. 2, $Z_e$ is the electrical impedance with the mechanical motion blocked, given by

$$Z_e = R_e + j \omega L_e,$$

where $\omega$ is the angular frequency.

The equivalent mechanical impedance, $Z_{e,M}$, measured without current in the electrical domain, is given by

$$Z_{e,M} = \frac{(Bl)^2}{R_m + j(\omega M_m - K_m/\omega)},$$

and $Z_{e,a}$, the equivalent impedance due to the acoustic loading in the electrical domain, is given by

$$Z_{e,a} = \frac{(Bl)^2}{S^2 Z_a}.$$  \hspace{1cm} (3)

The total electrical impedance at the driver, $Z_i$, is then expressed by

$$Z_i = Z_e + \frac{1}{1/Z_{e,M} + 1/Z_{e,a}} = Z_e + \frac{(Bl)^2}{R_m + j(\omega M_m - K_m/\omega) + S^2 Z_a} = Z_e + \frac{(Bl)^2}{Z_{ma}},$$

where

$$Z_{ma} = R_m + j \left( \omega M_m - \frac{K_m}{\omega} \right) + S^2 Z_a.$$ \hspace{1cm} (5)

The force provided by the driver is

$$F = Bl \cdot I = Z_{ma} \cdot u,$$

and therefore

$$\frac{u}{I} = \frac{Bl}{Z_{ma}}.$$ \hspace{1cm} (7)

Finally, the transfer function between the piston velocity and the voltage input to the driver may be calculated using Eqs. (4) and (7), and is given by

$$H_{u/v} = \frac{u}{I} \cdot \frac{1}{Z_i} = \frac{Bl/Z_{ma}}{Z_e + (Bl)^2/Z_{ma}}.$$ \hspace{1cm} (8)

The following expression results from Eq. (4):

$$\frac{Bl}{Z_{ma}} = \frac{Z_i - Z_e}{Bl}.$$ \hspace{1cm} (9)

Substitution of Eq. (9) into Eq. (8) yields

$$H_{u/v} = \frac{Bl/Z_{ma}}{Z_e + (Bl)^2/Z_{ma}} = \frac{(Z_i - Z_e)/Bl}{Z_e + [(Z_i - Z_e)/Bl]Bl}$$

$$= \frac{Z_i - Z_e}{Bl Z_i},$$ \hspace{1cm} (10)

which also supplies the relation,

$$H_{u/v} Bl Z_i = Z_i - Z_e = Z_i - R_e - j 2 \pi f L_e.$$ \hspace{1cm} (11)

Considering a pair of frequencies $f_1$ and $f_2$, with $f_1$ very close to the frequency of interest $f_2$, the frequency-dependent electrical resistance, $R_e$, and inductance, $L_e$, can be assumed to be constant over the range between $f_1$ and $f_2$. Thus the following equation can be obtained from Eq. (11):

$$(H_{u/v})_{f_1} |_{Z_i} - H_{u/v} |_{f_2} |_{Z_i} = Bl$$

$$= Z_i |_{f_1} - Z_i |_{f_2} - j 2 \pi (f_1 - f_2) L_e.$$ \hspace{1cm} (12)

Finally, the transduction coefficient, $Bl$, is given by the expression

$$Bl = \text{Re} \left( \frac{Z_i |_{f_1} - Z_i |_{f_2} - j 2 \pi (f_1 - f_2) L_e}{H_{u/v} |_{f_1} |_{Z_i} - H_{u/v} |_{f_2} |_{Z_i}} \right).$$ \hspace{1cm} (13)

Although $Bl$ should be a complex number, the lagging phase angle is very close to zero near the driver resonance frequency. In thermoacoustic coolers, the electroacoustic drivers are generally tuned to have resonance around the operating frequency. Therefore $Bl$ was assumed to be a real number in Eq. (13).

Equations for $R_e$ and $L_e$ can then be obtained directly from Eq. (11) and the expression for $Bl$. These are

$$R_e = \text{Re}(Z_i |_{f_2} - H_{u/v} |_{f_2} Bl Z_i |_{f_2})$$ \hspace{1cm} (14)

and

$$L_e = \frac{\text{Im}(Z_i |_{f_2} - H_{u/v} |_{f_2} Bl Z_i |_{f_2})}{2 \pi f_2}.$$ \hspace{1cm} (15)

The transduction factor, $Bl$, is initially obtained from Eq. (13) by assuming that the $L_e$ term is negligible compared with other terms. The parameters $R_e$ and $L_e$ are then evaluated using the approximate value for $Bl$. The calculation is iterated until $Bl$, $R_e$, and $L_e$ converge to satisfy Eqs. (13)–(15).

The input impedance of the mechanical circuit with acoustical load is given by

$$F = R_m + j \left( \omega M_m - \frac{K_m}{\omega} \right) + S^2 Z_a.$$ \hspace{1cm} (16)

From the well known relation,

$$F = Bl \cdot I,$$ \hspace{1cm} (17)

in conjunction with Eqs. (1) and (2), $F$ may be eliminated from Eq. (16) as follows:
\[ R_m + j \left( \omega M_m - \frac{K_m}{\omega} \right) = B l \frac{I}{Z_n} - S^2 Z_a = B l \frac{I}{Z_n H_{u/v}^2} - S^2 Z_a. \]  

(18)

Now, \( R_m \) can be obtained from the measured transfer functions at \( f_2 \) and \( B l \) as follows:

\[ M_m = \frac{\text{imag}(B l / Z_{m})_{f_2} H_{u/v}^2 f_2 - S^2 Z_a f_2 - \text{imag}(B l / Z_{m})_{f_2} H_{u/v}^2 f_2 - S^2 Z_a f_2) f_2}{2 \pi (f_1^2 - f_2^2)}. \]

(20)

Finally, the equivalent stiffness parameter, \( K_m \), is obtained using

\[ K_m = 4 \pi^2 f_2^2 M_m - \text{imag} \left( \frac{B l}{Z_{m}} f_2 H_{u/v}^2 f_2 - S^2 Z_a f_2 \right). \]

(21)

All the driver parameters are expressed in closed form in terms of the driving frequencies, \( f_1 \) and \( f_2 \), the total electrical impedance, \( Z_t \), the velocity-voltage transfer function, \( H_{u/v} \), and the acoustic impedance, \( Z_a \) with the exception of \( B l \); the latter is calculated in conjunction with \( R_e \) and \( L_e \) as previously described. The relations (13)–(15) and (19)–(21), although simple to derive, are not readily available in the literature. The indirect parameter identification method based on these equations will be referred to as "method A" in subsequent sections of this paper, for convenience.

According to Wakeland,\(^{17}\) the electroacoustic efficiency, \( \eta \), of the driver is related to the driver parameters and the acoustic impedance through

\[ \frac{1}{\eta} = \frac{R_e R_m}{(B l)^2 R_a R_m} \left( 1 + \frac{R_a}{R_m} \right)^2 + \left( 1 + \frac{R_m}{R_a} \right)^2 \left( \frac{R_m X^2}{(B l)^2 R_a R_m} \right), \]

(22)

where \( R_e = \text{Re}[S^2 Z_a] \) and \( X = \text{Im}[Z_{ma}] \). Equation (22) can be further simplified to

\[ \eta = \frac{R_a (B l)^2}{R_e (X^2 + (R_m + R_a)^2) + (B l)^2 (R_a + R_m)}. \]

(23)

Equation (23) was used to calculate electroacoustic efficiency using parameter values inferred from Eqs. (13)–(15) and (19)–(21), i.e., using method A.

### III. GUIDANCE IN SELECTING FREQUENCY DIFFERENTIALS

The direct formulation, method A, for the identification of \( R_e \), \( L_e \), and \( B l \) is valid when the two driving frequencies are nearly the same, and the piston displacement is fixed. This is because \( R_e \) and \( L_e \) are assumed constants, and the term including \( L_e \) in Eq. (13) is initially neglected to find \( B l \). Therefore, small frequency intervals that ensure the imaginary part of the term including \( L_e \) in Eq. (13) is much smaller than the imaginary part of its preceding term are essential for estimating \( R_e \), \( L_e \), and \( B l \).

The equivalent mass parameter, \( M_m \), and the stiffness, \( K_m \), are calculated from Eq. (18) with the help of the measured transfer functions at \( f_1 \):

\[ R_m = \text{real} \left( \frac{B l}{Z_{m}} f_2 H_{u/v}^2 f_2 - S^2 Z_a f_2 \right). \]

(19)

However, the use of two very similar frequencies could result in large errors in the identification of \( M_m \) and \( K_m \) if there are small errors in the source frequencies due to the term \( f_1^2 - f_2^2 \) in Eq. (20). The accuracy of the source frequency from the DSP board used for the experiments is about 0.02 Hz. A simulation was done assuming 0.02-Hz frequency errors to estimate uncertainties in the identified parameters. Relative errors for \( M_m \) and \( K_m \) of up to 45% can result when a 0.1-Hz frequency differential is used. The uncertainty in \( K_m \) was very close to \( M_m \) because \( M_m \) is included in the equation for \( K_m \). Errors for the other parameters associated with the added 0.02-Hz frequency errors were less than 6%. The relative errors of \( M_m \) and \( K_m \) were reduced to 2.3% when a 2-Hz frequency differential was used. Based on these results, a 2-Hz frequency increment was selected for \( M_m \) and \( K_m \), and a 0.1-Hz frequency increment was used for all the other parameters.

An approximate estimate of the uncertainty in \( M_m \) can be developed from Eq. (20). The uncertainty in \( K_m \) is approximately the same as that for \( M_m \). By assuming the uncertainty of the numerator is negligible compared with that of the denominator, Eq. (20) can be rewritten as

\[ \frac{M_m}{M_{m,\text{m}}^\text{m}} = \frac{C}{2 \pi (f_1^2 - f_2^2)}, \]

(24)

where \( C \) is a constant. The uncertainty in \( M_m \) is then given by

\[ \delta M_m = \sqrt{\left( \frac{\partial M_m}{\partial f_1} \delta f_1 \right)^2 + \left( \frac{\partial M_m}{\partial f_2} \delta f_2 \right)^2} = \frac{C \sqrt{f_1^2 \delta f_1^2 + f_2^2 \delta f_2^2}}{\pi (f_1^2 - f_2^2)^2}. \]

(25)

where \( \delta f_1 \) and \( \delta f_2 \) are uncertainties in \( f_1 \) and \( f_2 \), respectively. Finally, the relative uncertainty of \( M_m \) is

\[ \frac{\delta M_m}{M_m} = \frac{2 \sqrt{f_1^2 \delta f_1^2 + f_2^2 \delta f_2^2}}{f_1^2 - f_2^2}. \]

(26)

By assuming \( f_1 \) is close to \( f_2 \), Eq. (26) can be simplified to

\[ \frac{\delta M_m}{M_m} = R \frac{\sqrt{f}}{f}, \]

(27)
where $R$ is the relative uncertainty of the frequency, $f$ is the frequency of the source, and $\Delta f$ is the frequency differential. Equation (27) yields 28.9% uncertainty for a 0.1-Hz frequency differential and 1.4% uncertainty for a 2-Hz frequency interval when 170 Hz with the relative uncertainty of 0.012% is used. Although Eq. (27) is approximate, it can be used to select a proper frequency differential for $M_m$ and $K_m$ in the absence of knowledge of other parameters and transfer functions.

**IV. EXPERIMENTAL APPARATUS**

A thermoacoustic cooler prototype was used to investigate the driver parameter characterization measurement method. A schematic of the thermoacoustic cooler is shown in Fig. 3. The driver was a moving magnet linear actuator (CFIC, model B-300) mounted on an added metal leaf spring to provide additional suspension stiffness. The driver’s natural frequency without the added spring was near 33 Hz. The driver was designed to deliver more than 200 W of acoustic power at 33 Hz, with an electro-acoustic transduction efficiency of 70% and a maximum displacement amplitude of 6 mm. The driver’s natural frequency was increased to about 163 Hz, and its maximum displacement amplitude was reduced to less than 1 mm due to the added leaf spring. More information on the detailed driver characteristics is available in Ref. 21. The nominal driver parameters without the added spring provided from the manufacturer are given in Table I. The nominal parameters were directly measured by the manufacturer using various methods. The electrical resistance was measured using a four-wire technique, and a dc current from a dc power source. The stiffness and the force constant were measured statically using a force gauge and a dc current source. The mechanical resistance was measured using the free decay response method.

In the present study, an accelerometer (PCB, 353B15) was mounted on the driver piston to measure the axial acceleration. A dynamic pressure sensor (PCB, 102A07) was installed in a port near the piston to measure the driver pressure. An Agilent VXI mainframe (E8403A) and a DSP board (E1432A) were used for data acquisition. The input voltage, the resulting current, the piston velocity, and the driver pressure were measured simultaneously. One source channel was used to operate the moving magnet driver. A data acquisition and display program was developed using Agilent Vee, and was used for the measurement.

The piston displacement was monitored and the voltage input to the driver was adjusted to keep the piston displacement constant during the measurements. Seven frequencies in the range from 168 to 174 Hz, which encompassed the maximum electroacoustic efficiency of the driver, were chosen. The transfer functions were measured at frequencies both 0.1 and 2 Hz below and above the chosen frequencies, as well as at the chosen frequency. Two different sets of parameters were identified for each chosen frequency using forward and backward frequency steps. The two values were averaged.

**V. BENCH MEASUREMENTS OF DRIVER PARAMETERS**

To verify the parameters calculated using method A, the driver parameters were also measured directly with the exception that manufacturers’ electrical inductance was used.

The undamped natural frequency, $f_o$, is related to the moving mass and the mechanical stiffness through

$$f_o^2 = \frac{1}{4\pi^2} \frac{K_m}{M_m}$$

(28)

The relation between the undamped and the damped natural frequency is

$$f_d = f_o \sqrt{1 + \left( \frac{R_m}{4\pi f_d M_m} \right)^2}$$

(29)

where $f_d$ is the damped natural frequency. The free decay response of the driver was measured after adding a known mass to the piston, assuming that damping was small enough not to affect the natural frequency. The second term in the square root in Eq. (29) is much smaller than one, and $f_d$ is approximately equal to $f_d$. This allowed the moving mass and the mechanical stiffness to be estimated.

The mechanical resistance was obtained from measured decay curves of the piston displacement, $x$, which can be described by

$$x = x_o e^{-R_e x M_m \cos(2\pi f_d t)}$$

(30)

where $x_o$ is the initial piston displacement. The force constant, $B_l$, was obtained from the measured free decay curves of the acceleration and the voltage. The voltage leads the acceleration by 90°, and the ratio of the voltage and the integrated acceleration yields the force constant when the circuit is electrically open. This can be verified from Eq. (8) when $Z_e$ is zero.

The electrical resistance, $R_e$, was obtained from the electrical impedance measured at frequencies far different from the natural frequency of the driver. For operating frequencies either much greater or smaller than the natural frequency, the real part of the electrical impedance is about the same as the electrical resistance. This can be verified from

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**TABLE I. Driver parameters provided by manufacturer.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>$R_e$ (Ω)</td>
<td>0.11</td>
</tr>
<tr>
<td>$L_e$ (mH)</td>
<td>0.86</td>
</tr>
<tr>
<td>$B_l$ (N/A)</td>
<td>9</td>
</tr>
<tr>
<td>$M_m$ (kg)</td>
<td>1.6</td>
</tr>
<tr>
<td>$K_m$ (MN/m)</td>
<td>0.74</td>
</tr>
<tr>
<td>$R_m$ (N-s/m)</td>
<td>25</td>
</tr>
</tbody>
</table>
The measured electrical impedance values above and below the natural frequency were linearly interpolated to find the frequency-dependent electrical resistance.

The direct method described above will subsequently be referred to as “method B,” to distinguish it from “method A” described in Sec. II. The driver parameters measured with method B for three different piston displacements are given in Table II. The large mechanical stiffness value in Table II compared to that in Table I is due to the addition of a leaf spring to the driver. The electrical resistances were arithmetically averaged over the frequency range between 168 and 172 Hz. The mechanical resistance decreased with increasing piston displacement, as discussed later.

VI. RESULTS

A. Driver parameter estimation

The total electrical impedance and the velocity-voltage transfer functions were measured in vacuo over the frequency range from 168 to 174 Hz for different fixed piston peak displacements, \( x_p \), of 0.2, 0.3, and 0.4 mm, respectively. The six driver parameters were then calculated using method A.

The results are shown in Figs. 4–6. The identified electrical resistance and the inductance increase with frequency. The trend of an exponential increase has been reported and attributed to eddy current effects.\(^{24}\) Wright also observed this trend for frequencies up to 20 kHz.\(^{10}\) At low frequencies, the increase can be approximated as linear.\(^{7}\) The curves in Fig. 4 indicate a nearly linear trend. The resistance values, \( R_e \), obtained for the frequencies of interest are much larger than the dc resistance measured by the manufacturer (Table I). Therefore the maximum electroacoustic efficiency of the driver calculated with the nominal dc electrical resistance using the usual method\(^{17}\) may not be achieved for high frequencies.

The force constant, \( B_l \), and the mechanical resistance, \( R_m \), are shown in Fig. 5. The force constant increases slightly with the piston amplitude, and asymptotically approaches a constant for large piston displacements. Although not shown in Fig. 5, \( B_l \) is expected to sharply decrease with piston displacements for much larger piston displacements.\(^{7,13}\) This occurs when the moving magnet oscillation displacement is much larger than the stationary electromagnet structure, and thus portions of the moving magnet and the stator are less effective in creating a magnetic flux within the gap.\(^{13}\)

The resistance, \( R_m \), was obtained using Eq. (30) applied for added mass tests. The mechanical resistance \( R_m \) decreased as the piston displacement increased. The mechanical resistance lumps all dissipative phenomena such as internal damping in the spring element, or losses in the voice-coil centering mechanism (moving magnet in this case).\(^{4}\) Based on Fig. 5, both \( B_l \) and \( R_m \) are clearly functions of piston displacement, but they are almost independent of operating frequency. This is in apparent contradiction with Smith’s experimental observations, which report a decrease of \( R_m \) with increasing frequency.\(^{7}\)

The identified spring stiffness was amplitude and fre-

---

**TABLE II. Driver parameters obtained from method B.**

<table>
<thead>
<tr>
<th></th>
<th>0.2 mm</th>
<th>0.3 mm</th>
<th>0.4 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_e ) (( \Omega ))</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>( B_l )</td>
<td>8.5</td>
<td>8.6</td>
<td>8.6</td>
</tr>
<tr>
<td>( M_m ) (kg)</td>
<td>1.71</td>
<td>1.71</td>
<td>1.71</td>
</tr>
<tr>
<td>( K_m )</td>
<td>1.78</td>
<td>1.78</td>
<td>1.78</td>
</tr>
<tr>
<td>( R_m )</td>
<td>47.2</td>
<td>37.6</td>
<td>35.5</td>
</tr>
</tbody>
</table>

---

**FIG. 4.** Driver equivalent parameters identified using method A versus frequency: (a) electrical resistance and (b) electrical inductance. ○: \( x_p = 0.2 \) mm, □: \( x_p = 0.3 \) mm, and ◆: \( x_p = 0.4 \) mm.

**FIG. 5.** Driver equivalent parameters identified using method A versus frequency: (a) force constant and (b) mechanical resistance. ○: \( x_p = 0.2 \) mm, □: \( x_p = 0.3 \) mm, and ◆: \( x_p = 0.4 \) mm.

**FIG. 6.** Driver equivalent parameters identified using method A versus frequency: (a) mechanical moving mass and (b) mechanical stiffness. ○: \( x_p = 0.2 \) mm, □: \( x_p = 0.3 \) mm, and ◆: \( x_p = 0.4 \) mm.
frequency independent, as shown in Fig. 6. This is in contrast with earlier observations.11,12 The values of the mechanical stiffness obtained using method B (Table II) did not increase with amplitude, either. The amplitude-dependent characteristics of the spring may be truly linear because the piston displacement is small due to the added leaf spring. Both the mechanical stiffness and the moving mass obtained with method A were within 15% of the values obtained with method B. The effective moving mass was also amplitude and frequency independent as shown in Fig. 6.

B. Electroacoustic efficiency predictions

The equivalent parameters obtained using method A were arithmetically averaged over the frequency span, neglecting the effects of frequency, for electroacoustic efficiency predictions. A model for the effects of a back cavity is needed in addition to the parameters measured in vacuum to predict the driver efficiency in a pressurized gas mixture. When the product $kl$ is much less than unity, the fluid in the back cavity can be considered as a lumped element.25 Therefore the gas spring stiffness can be calculated using

$$K_{gas} = \frac{\gamma \cdot P_o \cdot V^2}{S_p^2},$$

(31)

where $\gamma$ is the specific heat ratio, $P_o$ is the mean pressure of the gas, $V$ is the volume of the cavity, and $S_p$ is the cross-sectional area of the cavity. The calculated gas-spring stiffness was about 10,000 N/m. This value was added to the mechanical stiffness in the model. The frequency-averaged parameters combined with the measured acoustic impedance yield the electroacoustic efficiency of the driver. The frequency-averaged parameters are shown in Table III.

The electroacoustic efficiency was calculated using Eq. (23), with identified parameter values from method A, and the acoustic load impedance measured in the pressurized system. The calculated efficiency was compared with directly measured efficiencies for operating conditions near the resonance frequency. The electroacoustic efficiency was also calculated from parameters obtained using method B, i.e., the parameter values of Table II, along with the nominal electrical inductance of Table I. Figures 7–9 show the predicted and the measured efficiency with respect to frequency for 0.2, 0.3, and 0.4 mm piston peak displacements, respectively. For all cases, the efficiency predictions using parameters obtained from method A yielded less than 17% relative errors from the measured efficiency. The efficiency calculated using the parameters obtained from method B also yielded efficiency predictions within 17%. For a 0.4-mm piston amplitude, the efficiency calculated from the frequency-averaged equivalent parameters using method A yielded a slightly better estimate of the measured efficiency than that calculated from method B.

### Table III. Frequency-averaged driver parameters obtained from method A.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>0.2 mm</th>
<th>0.3 mm</th>
<th>0.4 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_e$ (Ω)</td>
<td>0.19</td>
<td>0.19</td>
<td>0.19</td>
</tr>
<tr>
<td>$L_e$ (mH)</td>
<td>0.86</td>
<td>0.87</td>
<td>0.87</td>
</tr>
<tr>
<td>$B_l$</td>
<td>8.7</td>
<td>8.7</td>
<td>8.7</td>
</tr>
<tr>
<td>$M_m$ (kg)</td>
<td>1.89</td>
<td>1.88</td>
<td>1.92</td>
</tr>
<tr>
<td>$K_m$</td>
<td>1.96</td>
<td>1.95</td>
<td>2.00</td>
</tr>
<tr>
<td>$R_m$</td>
<td>42</td>
<td>36</td>
<td>34</td>
</tr>
</tbody>
</table>

FIG. 7. Electroacoustic driver efficiency versus frequency when $x_p = 0.2$ mm. •: calculated efficiency using the averaged identified parameters obtained from method A in vacuum, ◦: measured efficiency in pressure, and □: calculated efficiency using the parameters obtained from method B.

FIG. 8. Electroacoustic driver efficiency versus frequency when $x_p = 0.3$ mm. •: calculated efficiency using the averaged identified parameters obtained from method A in vacuum, ◦: measured efficiency in pressure, and □: calculated efficiency using the parameters obtained from method B.

FIG. 9. Electroacoustic driver efficiency versus frequency when $x_p = 0.4$ mm. •: calculated efficiency using the averaged identified parameters obtained from method A in vacuum, ◦: measured efficiency in pressure, and □: calculated efficiency using the parameters obtained from method B.
VII. DISCUSSION

A. Effects of piston leakage

The transfer functions measured in the pressurized system for a 0.4-mm piston displacement are compared with the same quantities calculated from in vacuo parameters identified using method A in Fig. 10. The calculated values were slightly underpredicted compared to the measured values around the frequency of peak electroacoustic efficiency. One possibility that could explain the slight discrepancy is the blow-by flow that might occur through a small gap between the piston and the piston housing. To investigate this phenomenon, an unknown blow-by impedance, $Z_b$, was added in parallel to the acoustic impedance, as shown in Fig. 11.

If the acoustic impedance, $Z_a$, is zero (like in vacuum), all the flow occurs through $u_1$, and there is no blow-by flow, i.e., $u_2 = 0$. When $Z_b$ is much larger than $S^2 Z_a$, all the flow occurs through $u_1$, and this again corresponds to no blow-by flow. For an arbitrary, nonzero acoustic load, $Z_a$, flow through, $u_2$, and an additional black box impedance, $Z_b$, contribute to the measured impedance.

The total electrical impedance and the velocity-voltage transfer function including the unknown impedance are given by

$$Z_e = Z_a + \frac{(Bl)^2}{Z_{mb}},$$

$$H_{u/v} = \frac{Bl}{Z_e Z_{mb} + (Bl)^2},$$

where

$$Z_{mb} = R_m + j \left( \omega M_m - \frac{K_m}{\omega} \right) + \frac{S^2 Z_a \times Z_b}{S^2 Z_a + Z_b}.$$  (34)

The identified parameters in vacuum for a 0.4-mm piston displacement were substituted into Eqs. (32) and (33) and compared with the measured transfer functions in the pressurized system. The blow-by impedance, $Z_b$, was artificially adjusted to see its effect on the transfer functions. Figure 12 shows the results for $Z_b$ equal to zero. This means that the blow-by impedance is much smaller than the acoustic impedance, and all the flow occurs through the blow-by impedance. The fluid in front of the piston is not compressed by the oscillating piston but flows back and forth through the gap between the piston and the piston housing. As expected, there is no peak in the total electrical impedance due to the acoustic resonance. The same results can be obtained when $Z_a$ is zero, as occurs in a vacuum. When the acoustic impedance is not zero, and the blow-by impedance, $Z_b$, is twice $S^2 Z_a$, the calculated transfer functions approaches the shape of the measured transfer functions as shown in Fig. 13. When $Z_b$ reaches about ten times $S^2 Z_a$, the plot of the measured and the calculated transfer functions are the same as that without blow-by flow as shown in Fig. 10. This implies that, if blow-by occurs, the calculated transfer function values would have been greater than the measured values, and not smaller as observed in Fig. 10. Based on this model, it was concluded that piston leakage effects were insignificant, and that other unknown reasons were responsible for the discrepancies in the model predictions.

VIII. CONCLUSIONS

A method to identify the driver parameters was developed based on the assumption that the driver parameters are constant for two closely spaced, distinct frequencies and for a fixed piston amplitude. The total electrical impedance and the velocity-voltage transfer function were simultaneously measured to enable the evaluation of the six driver parameters in terms of known quantities. The parameter identifica-
The proposed driver parameter identification method is useful in estimating driver parameters and efficiency at an amplitude and frequency of interest. A knowledge of these parameters is useful in achieving optimal electroacoustic efficiency, with relative errors within 17%.

The averaged identified driver parameters were used to calculate the electroacoustic efficiency in the pressurized vessel. These supplied a good estimate of the measured electroacoustic efficiency. The averaged identified driver parameters were used to calculate the electroacoustic efficiency in the pressurized vessel. These supplied a good estimate of the measured electroacoustic efficiency. The averaged identified driver parameters were used to calculate the electroacoustic efficiency in the pressurized vessel. These supplied a good estimate of the measured electroacoustic efficiency.

The averaged identified driver parameters were used to calculate the electroacoustic efficiency in the pressurized vessel. These supplied a good estimate of the measured electroacoustic efficiency.

FIG. 13. Measured and calculated transfer functions when \( x_p = 0.4 \) mm, and the blow-by impedance is twice \( S^2Z_a \). (a) Real part of total electrical impedance; (b) imaginary part of total electrical impedance; (c) real part of velocity-voltage transfer function; and (d) imaginary part of velocity-voltage transfer function. 


22 Private communication.

