Exam #3: Rotational Motion Solutions

1) The film has a linear speed of v = ω(2R) when it leaves the back spool. Its speed doesn't change along its linear trajectory between the two spools and so it enters the front spool at v = 2ωR. Since the radius of the front spool is R, the front spool must rotate at a rate of 2ω to keep up with the speed of the film.

2) For constant angular acceleration we know the equations of motion.

a. To find the angular acceleration we have to convert the linear speeds to angular speeds (v = ωR). The speeds are

$$\begin{aligned}\omega_0 &= \frac{v_0}{R} = \frac{17.8 \text{ m/s}}{0.20 \text{ m}} = 89.4 \text{ rad/s} \\ \omega &= \frac{v}{r} = \frac{15.6 \text{ m/s}}{0.20 \text{ m}} = 78.2 \text{ rad/s}\end{aligned}$$

With these we can find the angular acceleration.

$$\begin{aligned}\omega(t) &= \omega_0 + \alpha t \\ \alpha &= \frac{\omega(t) - \omega_0}{t} \\ &= \frac{78.2 \text{ rad/s} - 89.4 \text{ rad/s}}{15 \text{ s}} \\ &= -0.75 \text{ rad/s}^2\end{aligned}$$

b. The torque is

$$\begin{aligned}\tau &= I\alpha \\ &= \frac{1}{2}MR^2\alpha \\ &= \frac{1}{2}(20 \text{ kg})(0.20 \text{ m})^2 \left(0.75 \text{ rad/s}^2\right) \\ &= 0.30 \text{ Nm}\end{aligned}$$

3)

a. The moment of inertia of a cylinder spun along its long axis is

$$\begin{aligned}I &= \frac{1}{2}MR^2 \\ &= \frac{1}{2}(2.6 \text{ kg})(0.065 \text{ m})^2 \\ &= 5.49\text{x}10^{-3}\text{kgm}^2\end{aligned}$$

So that our units are consisten we must change out angular velocity into rad/s

$$\begin{aligned}\omega &= \frac{12 \text{ rev}}{1 \text{ s}}\left(\frac{2\pi}{1 \text{ rev}}\right) \\ &= 75.4 \text{ rad/s}\end{aligned}$$

The rotational kinetic energy is

$$\begin{aligned} K &= \frac{1}{2}I\omega^2 \\ &= \frac{1}{2}\left(5.49\text{x}10^{-3}\text{kgm}^2\right)\left(75.4\text{ rad/s}\right)^2 \\ &= 15.6\text{ J} \end{aligned}$$

b. The linear kinetic energy is

$$\begin{aligned} K &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}m\left(\omega R\right) \\ &= \frac{1}{2}\left(2.6\text{ kg}\right)\left(75.4\text{ rad/s x }0.065\text{ m}\right)^2 \\ &= 31.2\text{ J} \end{aligned}$$

c. The total energy is the sum of a) and b) or 46.8 J.

4) When the object starts from rest on an incline, we showed what its velocity at the bottom of the incline should be

$$\begin{aligned} v &= \sqrt{\frac{2gh_i}{1+\frac{I}{mr^2}}} \\ &= \sqrt{\frac{2gh_i}{1+\frac{\frac{2}{5}mr^2}{mr^2}}} \\ &= \sqrt{\frac{10}{7}gh_i} \end{aligned}$$

This speed is the speed that it comes off the table. This is a zero-launch angle problem where the range is

$$\begin{aligned} R &= v\sqrt{\frac{2h}{g}} \\ &= \sqrt{\frac{10}{7}gh_i}\sqrt{\frac{2h_t}{g}} \\ &= \sqrt{\frac{20}{7}h_ih_t} \\ &= \sqrt{\frac{20}{7}\left(0.35\text{ m}\right)\left(1.2\text{ m}\right)} \\ &= 1.1\text{ m} \end{aligned}$$

5) The problem is similar to what we did in class but we have added a third mass. Let's say 1 is on the left 2 is in between and 3 is on the right. We will chose the center-of-mass to be our pivot and calculate the torques of each of the weights about this pivot. Let's assume the center of mass is between 1 and 2. Then 1 will produce a positive torque and the other two will produce a negative torque.

$$\begin{aligned} \tau_1 &= m_1g\ell \\ \tau_2 &= -m_2g\left(L_1-\ell\right) \\ \tau_3 &= -m_3g\left(L_2+L_1-\ell\right) \end{aligned}$$

The sum of torques must be zero in this case

$$\begin{aligned} m_1g\ell - m_2g\left(L_1-\ell\right) - m_3g\left(L_2+L_1-\ell\right) &= 0 \\ \left(m_1+m_2+m_3\right)\ell - \left(m_2+m_3\right)L_1 - m_3L_2 &= 0 \\ \frac{\left(m_2+m_3\right)L_1+m_3L_2}{m_1+m_2+m_3} &= \ell \end{aligned}$$