Effectiveness of parallel-plate heat exchangers in thermoacoustic devices

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Measurements are made of the heat transferred between two identical parallel-plate heat exchangers under conditions of oscillating flow over a range of frequencies and amplitudes. The results are analyzed and summarized in terms of heat-exchanger effectiveness, the ratio of the actual heat transfer rate to the maximum possible heat transfer rate. Measured results are compared to the DELTAE model that is often used in the design of conventional thermoacoustic devices, and possible improvements to the model are offered. © 2004 Acoustical Society of America.

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I. INTRODUCTION

Heat exchangers are important components of thermoacoustic engines and refrigerators. This paper reports measurements of heat transfer between identical heat exchangers in oscillating flow. The results are analyzed in a way that is intended to be useful in the design of thermoacoustic devices.

The authors are not aware of any previously published experiments on the performance of heat exchangers in purely oscillatory flow without externally imposed pressure oscillations. Cooper, Yang, and Nee\(^1\) carried out a review of “fluid mechanics of oscillatory and modulated flows,” resulting in 63 references in the category “convective heat transfer.” Almost all of the cited studies, however, concern “modulated flows,” i.e., steady flows with small oscillatory components superimposed. Most of these papers are concerned with the use of flow pulsations as an enhancement mechanism. Of the papers cited in their review, the only experimental investigation of a situation similar to that encountered in thermoacoustics is the reviewers’ own study.\(^2\) In this experimental paper they note that, “to the knowledge of the authors, no papers have been published dealing strictly with heat transfer to oscillatory flows [with no mean flow].” Details of their experiment, however, make it difficult to apply their results to heat exchangers for thermoacoustics. In particular, Cooper \textit{et al.} used a complicated system of “doors” and “back flow preventers” in order “to provide fresh air every half oscillation cycle,” so that their experiment is more like the repeated measurement of a time-varying unidirectional flow than a true oscillatory flow. One paper on heat exchange in oscillating flow that escaped the attention of Cooper \textit{et al.} is that of Hwang and Dybbs.\(^3\) Once again, however, the details are such that we have not yet found a way to relate Hwang and Dybbs’s results to the results of the present study.

A conventional method of testing a heat exchanger is depicted in Fig. 1(a). Fluid at a well-defined temperature \(T_{\text{in}}\) flows down a duct and through a heat exchanger that is held at temperature \(T_{\text{hx}}\). The fluid exits the exchanger at a different temperature \(T_{\text{out}}\). Meanwhile, the rate at which heat is delivered to the exchanger is monitored. A heat-transfer coefficient is then determined in terms of the temperatures and heat.

The situation is not so straightforward for the case of oscillating flow. Consider a single heat exchanger in an oscillating flow in a duct, shown in Fig. 1(b). The exchanger is at temperature \(T_{\text{hx}}\), and the fluid in the duct is initially at \(T_\infty\). After a few passes through the exchanger, however, all the fluid near the exchanger approaches \(T_{\text{hx}}\). At this point, the temperature of the fluid entering the exchanger is not well known, and the rate of heat delivery to the fluid has more to do with the way heat is transported along the length of the duct than it has to do with the characteristics of the exchanger itself. This configuration has been studied experimentally by Peattie and Budwig.\(^4\) The interaction between the oscillating boundary layer and the temperature gradient along the duct causes a type of thermoacoustic heat pumping examined in several papers by Kurzweg and collaborators (Refs. 5 and 6, and references therein). While there are applications for this type of transport, it is not particularly relevant to the study of heat exchanger performance in thermoacoustics.

By outfitting an exchanger and a stack (or regenerator) with temperature sensors, it is possible to study heat exchangers in the context of a thermoacoustic device, as in Fig. 1(c), where a heat exchanger is seen placed adjacent to a stack. Brewster, Raspet, and Bass\(^7\) report some results from this sort of measurement, as do Braun \textit{et al.},\(^8\) and a particularly careful study of this type was undertaken recently by Mozurkewich.\(^9\) For all measurements of this sort, the experimental situation is complicated, and interpretation of the results is difficult. In particular, the stack only functions in the presence of pressure oscillations. As discussed in Sec. VIII B, however, we believe that the temperature oscillations caused by these pressure swings may significantly alter the heat transfer within the heat exchanger. Furthermore, the way in which heat transfer occurs at the end of the stack is no better understood than within the heat exchanger, and

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may be different in nature. These phenomena are, of course, part of the reality of heat exchanger performance within real thermoacoustic devices. However, it is important to know the isolated effects of oscillating flow on the exchanger itself, without additional complications.

The approach taken in the present measurement is to place two exchangers at different temperatures close together, and to move fluid in an oscillatory manner between the two exchangers, as depicted in Fig. 1(d). The situation is now well defined, and involves only heat exchangers; if the duct is insulated, then, in steady state, this is a measurement of the exchangers, not of transport in the duct. Furthermore, if the exchangers are identical, then the average temperature in the plane centered between the exchangers is known to be \((T_h + T_c)/2\).

II. EXPERIMENTAL APPARATUS

The core of the measurement apparatus is shown in Fig. 2. A test section, consisting of two test heat exchangers separated by a spacer, is inserted into the middle of the apparatus, which is vertically symmetrical about the test section except for the shaker at the bottom. An APS Dynamics\textsuperscript{10} shaker moves a metal end-plate that is connected to the stationary parts of the duct via a square polyurethane bellows. Just above the bellows is a heat exchanger, which we call a “guard heat exchanger” discussed in the following. The maximum peak gas displacement amplitude is 70 mm, with the rest of the apparatus held stationary by a strut structure (not shown). The moving parts are shown here in their lowest position, so that the upper bellows is compressed and the lower bellows is expanded.

Also attached to the shaker (but not shown in the diagram) is a strut structure that supports suspension springs. The spring stiffness combines with the mass of the moving parts to give a resonance frequency around 4 Hz. This choice of spring stiffness allows the shaker to oscillate the end plates to its full stroke at up to 5 Hz while still allowing the full stroke at 0.125 Hz. The amplitude diminishes as the frequency is increased above 5 Hz due to amplifier limitations.

All experiments are conducted in air at ambient atmospheric pressure, which is measured with a mercury barometer for determination of the air density \(\rho_m\). The thermal conductivity \(k_0\) and dynamic viscosity \(\mu\) of air are calculated from the mean temperature of the gas in the duct using formulas from Pierce\textsuperscript{11} and the kinematic viscosity is \(\nu = \mu/\rho_m\). The specific heat of air is \(c_p = 1005 \text{ J kg}^{-1} \text{ K}^{-1}\).

The position of the end plates is measured with a linear variable differential transformer (LVDT). The gas displacement is inferred from the end-plate position measurement. The maximum peak gas displacement amplitude is 70 mm, which at 5 Hz gives a maximum air speed of 2.2 m/s. The pressure drop across the test section is also measured, but pressure measurements are not reported in the present paper. Data acquisition, validation, and error analysis of the pressure and position sensors are described in detail in Ref. 12.
A. Flow loops, chillers, and pumps

The primary goal of the experiments is the measurement of the amount of heat transferred between two heat exchangers in oscillating flow for various conditions of oscillation amplitude and frequency. The heat transferred between the two exchangers is found by measuring the amount of temperature change of water flowing though the heat exchanger tubes. There are two flow loops, one for the hot exchanger and one for the cold. In each loop, the temperature is maintained by a “recirculating chiller.” The “chiller” on the hot side includes an electric heater so that it may be used above room temperature. In each loop, water is pumped from the chiller through a reference heater (described in the following), the heat exchanger, a filter, a turbine flow meter, the guard heat exchanger, and back to the chiller. The flow rate is adjusted to produce a reasonable temperature increase as the water passes through the heat exchanger, with flow rates of about 0.3–0.6 kg/min. Distilled water is used for all measurements, with the addition of a few drops of a biocide and a few grams of corrosion inhibitor.

The guard heat exchangers are off-the-shelf units intended for the cooling of truck transmission fluid. Their form is similar to that of a car radiator. The primary function of the guard heat exchangers is to insulate the air inside the duct from the room. Almost all of the core of the test apparatus is well insulated with 2.5–10 cm of polystyrene foam, but it is impossible to insulate the bellows. The guard heat exchanger acts as a substitute for insulation, holding the air that passes from the bellows into the test duct at a temperature that is near to that of the nearest heat-exchanger-under-test. The guard exchanger also acts to straighten the flow of the air coming into the duct from the compressing bellows.

B. Temperature sensors

Water temperatures are measured with thermistor probes manufactured by RDP Corporation. Each probe contains a 2252 Ω YSI series 55000 glass-encapsulated thermistor, sheathed in a stainless steel tube that is 15 cm long and 3.175 mm in diameter. Attached to each heat exchanger are two 6.35-mm-o.d. tubes for connecting the hoses for entering and exiting water. These tubes are made quite long, about 13 cm. The long sheath of a thermistor probe is inserted through the straight leg of a Swagelok tee-fitting and into a connecting tube. Water enters through the side branch of the tee, so that water flows past the entire length of the probe sheath on its way into the exchanger (and vice versa for exiting water). By this connection method, the probe is effectively immersed in the water to a depth of 15 cm. Tests in the recirculating chiller bath show that 10 cm of immersion is sufficient to eliminate any detectable effect from heat leak down the sheath. An additional advantage of this arrangement, in which the water plumbing connections are made far from the exchanger, is that insulation can be tightly and permanently attached to the heat exchanger connecter tube right up to the exchanger manifold, while at the same time allowing the thermistor, which is at the tip of the long probe, to be placed very close to the exchanger entrance or exit. This is especially important for the cold heat exchanger, since the circulating cold water is sometimes cold enough to condense moisture from the ambient air. Additional insulation is placed over the Swagelok tee-fitting after the water connections are made.

The present measurements are not very sensitive to absolute temperature accuracy, but are extremely sensitive to the accuracy of temperature differences. Response differences between the probes are corrected to within 2 mK by calibrating them simultaneously in a variable-temperature water bath.

C. Flow meters and reference heaters

A DigiFlow DFS-2W turbine flow meter is placed in each flow loop to measure the rate of water flow. Temperatures and flow rates are combined to give heat transfer rates. For the determination of heats based on the flow meters, \( \dot{Q}_{\text{raw}} = \dot{m}_w c_w \Delta T_{\text{hx}} \), where \( \dot{m}_w \) is the mass flow rate of the water, \( c_w = 4180 \text{ J kg}^{-1} \text{ K}^{-1} \) is the specific heat of water, which is constant within the accuracy of these measurements, and \( \Delta T_{\text{hx}} \) is the change of temperature of the water as it passes through the exchanger.

An electric heating element inserted in the flow loop provides a second method of measuring the heats. The method is to measure the temperature increase in the circulating fluid due to a well-known rate of heating, provided by an electric heating element, and to compare the temperature change at the test exchanger to that at the reference heater, measured with probes like those used to measure the heat–exchanger temperature differences (see Ref. 16, Sec. 9.2.2). The heater-method uses \( \dot{Q}_{\text{raw}} = \dot{Q}_{\text{ref}} (\Delta T_{\text{hx}}) / \Delta T_{\text{ref}} \), where \( \Delta T_{\text{ref}} \) is the temperature increase of the water as it passes through the heater, and \( \dot{Q}_{\text{ref}} \), the known electrical heat supplied to the reference heater, is calculated from the resistance of the heater element and the voltage at the heater measured by four-wire technique.

III. THE PARALLEL-PLATE HEAT EXCHANGERS

The parallel-plate heat exchangers used in these measurements are made from flat extruded aluminum tubing of a type used in automotive air-conditioner condensers. The tubes are 2.0 mm × 22.0 mm in external cross section, having rounded ends. The interior of the tube is divided into nine rectangular channels, as shown in Fig. 3. Interior and side walls are all 0.50 mm thick. These tubes are analogous to the parallel fins of a tube-and-fin heat exchanger, with nearly 100% fin efficiency (no conduction loss) due to the water flowing through them. That is, in these exchangers, the “tubes” and the “plates” are the same.

Each 292 mm × 292 mm exchanger is made up of 35 of these tubes separated by 6.35 mm gaps, for a center-to-center tube spacing of 8.35 mm and hydraulic radius of \( r_h = \gamma_0 = 3.175 \text{ mm} \). (The hydraulic radius for parallel plates is half of the plate separation. This half spacing is often referred to as \( \gamma_0 \) in thermoacoustics, following the notation of Swift.) The total wetted perimeter of the tubes is \( \pi L = 20.45 \text{ m} \). The tubes are mounted at each end into a manifold made from 25.4 mm square aluminum tube. The tubes are centered on the manifolds, so that each tube is set (25.4–22.0)/2

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= 1.7 mm back from the edges of the manifold. That is, when the two heat exchangers are brought together with their manifolds touching, there is a 3.4 mm gap between the two sets of tubes. This 3.4 mm is then the minimum possible value of the distance between the edges of the tubes of the two exchangers, which is the distance that is meant by the term “heat exchanger separation” and indicated by the symbol $2x_{hx}$.

When the exchangers are installed in the test rig, each is placed into a mounting structure made from polystyrene foam board of the type used to insulate houses. This mounting insulates the exchangers from the test duct and from the outside air, and also forms the two sides of each exchanger that are perpendicular to the manifolds. An additional piece of foam is the “spacer,” used to insulate the exchanger manifolds from each other, and to establish the exchanger separation spacing.

The hot exchanger is placed above the cold exchanger for gravitational stability of the air inside the duct. Care is taken to align the exchangers so that, if one could look axially down the duct, the tubes of the closer exchanger would lie directly in front of the tubes of the far exchanger, with a maximum amount of “free flow area.” Water passes through the exchangers in counter-flow with respect to each other, so that the temperature difference between the exchangers should be nearly the same at each location along the length of the tubes.

The adjustment of oscillation amplitude $x_1$ and frequency $f$ (and angular frequency $\omega = 2\pi f$) is under computer control. The control program reads a desired oscillation amplitude from a list and adjusts the shaker drive level until the measured amplitude is within about 1% of the prescribed value. The temperature difference across each exchanger is measured, and then measured again about 1 min later. If either $\Delta T$ has changed by more than 3 mK (the smallest practical noise-limited value), the process is repeated until both heat exchangers meet the 3 mK tolerance during a single measurement cycle, after which the computer records all measured values and moves to the next desired amplitude or frequency.

### IV. RAW RESULTS AND UNCERTAINTIES

Measurements at several sample frequencies are shown in Fig. 4. In this measurement, the exchangers are placed as close together as possible: the insulating spacer between the manifolds is 2.1 mm thick, so that the separation of the exchanger tubes is $2x_{hx}=5.5$ mm. Plotted along the horizontal axis is $x_1$, the peak displacement amplitude of the oscillating gas. The two solid vertical lines are aids to the eye that show the “position” of the edges of the exchanger. The first is at $x_{hx}$, the amplitude at which a parcel of gas that begins at the center of the inter-exchanger gap barely enters each exchanger at the limits of its excursion. The second line represents the amplitude at which this parcel barely traverses both exchangers. This occurs when $x_1 = x_{hx} + \sigma L_{hx}$, where $L_{hx}$ is the length of each exchanger in the oscillation direction (22 mm) and $\sigma$ is the porosity (void volume divided by total volume) of the exchangers (0.760).

The curve for each frequency in Fig. 4(a) is actually a cluster of four curves, one for each measurement method, as indicated by the legend. A significant difference between these would indicate a problem either with a flow meter or with one or both of the temperature sensors in a reference heater, or with the reference heater insulation. The flowmeter and reference-heat methods of measuring the heat are not completely independent, however, since both depend...
\(\Delta T_{bh}\). These are basically two ways of measuring mass flow rate,\(^{18}\) though the reference heater is actually measuring heat capacity rate. The spread in these four curves is an indication of the uncertainty of the flow measurements. The standard error in the mean\(^{19}\) of the four measurements is plotted in Fig. 4(b). The error is generally about 0.5% for amplitudes that take the gas beyond both exchangers, 1% for amplitudes within the exchangers, growing to as much as 3% for the smallest amplitudes.

Any systematic offset in the measured temperature differences can be detected by reversing the direction of the flow through both exchangers and then repeating the measurements. (The direction of flow through the reference heaters is not reversed.) Once again, typical discrepancies are about 0.5% for large amplitudes, 1% for medium amplitudes, and 3% for the smallest amplitudes. The mean of all of these points is \(-0.2\%\), indicating that the overall systematic discrepancy of the temperature differences measured across the exchangers is small.

The objective of the experiment is to measure the heat transfer between the heat exchangers due to oscillating flow. In the absence of oscillation, however, there is still some heat transfer between the exchangers due to simple conduction through the air, and between the manifolds of the exchangers through the spacer. There is an additional heat leak out of the hot exchanger into the room and from the room onto the cold exchanger, via the foam that insulates the sides of the exchangers from the room air. After reducing heat leaks as much as practical with insulation, the strategy has been to make the hot and cold heat leaks equal by putting the hot exchanger above room temperature and the cold exchanger below room temperature, typically 10–15 K. The baseline heat \(Q_{0}\), which is observed, for example, as the nonzero value of the heat at zero amplitude in Fig. 4(a) is then subtracted from all measured heats \(Q_{\text{raw}}\) to give the amount of heat flow attributed to oscillating flow advection, \(\dot{Q} = Q_{\text{raw}} - Q_{0}\), before subsequent analysis.

The temperature of the room fluctuates over the course of the measurement. It took 4.75 days to collect the small-gap data, during which time the room temperature varied over a range of \(\pm 2\) K. The variation in the small-amplitude heat leak over this time, however, was only \(\pm 0.02\) W/K.

The oscillating flows within the exchangers are very likely laminar. The peak Reynolds number based on hydraulic diameter \((=4r_{h})\), \(Re_{\text{hyd}} = \omega x_{1} r_{h} / \nu\), has a maximum value of 2020. The acoustic, oscillating-flow Reynolds number, \(Re_{\text{ac}} = \omega x_{1} \delta_{x} / \nu\), depends on frequency, since the viscous penetration depth is \(\delta_{x} = \sqrt{2
u / \omega}\). Its maximum value for any of these measurements is 160. With this combination of \(Re_{\text{hyd}}\) and \(Re_{\text{ac}}\), the flow within the exchanger would be expected to be strictly laminar if the exchangers were long (see Ref. 16, Sec. 7.2). The shortness of the exchangers puts this conclusion in some doubt, but the low value of \(Re_{\text{hyd}} < 2020\) itself also suggests that laminar conditions are likely.

In this paper, measured heats are normalized by such quantities as thermal penetration depth \(\delta_{x} = \sqrt{2k_{p} / \rho_{i} c_{p} \nu \omega}\) and heat capacity rate \(\dot{m} c_{p}\). The reader should keep in mind that in the experiments, only two quantities were varied, amplitude and frequency. Thus, normalization by “\(\delta_{s}\)” is really normalization by \(f^{-0.5}\), for example. Additional factors involving \(\rho_{i}, c_{p}, \gamma_{0}, \text{and } L_{bh}\) are included to nondimensionalize the results, and there is reason to believe that the results should be scalable to other gases at other pressures. In the present study, however, no experiments were carried out using gases of different properties or exchangers of different geometries.

### V. Normalized Results

Normalized results are shown in Fig. 5 for small, medium, and large exchanger separations. In these plots, the results of the different measurement methods have been averaged together, four measurements for each point in Figs. 5(a) and (b), and two measurements each in Fig. 5(c), for which reference-heater data are not available. The baseline heats have been subtracted. The average \(\dot{Q}\) for each amplitude and frequency is divided by \(\Delta T_{ave} = (T_{h} - T_{c})/2\), where \(T_{h}\) and \(T_{c}\) are the temperatures of the hot exchanger and of the cold exchanger (that is, each is the average of the inlet and the outlet temperature for that exchanger.) This \(\Delta T_{ave}\) is the difference between either exchanger and the average temperature in the plane that is halfway between the two exchangers.

Besides division by \(\Delta T_{ave}\), \(\dot{Q}\) is nondimensionalized by normalizing by \(k_{0} and \Pi = 20.45\) m, the wetted perimeter of the exchanger tubes. Since the hydraulic radius \(r_{h}\) is related to the minimum free-flow area \(A_{h}\) and the total frontal (dust) area \(A_{b}\) by \(r_{h} = A_{h} / \Pi = \sigma A_{h} / \Pi\), normalizing by \(\Pi\) is equivalent to normalizing by \(\sigma A_{h} / r_{h}\).

The oscillation amplitude \(x_{1}\) on the abscissa is normalized two ways: on the bottom axis \(x_{1}\) is normalized by \(y_{0} = r_{h}\). On the top axis, the oscillation amplitude is normalized in terms of exchanger length. The dotted vertical line represents the center of the exchanger. The factor of \(\sigma\) has been included in this normalization so that an increase in normalized amplitude of 1.0 causes the fluid within the exchanger to oscillate an additional exchanger length in each direction. This complication is unavoidable in any real exchanger, which necessarily has porosity less than 1.0.

The curves for different frequencies are labeled in terms of the nondimensional ratio \(y_{0} / \delta_{x} = r_{h} / \delta_{x}\). Researchers in other fields use Valensi number (usually taken to be \(4r_{h}^{2}\omega / \nu = 8r_{h}^{2} / \delta_{x}^{2}\)) or Womersley number (usually \(\sqrt{2r_{h}} / \delta_{x}\) or \(2 \sqrt{2} r_{h} / \delta_{x}\)) in conjunction with Prandtl number \((Pr = \delta_{x}^{2} / \delta_{p}^{2})\). The 5 Hz curve (\(y_{0} / \delta_{x} = 2.65\)) in Fig. 5(a) appears more jagged than the others because it includes points from two separate sweeps of amplitude at this frequency that were separated in time by 28 h.

### VI. THE DELTAE HEAT EXCHANGER MODEL

The de facto standard for designing thermoacoustic devices is a piece of software called DELTAE.\(^{20}\) One of the major reasons for carrying out the heat exchanger measurements has been to test, examine, and possibly improve upon DELTAE’s parallel-plate heat exchanger model, a very simple model that is probably the weakest aspect of the software. Before analyzing the data, it is useful to review DELTAE’s...
parallel-plate heat exchanger model to provide context and motivate the type of analysis that is carried out.

The DELTA E model for parallel-plate heat exchangers (Ref. 20, p. 113) can be cast in terms of the present notation as

\[ \dot{Q}_{\text{DELTA E}} = (\Delta T_{\text{ave}} k_0 \Pi) C \frac{x_{\text{eff}}}{y_{\text{eff}}} \]

with

\[ x_{\text{eff}} = \min \{ 2(x_1 - x_{hx})/\sigma, L_{hx} \}, \]

\[ y_{\text{eff}} = \min \{ \delta, y_0 \}, \]

where the constant \( C \), which is 1 in the DELTA E model, has been added as a parameter to adjust. The general form of Eqs. (1)–(3) was postulated by Swift.\(^{21}\)

DELTA E assumes that the exchanger is directly adjacent to a stack or regenerator. In the present experiment, the exchangers are separated by a distance \( 2x_{hx} \), and oscillation within this gap does not contribute to the effective transfer area, hence the use of \( 2(x_1 - x_{hx})/\sigma \) rather than \( 2x_1/\sigma \) in Eq. (2). The analysis will concentrate on the small-gap data, so that the situation is as much like that in a thermoacoustic device as possible.

At high frequencies \( (y_0/\delta > 1) \), \( \dot{Q}_{\text{DELTA E}} \) is inversely proportional to \( \delta_{L} x^{-1/2} \); at low frequencies \( (y_0/\delta < 1) \), \( \dot{Q}_{\text{DELTA E}} \) is independent of frequency. Specifically, for \( y_0/\delta > 1 \),

\[ \left( \frac{\dot{Q}_{\text{DELTA E}}}{\Delta T_{\text{ave}} k_0 \Pi} \right) \frac{\delta_{L}}{L_{hx}} = 2C \frac{x_1 - x_{hx}}{\sigma L_{hx}}, \]

which indicates that, when plotted against \( (x_1 - x_{hx})/\sigma L_{hx} \), \( (\delta_{L}/L_{hx}) \dot{Q}_{\text{DELTA E}} / \Delta T_{\text{ave}} k_0 \Pi \) gives a straight line with slope \( 2C \) for all \( y_0/\delta < 1 \), whereas for \( y_0/\delta > 1 \), \( \dot{Q}_{\text{DELTA E}} / \Delta T_{\text{ave}} k_0 \Pi \) is the quantity that produces a single straight line, in this case with slope \( 2CL_{hx}/y_0 \), which for these exchangers is 13.9\( C \). The DELTA E model is shown together with data for these two normalizations in Fig. 6. Figure 6(a) shows the small-gap data at the twelve highest frequencies \( (y_0/\delta > 2) \) using the \( \delta/\dot{Q} \) normalization. The curves are all nearly the same shape, and are remarkably straight in the region between the vertical lines that indicate amplitudes corresponding to the edges of the exchangers. The dashed line is the prediction of the DELTA E model, with \( C = 1 \). The heavy solid line shows the DELTA E model with \( C = 0.45 \). If the frequency range of the data is extended down to \( y_0/\delta = 1 \), these lower-frequency curves have the same slope and straight-line shape up to about \( (x_1 - x_{hx})/\sigma L_{hx} = 1 \), above which they diverge (upward) somewhat from the higher-frequency data.

Figure 6(b) shows the DELTA E model together with the small-gap data for all frequencies low enough that \( y_0/\delta < 1 \). The amplitude coordinate is the same as in Fig. 6(a), but \( \dot{Q} \) is normalized by \( \Delta T_{\text{ave}} k_0 \Pi \), independent of frequency. The result is disturbing: even in the low-amplitude region, the data curves decrease with frequency, but the DELTA E model does not. As a consequence, even with \( C = 0.45 \) the model seriously over-predicts heat transfer at the lowest frequencies.

It turns out that there is a fundamental problem with the model defined by Eqs. (1)–(3). The problem is revealed by considering the case of perfect heat exchange, for which all of the working gas undergoes the full temperature swing...
The problem is now evident. For any exchanger plate spacing tighter than \( y_0 / \delta_k = \pi/2 = 1.57 \), the \( \Delta E \) model with \( C = 1 \) predicts \( Q_{\Delta E} / Q_{\text{perfect}} > 1 \), a greater rate of heat transfer than can possibly occur with fluid of this heat capacity at this frequency. While a value of \( C = 0.45 \) shifts the value at which this problem occurs down to \( y_0 / \delta_k = 0.71 \), for sufficiently tight spacings this problem will arise for any constant value of \( C \). This problem was pointed out six years ago by Brewster et al., but the importance of this result does not seem to have been appreciated by the thermoacoustics community.

If one defines a sort of “effective heat transfer coefficient” \( h_{\text{eff}} \) by

\[
\hat{Q}_{\Delta E} = h_{\text{eff}} y_{\text{eff}}^2 \delta_k T_{\text{ave}},
\]

then \( h_{\text{eff}} = C k_0 / y_{\text{eff}} \). With \( C = 1 \),

\[
h_{\text{eff}} = \frac{k_0}{\delta_k} \quad \text{for} \quad \frac{y_0}{\delta_k} \gg 1,
\]

and

\[
h_{\text{eff}} = \frac{k_0}{y_0} \quad \text{for} \quad \frac{y_0}{\delta_k} \ll 1.
\]

We know from Eq. (10) that there is some error in Eqs. (11) and (13). The form \( h_{\text{eff}} = k_0 / y_0 \) for \( y_0 / \delta_k < 1 \) appears quite reasonable, however. After all, how can the effective film thickness be greater than half the plate spacing? The problem is not in \( h_{\text{eff}} \), per se. Rather, it is with the notion that a heat transfer coefficient can be used in conjunction with an initial temperature difference such as \( \Delta T_{\text{ave}} \). In heat exchangers with \( y_0 / \delta_k < 1 \), the instantaneous gas-to-exchanger temperature difference \( \Delta T(t) \) becomes small enough that the rate of heat transfer is small even though the heat transfer coefficient is large. In cases where \( \Delta T(t) \) is not approximately constant, use of the concept of “heat transfer coefficient” requires detailed knowledge of the time history of \( \Delta T(t) \). An alternative approach that allows us to retain use of \( \Delta T_{\text{ave}} \) is to use the concept of “effectiveness,” discussed next.

**VII. EFFECTIVENESS**

Effectiveness \( \varepsilon \) is \( \frac{\hat{Q}}{Q_{\text{perfect}}} \) “the ratio of the actual heat transfer rate for a heat exchanger to the maximum possible heat transfer rate.”

\[
\varepsilon = \frac{\hat{Q}}{Q_{\text{perfect}}}.
\]

In Fig. 7, \( \hat{Q} \) is normalized by \( 2 \Delta T_{\text{ave}} \rho_m c_m A_{\text{eff}} \sigma L_{\text{hh}} f \). This heat-capacity-based normalization results in the plot of \( Q_{\text{perfect}} \) vs \( (x_1 - x_{\text{hh}}) / \sigma L_{\text{hh}} \) having a constant slope of 2, shown by the heavy line. At the lowest frequency, \( y_0 / \delta_k = 0.418 \), the exchangers are nearly perfectly effective, even for amplitudes that take the gas well beyond both exchangers. At \( y_0 / \delta_k = 0.835 \), the effectiveness is still quite high at
low amplitudes, around 80%, but drops to 60% for the highest amplitude. At \( y_0 / \delta_{x} = 2.05 \), the effectiveness is never more than about 30%, even at low amplitudes.

A model based on effectiveness, rather than on an effective heat transfer coefficient, would have the advantage that it would naturally incorporate heat capacity into the heat transfer model, and is attractive because the oscillating heat capacity rate is well known. In the following, the data are cast in terms of effectiveness, fits are made, and an effectiveness-based heat transfer model is proposed for use in design. Such a model might form the basis of an improved DELTAE parallel-plate heat exchanger segment.

### A. Low-amplitude conduction enhancement

Before completing the analysis, it is necessary to discuss a difficulty arising from the fact that \( x_{hx} > 0 \). In Eq. (6), which is the simplest possible model of “perfect” heat transfer between the exchangers, \( \dot{Q} = 0 \) for any \( x_1 < x_{hx} \), since no gas actually enters both exchangers. In Fig. 5, however, we see that in fact \( \dot{Q} > 0 \) for \( x_1 = x_{hx} \). Even when there is an oscillation that is too small to carry any gas over the full distance between the exchangers, gas that has its equilibrium position barely within one exchanger spends some time closer to the other exchanger than it would have had there been no oscillation. The effect is actually not so much one of increased effective conductivity as of decreased effective exchanger separation. Since this phenomenon is not understood quantitatively, no attempt has been made to correct for it. Unfortunately, this “low-amplitude enhancement effect” causes the measured effectiveness to exceed unity at the lowest amplitudes.

The small-amplitude enhancement effect becomes more important as \( x_{hx} \) is increased, as can be seen by comparing Figs. 5(a)–(c). Collecting data using the smallest practical gap reduces the effect, but does not eliminate it. Using a tiny gap introduces its own problems, however, since it increases the baseline (zero-amplitude) heat that is subtracted from all data. At the lowest frequencies, for which the oscillation-induced \( \dot{Q} \) is very small, this means subtracting a very large fraction of the total measured heat, resulting in errors. We judge the lowest-frequency data for the small gap configuration to be unreliable except at the very highest amplitudes. For this reason, medium-gap data are used in the ensuing analysis for determination of the low-amplitude effectiveness at the lowest frequencies. Medium-gap data are indicated by triangular markers. A complete discussion of this issue is found in Ref. 23.

### B. Effectiveness data and low-amplitude fit

In the present oscillating flow study, where two exchangers interact, the notation must distinguish between the effectiveness of a single exchanger and that of a two-exchanger system. In the analysis carried out later, the symbol \( e_{hx} \) is used to indicate the effectiveness of a single exchanger of type “hx,” the symbol \( e_T \) refers to the total effectiveness of a two-exchanger system, and the symbol \( [e_T]_{hx} \) refers to the total effectiveness of two identical exchangers of type “hx.” The measured effectiveness of the experimental parallel-plate exchangers is of this last type, so the results are referred to using the \( [e_T]_{hx} \) notation, for compatibility with the later analysis.

The measured effectiveness of the heat transfer between the two exchangers \( [e_T]_{hx} \) is plotted in Fig. 8 for many different frequencies. This plot includes all of the small-gap data above 0.5 Hz \( (y_0 / \delta_x > 0.838) \), plus medium-gap at 0.5 Hz and two lower frequencies, indicated with triangles.
Below \((x_1 - x_{hx})/\sigma L_{hx} = 1\), the effectiveness at each frequency is at least roughly constant, but because \(Q_{\text{perf}}\) goes
to zero at \((x_1 - x_{hx})/\sigma L_{hx} = 0\), the measured \([e_T]_{hx}\) is not very stable in this region. For the purposes of proceeding
with the analysis, the value of \([e_T]_{hx}\) nearest \((x_1 - x_{hx})/
\sigma L_{hx} = 1\) is chosen as representative of the low-amplitude
effectiveness. This is a somewhat arbitrary choice, and it probably
underestimates \([e_T]_{hx}\) slightly, but we have concluded that the values of \([e_T]_{hx}\) at lower amplitudes are simply
less reliable, even when considering some scheme of averaging. The points selected for analysis are highlighted with circles.

The selected points are collected into a single plot of
\([e_T]_{hx}\) vs \(y_0/\delta_x\) in Fig. 9. The log–log rendering reveals a
very straight line connecting the points at the higher frequencies.

The function chosen to fit the data, indicated by the curve, has two parts. At high frequencies the fit is
\[
[e_T]_{hx} = \frac{0.7}{y_0/\delta_x} \quad \text{for } y_0/\delta_x \geq 1 \text{ and } \frac{x_1 - x_{hx}}{\sigma L_{hx}} < 1.
\] (15)

This is not actually new information, corresponding as it does to the fact that \(Q \approx \delta_x^{-1}\) at high frequencies, as established by Fig. 6(a), and to be discussed further in Sec. VII C. The new information to come out of Fig. 9 is the low-frequency fit,
\[
[e_T]_{hx} = 1 - 0.3(y_0/\delta_x)^{2.5} \quad \text{for } y_0/\delta_x \leq 1 \text{ and } \frac{x_1 - x_{hx}}{\sigma L_{hx}} < 1.
\] (16)

Unfortunately, there are few reliable data points in this region, and Eq. (16) is both strictly empirical and not very tightly constrained by the data. Fortunately, having the exact power in Eq. (16) is not particularly important, because \([e_T]_{hx}\) is so close to 1.0 in this region. Equation (16) simply provides a convenient way of connecting the known zero frequency limit, \([e_T]_{hx} = 1\), and the physically motivated high-frequency form \([e_T]_{hx} \propto (y_0/\delta_x)^{-1}\), which has the measured value \([e_T]_{hx} = 0.7\) at \(y_0/\delta_x = 1\).

FIG. 9. Circled data points from Fig. 8, together with a bipartite fit function (solid curve) defined by Eqs. (15) and (16).

C. Effectiveness in oscillating flow

In this section, the measured effectiveness is related to \(\Delta T_{\text{ave}}\), so that we can use the measurements in a model that has the same form as the existing \textsc{deltae} model. The derivation is greatly simplified by the fact that the heat capacity of an exchanger is effectively infinite compared to that of the oscillating gas. Under these circumstances, and if the specific heat of the gas is constant, effectiveness can be expressed simply in terms of temperatures,
\[
e_{hx} = \frac{T_{\text{in}} - T_{\text{out}}}{T_{\text{in}} - T_s},
\] (17)

where \(T_{\text{in}}\) is the temperature of the gas as it enters the exchanger, \(T_{\text{out}}\) is the temperature of the gas as it exits the exchanger, and \(T_s\) is the (constant) exchanger surface temperature.

Figure 10 shows a sketch of temperature versus position for a parcel of gas as it oscillates between hot and cold exchangers at temperatures \(T_h\) and \(T_c\). Gas enters the cold exchanger at \(T_{\text{in}} = T_1\) and exits at \(T_{\text{out}} = T_2\), so the effectiveness \(e_c\) of the cold exchanger is
\[
e_c = \frac{T_1 - T_2}{T_1 - T_s}.
\] (18)

Similarly, for the hot exchanger,
\[
e_h = \frac{T_2 - T_1}{T_2 - T_h} = \frac{T_1 - T_2}{T_h - T_2}.
\] (19)

Now consider a combined system of hot and cold heat exchangers. If both exchangers were perfect, then the gas would cover the full span between \(T_h\) and \(T_c\) as it oscillated, so for the nonideal exchangers of Fig. 10, the total effectiveness \(e_T\) is
\[
e_T = \frac{T_1 - T_2}{T_h - T_c}.
\] (20)

The relationship between the effectiveness of the individual exchangers and the effectiveness of the two exchanger system is, then,
\[
\frac{1}{e_h} + \frac{1}{e_c} = \frac{1}{e_T} + 1.
\] (21)

For the case of two identical heat exchangers, each with effectiveness \(e_{hx}\), the total effectiveness is
\[ [\varepsilon_T]_{\text{hx}} = \frac{\varepsilon_{\text{hx}}}{2 - \varepsilon_{\text{hx}}}. \]  

For purposes of comparing to the DELTAE model, it is useful to put these results in terms of \( T_{\text{ave}} \), the average temperature of the gas at the interface between a stack (or regenerator) and a heat exchanger. Consider Fig. 10, interpreting the heavy line labeled \( T_h \) to be the end of a stack or regenerator of unknown effectiveness \( \varepsilon_{\text{hx}} \). This unknown effectiveness affects the average temperature of the gas at the interface, and thus the amount of heat transfer. For example, if the line \( T_h \) represents a regenerator in good thermal contact with the gas, the higher \( \varepsilon_h \) of the regenerator will result in an average temperature closer to \( T_h \), and more total heat transfer for a given \( \varepsilon_e \), than would the lower \( \varepsilon_h \) of a stack. The average temperature of the gas at the interface \( T_{\text{ave}} \) is

\[ T_{\text{ave}} = \frac{T_1 + T_2}{2}. \]  

The difference between \( T_{\text{ave}} \) and \( T_e \) is

\[ [\Delta T_{\text{ave}}] = T_{\text{ave}} - T_e. \]  

Using Eqs. (18), (22), and (24) it can be seen that

\[ [\Delta T_{\text{ave}}]_h \varepsilon_e^2 = T_1 - T_2. \]  

Similar algebra results in a similar expression of the hot exchanger, so that, in general,

\[ [\Delta T_{\text{ave}}]_h \varepsilon_e^2 = T_1 - T_2, \]  

where \([\Delta T_{\text{ave}}]_h\) is the difference between the average gas temperature at the interface and the heat exchanger temperature, whether the exchanger is hotter or colder than the stack. That is, \([\Delta T_{\text{ave}}]_h\) is the very “\( \Delta T_{\text{ave}} \)” used in the DELTAE model. Equation (26) is the desired relationship between \( \Delta T_{\text{ave}} \), the total effectiveness of two identical exchangers \([\varepsilon_e]_h\), which is the effectiveness measured in the present experiment, and \( T_1 - T_2 \), which is necessary to calculate the amount of heat transferred.

The rate of heat transfer in Fig. 10 is

\[ \dot{Q} = mc_p(T_1 - T_2) \]  

\[ = \rho_m c_p \frac{2(x_1 - x_{\text{hx}})}{\sigma} \]  

\[ \left( \Delta T_{\text{ave}} \right)_{\text{eff}} \]  

\[ \left( \Delta T_{\text{ave}} \right)_{\text{eff}} \]  

\[ = (\Delta T_{\text{ave}} k_0 \Pi) \frac{2([\varepsilon_e]_h \varepsilon_{\text{eff}} y_0)}{\pi} \]  

\[ x_{\text{eff}} \]  

where \( x_{\text{eff}} \) is different from \( x_{\text{eff}} \) in that

\[ x_{\text{eff}} = \frac{2(x_1 - x_{\text{hx}})}{\sigma} \]  

for all \( x_1 \),

\[ \text{for } y_0/\delta_{\xi} \geq 1 \text{ and } x_{\text{eff}} < x_{\text{eff}} \]  

\[ \text{for } y_0/\delta_{\xi} \leq 1 \text{ and } x_{\text{eff}} < x_{\text{eff}} \]  

Comparing Eq. (31) to Eq. (1), we see that we have recovered the high-frequency, low-amplitude DELTAE model, with \( C = 0.7(2/\pi) = 0.45 \). The result applies up to \( x_{\text{eff}} = 2L_{\text{hx}} \), not just to \( x_{\text{eff}} = L_{\text{hx}} \), however.

With the effectiveness-based model of \( \dot{Q} \), in the form of either Eq. (28) or Eq. (29), and the low-frequency effectiveness fit of Eq. (16), we can immediately extend the model to low frequencies,

\[ \dot{Q} = (\Delta T_{\text{ave}} k_0 \Pi) \frac{2}{\pi} \left(1 - 0.3(y_0/\delta_{\xi})^{2.5}\right) \frac{x_{\text{eff}} y_0}{\delta_{\xi}^2} \]  

\[ \text{for } y_0/\delta_{\xi} \leq 1 \text{ and } x_{\text{eff}} < x_{\text{eff}} < 1. \]  

with the assurance that the result will not exceed that which is physically possible.

VIII. EXTENDING TO HIGHER AMPLITUDES

A. Empirical fit

So far, we have a model that works well at the lower amplitudes (\( x_{\text{eff}} \leq 2L_{\text{hx}} \)). One of the more important features observed in the data is that heat transfer continues to increase significantly as the amplitude increases beyond \( x_{\text{eff}} = 2L_{\text{hx}} \). For design purposes, it would be very useful to have a fit that follows the data into the high amplitude region. This fit is to be used to calculate \( \dot{Q} \) in the form

\[ \dot{Q}_{\text{fit}} = \rho m c_p x_{\text{eff}} y_0 / 2 \Delta T_{\text{ave}} F_e, \]  

\[ \text{or } \]  

\[ \dot{Q}_{\text{fit}} = (\Delta T_{\text{ave}} k_0 \Pi) x_{\text{eff}} y_0 \frac{2}{\delta_{\xi}^2} \frac{\varepsilon_{\text{eff}}}{\pi} F_e. \]  

where \( x_{\text{eff}} \) is defined in Eq. (30), \( \varepsilon_{\text{eff}} \) is the low-amplitude effectiveness fit of Eqs. (15) and (16), namely

\[ \varepsilon_{\text{eff}} = 1 - 0.3(y_0/\delta_{\xi})^{2.5} \text{ if } y_0/\delta_{\xi} \leq 1, \]  

\[ \varepsilon_{\text{eff}} = 0.7(y_0/\delta_{\xi})^{-1} \text{ if } y_0/\delta_{\xi} > 1, \]  

and \( F_e \) is a function of \( x_{\text{eff}} \) that will fit the data at high amplitudes. A fit that works fairly well is

\[ F_e = 1 \text{ if } x_{\text{eff}} < 2L_{\text{hx}}, \]  

\[ F_e = 1 + 0.7(y_0/\delta_{\xi})^{(x_{\text{eff}}/2L_{\text{hx}})^{0.7}} \text{ if } x_{\text{eff}} > 2L_{\text{hx}}. \]  

Data curves are shown in Fig. 11, together with \( \dot{Q}_{\text{fit}} \) curves determined from Eqs. (33) to (38). Not all frequencies are shown in Fig. 11 to make it easier to distinguish which fit curve goes with which data. An encouraging aspect of the fit is the lowest curve on the plot, \( y_0/\delta_{\xi} = 0.296 \), measured at 1/16 Hz using the small gap. Recall that these data were excluded from the fit to \([\varepsilon_e]_{\text{hx}}\), because they are deemed unreliable below \( x_{\text{eff}} = 2L_{\text{hx}} \). The fit curve does, however, approach the data at the highest amplitude, where these data
are most reliable. Generally speaking, the fits are within 10% of
the data for amplitudes above \((x_1 - x_{hx})/\sigma L_{hx} = 0.5\)
(which is \(x_EFF = L_{hx}\)), below which the low-amplitude en-
hancement effect causes the errors to grow and eventually to
blow up at \(x_1 - x_{hx} = 0\).

The proposed form of Eq. (38), which is basically a variation on \(1/(1 + x^a)\) with \(0 < a < 1\), while empirical, is not
entirely arbitrary. We have endeavored to find a function that
is physically plausible at higher amplitudes, i.e., that results
in a \(Q_{fit}\) that does not turn over and go to zero (or even negative!) at amplitudes just above the limits of the fitted
data. This rules out most forms involving exponential decays
and most power laws. The function in Eq. (38) becomes propor-
tional to \(x_EFF^{-0.7}\) at high amplitudes. When multiplied by
\(x_EFF\) in Eq. (33) or (34), the result is \(Q \propto x_EFF\) for large \(x_EFF\). The search for a function with a high-amplitude limit of \(x_EFF\)
with \(0 < b < 1\) was motivated by the thought that at high
amplitudes the heat transfer should be dominated by the
high-velocity portion of the cycle when the boundary layer is
similar to the steady-flow entrance-region result. The velocity
dependence of Nusselt number for the steady-flow prob-
lem depends on the boundary conditions. For constant wall
temperature under steady laminar flow, “thermal entry
length” conditions give \(Nu \propto Re^{0.7}\) and “simultaneously de-
veloping” thermal and hydrodynamic boundary layers \(34\)
give \(Nu \propto Re^{0.3}\). The power 0.7 in Eq. (38) results in \(Nu \propto Re^{0.3}\),
which may not be exactly correct, but it is probably the right
idea. This power sets the rate of curvature at the higher am-
plitudes. Referring to Fig. 11, the choice of 0.7 was obvi-
ously a compromise, with the middle frequencies exhibiting
sharper curvature than either of the frequency extremes. The
remaining details of Eq. (38) were chosen simply to improve
the fit. Presumably, it is mere coincidence that the coefficient
0.7 on the factors \(0.7(y_0/\delta_x)\) is equal to the power 0.7 on the
factor \((x_EFF/2L_{hx})^{0.7}\). The coefficients, as well as the power 1
on \((y_0/\delta_x)\), help match the slopes of the curves at \((x_1 - x_{hx})/\sigma L_{hx} = 1\). It seems likely that some or all of these
parameters depend in some way on the ratio \(\sigma L_{hx}/y_0\). The
test heat exchangers are fairly “short” compared to exchang-
ers in some devices, with \(\sigma L_{hx}/y_0 = 5.3\). Measurements on
heat exchangers with different dimensions would be neces-
sary to clarify this matter.

**B. Discussion**

Hofler carried out an effectiveness analysis eight years
ago \(25\) and concluded that “Heat exchangers with \(x_1/L_{hx}\) in
the range of 4 to 8 can be thermally effective if \(y_0/\delta_x\) is in
the range 0.75 to 0.5,” where “thermally effective” means
that “thermal effectiveness is between 77% and 93%.” Hof-
ler was concerned about minimizing both thermal and vis-
cous losses in the heat exchangers. “Thermal losses” peak
when the plate spacing is around \(y_0/\delta_x = 1\) for the same
reason that stacks operate in this range. Of course, viscous
losses simply increase as \(y_0/\delta_x\) decreases. Based on this
reasoning, Hofler concluded that “it is apparent that some
appropriate ‘figure of merit’ function used in optimizing heat
exchanger geometries would peak strongly at a value of
about \(y_0/\delta_x = 0.5\).” Hofler, then, advocated exchangers that
had smaller \(y_0/\delta_x\) and shorter \(L_{hx}\) than the dimensions
\(y_0/\delta_x = 1\) and \(L_{hx} = 2x_1/\sigma\) that tend to emerge from designs
developed with DELTAE.

For the most part, the present measurements support the
viewpoint put forward by Hofler in 1994. In particular, the
effectiveness of tightly spaced plates is quite high, and heat
transfer does continue to increase even as the amplitude in-
creases to take the gas well beyond the far edges of both
exchangers. Two factors unknown to the thermoacoustics community at the time of Hofler’s analysis may result in modifying his conclusion. These are the importance of minor losses and the effect of externally imposed pressure oscillations.

Minor losses\(^{16,26}\) result from flow past abrupt changes in geometry, such as the sudden change in cross-sectional area between a heat exchanger and the adjacent duct. The pressure loss from the acoustic oscillation due to this type of loss depends roughly on \((1 - \sigma)^2\) and on the square of the velocity within the exchanger, which is greater than that outside the exchanger by \(1/\sigma^2\). The result is that the minor loss goes up sharply as porosity decreases. It is difficult to make a heat exchanger with small \(y_0/\delta_0\) that does not also have a small porosity. This is partly due to the difficulty manufacturing very thin fins, and also due to problems getting heat off the fins and onto a secondary flow loop (i.e., a problem with fin efficiency).

The other consideration, the influence on heat exchangers of the temperature oscillations that are caused by the oscillating pressure in thermoacoustic devices, is more speculative. We have carried out numerical studies that indicate that, for spacings typical of parallel-plate heat exchangers used in thermoacoustic devices (\(y_0/\delta_0 = 2/3\)) and pressure amplitudes of 5% of mean pressure, the pressure-driven temperature oscillations in standing-wave devices might increase or decrease heat transfer by as much as 30% for refrigerators and engines, respectively. There is some experimental evidence for this idea. Mozurkewich\(^9\) measured heat transfer between a thermoacoustic stack and a tube heat exchanger in a standing-wave refrigerator, which exchanger was the one nearer the velocity node, the “hot” transfer between a thermoacoustic stack and a tube heat exchanger. One of his fins and onto a secondary flow loop very thin fins, and also due to problems getting heat off the fins and onto a secondary flow loop (i.e., a problem with fin efficiency).

This effect depends on plate spacing, disappearing as \(y_0/\delta_0 \rightarrow 0\), and, of course, it depends upon the gas’s being within the exchanger. Thus, pressure-oscillation-driven enhancement or degradation of heat transfer would be considerably less in a “Hofler-style” (short, tight) exchanger. While this would appear to be an advantage in a standing-wave engine, it might be a disadvantage in an standing-wave refrigerator. We have yet to study how this effect would manifest itself in the exchangers of a regenerator-based device, for which the oscillations at the heat exchangers have a mixture of standing- and traveling-wave phasing.

**IX. Conclusions**

Measurements of the heat transferred between two identical parallel-plate heat exchangers, made under conditions of oscillating flow over a range of frequencies and amplitudes, have been analyzed with the goal of producing an improved model for use in the design of thermoacoustic devices. The proposed model is summarized by Eqs. (30) and (33)–(38). Qualitative conclusions are:

1. For \((x_1-x_{hx})/\sigma L_{hx} \leq 0.5\) and \(y_0/\delta_0 \geq 1\), the idea that \(Q \approx \delta_e^{-1}\) seems to be correct.
2. Figure 6(a) makes it appear that for \(y_0/\delta_0 > 2\), the product \(Q \delta_e\) collapses to a single curve, independent of frequency, even at high amplitudes. This may be an illusion, however, since the upper amplitude limit decreases with frequency. It could be that the high frequency transport is actually frequency dependent at higher amplitudes.
3. The idea that \(Q \approx x_1-x_{hx}\) for \((x_1-x_{hx})/\sigma L_{hx} \leq 0.5\) seems to be correct.
4. The value of the constant \(C\) indicated by the present measurements is 0.45. This value may depend on parameters not varied in these measurements, such as the porosity. Recall also that the leading edges of the parallel plates used in these heat exchangers are rounded, unlike the blunt-edged fins that have often been used in thermoacoustic devices, which may affect \(Q\). Mozurkewich studied heat exchange in oscillating flow analytically\(^{27}\) using an eigenfunction approach. His analysis concluded by suggesting the value \(C = 0.61\).
5. The form \(h_{eff} \approx k_0 y_0\) for \(y_0/\delta_0 \leq 1\) is incorrect, at least when used in conjunction with an equation like \(Q = h_{eff} L_{hx} \Delta T_{ave}\). For \(y_0/\delta_0 \leq 1\) the cycle-averaged heat transfer is limited by heat capacity, not by the heat transfer coefficient.
6. As a result, it is not correct that “you can always increase the heat transfer by decreasing the plate spacing.” One can always make the effectiveness close to 1 by decreasing plate spacing. In this limit, the amount of heat transferred for a given temperature difference is set by the oscillating heat capacity rate, the calculation of which might be useful in making design decisions.
7. It is quite clear that the idea that \(x_{eff}\) has a maximum value of \(L_{hx}\) is not correct. The slope of \(Q\) is nearly constant up to an amplitude of about \(2(x_1-x_{hx}) = 2\sigma L_{hx}\) [rather than \(2(x_1-x_{hx}) = \sigma L_{hx}\)], and has a significant positive value well beyond this value. That is, effectiveness decreases as the amplitude exceeds \(x_{eff} = 2L_{hx}\), but not abruptly. This suggests that it might be possible to get almost equal performance from exchangers that are only half as long as what has conventionally been suggested. A relatively simple empirical fit function [Eqs. (37) and (38)] describes the performance of the present experimental heat exchangers fairly well. This fit
can probably be used in a general model to give approximate results, but it is likely that the fit parameters depend on \( aL_{eq}/y_0 \) or some other geometrical parameter that was not varied in the present measurements.

(8) The concept of “effectiveness” is useful in the study of oscillating-flow heat exchangers, in part because it incorporates heat capacity into the analysis.

(9) The total combined effectiveness of two identical heat exchangers (separated by little or no gap) in oscillating flow, as measured in the present experiments, can be incorporated easily and directly into an effectiveness-based oscillating flow model.

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APPENDIX: ROUND TUBE HEAT EXCHANGERS

In addition to the parallel-plate heat exchanger measurements that are analyzed in detail in the main body of the text, earlier measurements were carried out on round tube heat exchangers, some of which are presented here, in Fig. 12. These measurements are relegated to an appendix because we have less confidence in them, and also because it is not at all evident what sort of analysis should be carried out on these data.

Each round tube heat exchanger is made up of a single row of hollow circular brass tubes with an external diameter of 6.35 mm, separated by spaces of 3.175 mm, for a center-to-center spacing of 9.525 mm. It takes 30 such tubes to span the 292.1 mm duct, with 6.35 mm of extra space left at the end. Results are normalized by the “length” \( L_{eq} \) of an exchanger, taken to be the tube external diameter, and hydraulic radius \( r_h \), defined by

\[
\frac{V_{\text{void}}}{A},
\]

where \( V_{\text{void}} \) is the void volume (of the air) and \( A \) is the total wetted area. For the heat exchangers examined in this appendix, where the space between tubes is half the tube diameter, \( r_h = 0.227 \, \text{D}_{\text{tube}} \), or 1.44 mm.

For these exchangers, the low-amplitude conduction enhancement effect (see Sec. VII A) is very large. At the highest frequencies, there is a noticeable sudden increase in heat transfer at around \( x_1/r_h = 2 \), about half the amplitude required for any parcel of the gas to traverse the full interchanger gap. Apparently some additional mechanism, such as jet formation, is further enhancing conduction at the higher frequencies. This unknown but interesting phenomenon makes these curves particularly difficult to analyze.

One problem with the round tube exchangers is that the manifolds are too small to provide for an even flow of water through all of the tubes. Fortunately, we were alerted to this issue before construction of the parallel plate exchangers, which were made to have much larger manifolds as a result. The manifolds on the round tube exchangers are 9.525 mm square tubes. The inlet and outlet are also at opposite corners, with the intention of equalizing the lengths of the various paths through the exchanger. This turns out to be a bad idea because of Bernoulli pressure changes in the manifolds. Evidence that uneven flow distribution is a problem in the tube heat exchangers but not in the parallel plate heat exchangers comes from reversing the flow direction through both exchangers in each case. The reversal should not change the total amount of heat transfer, and it does not in the parallel plate exchangers. Reversing the flow direction through both of the round tube exchangers, in contrast, reduces the measured heat transfer by as much as 10%. Because of this, and because the entire set of validation tests was never completed for these exchangers, we place the uncertainty on these results at +10%, −20%.


17 This is the air-to-tube wetted perimeter in the part of the exchanger that is constant in cross section. In this approximation, the hydraulic diameter is simply \( 4y_0 = 0.5 \, \text{in.} = 12.7 \, \text{mm} \). This does not take into account the transverse surface area at the round ends of the tubes. If the total gas volume on the gas side of the exchanger is divided by the total surface area of the tubes, the resulting hydraulic diameter is 5% smaller. Where the tubes enter the manifold, there are 6.35 mm sections of aluminum manifold between each pair of tubes, which add an additional 0.44 m, or 2%, to the total exchange perimeter.

18 Turbine flow meters are often considered to measure volume flow rate. However, in our lab these meters are always calibrated by weighing the
water passing through the meter in some time interval, so the calibration is actually of mass flow rate.

19 Also called “the standard deviation of the mean,” the standard error is the square root of the variance divided by the \( \sqrt{N} \), where \( N \) is the number of samples (\( N = 4 \) in this case). See Eqs. (4.9) and (4.14) in J. R. Taylor, An Introduction to Error Analysis (University Science Books, Mill Valley, CA, 1982).


