Answers to Example Exam #5: Simple Harmonic Motion and Wave Mechanics

1) The motion c) is not periodic. As a car turns the corner it is not repetitive. There is no pattern of motion that is repeated.
2)

a. The period of an object in periodic motion is

$$T = \frac{2\pi}{\omega}$$

The equation of motion

$$x(t) = A\cos(\omega \mathrm{t})$$

allows us to identify the angular frequency as 2.6 s^{-1}. So the period is

$$\begin{aligned} T &= \frac{2\pi}{2.6\ \mathrm{s}^{-1}} \\ &= 2.4\ \mathrm{s} \end{aligned}$$

b. The spring constant can be found from the angular frequency of a mass on a spring

$$\begin{aligned} \omega &= \sqrt{\frac{k}{m}} \\ k &= \omega^2 m \\ &= \left(2.6\ \mathrm{s}^{-1}\right)^2 (0.460\ \mathrm{kg}) \\ &= 3.1\ \mathrm{N/m} \end{aligned}$$

c. The maximum velocity of the mass is given by the amplitude and the angular frequency of the oscillation

$$\begin{aligned} v_{max} &= A\omega \\ &= (0.082\ \mathrm{m})\left(2.6\ \mathrm{s}^{-1}\right) \\ &= 0.21\ \mathrm{m/s} \end{aligned}$$

d. The velocity of an oscillating spring is

$$\begin{aligned} v(t) &= v_{max}\sin(\omega \mathrm{t}) \\ &= (0.21\ \mathrm{m/s})\sin\left(\left(2.6\ \mathrm{s}^{-1}\right)\mathrm{t}\right) \end{aligned}$$

3) This is a conservation of energy/conservation of momentum problem in three parts. First we need to find the velocity of the first mass when it reaches the vertical. But we know this result: v = $\sqrt{2gh}$. Next we need to determine the velocity of the second mass after a collision with the first. The initial momentum of the system is the momentum of the first mass since the second mass is at rest. The final momentum is the momentum of the second mass since the first is now at rest. By conservation of momentum

$$\begin{aligned} mv_{init} &= 2mv_{final} \\ v_{final} &= \frac{1}{2}v_{init} \\ &= \frac{1}{2}\sqrt{2gh} \end{aligned}$$

Finally we calculate the height that the second mass will travel after when initially travelling at the speed we just calculated. This is done by conservation of energy where the initial potential and final kinetic energy are identically 0. So the initial kinetic energy must equal the final potential energy.

$$\begin{aligned}\frac{1}{2}(2m)\,v_{final}^2 &= (2m)\,gh_{final}\\ h_{final} &= \frac{v_{final}^2}{2g}\\ &= \frac{\frac{1}{4}(2gh)}{2g}\\ &= \frac{1}{4}h \qquad (1)\end{aligned}$$

4)

a. The intensity of sound is given as

$$\begin{aligned}I &= \frac{P}{4\pi R^2}\\ &= \frac{(1.0\ \mathrm{W}}{4\pi(3.0\ \mathrm{m})^2}\\ &= 8.8\mathrm{x}10^{-3}\ \mathrm{W/m^2}\end{aligned}$$

To find the decibels we use the definition

$$\begin{aligned}\beta &= 10\log\left(\frac{\mathrm{I}}{\mathrm{I_0}}\right)\\ &= 10\log\left(\frac{8.8\mathrm{x}10^{-3}\ \mathrm{W/m^2}}{1.0\mathrm{x}10^{-12}\ \mathrm{W/m^2}}\right)\\ &= 99dB\end{aligned}$$

b. The different distances between listeners is related to the difference in sound intensities by

$$\begin{aligned}\frac{r_2}{r_1} &= 10^{\frac{\beta_1-\beta_2}{20}}\\ r_2 &= r_1 10^{\frac{\beta_1-\beta_2}{20}}\\ &= (3.0\ \mathrm{m})10^{\frac{99\ \mathrm{dB}-69\ \mathrm{dB}}{20}}\\ &= 95\ \mathrm{m}\end{aligned}$$

c. The time difference is given by

$$\begin{aligned}t_2 - t_1 &= \frac{r_2}{v_s} - \frac{r_1}{v_s}\\ &= \frac{r_2 - r_1}{v_s}\\ &= \frac{95\ \mathrm{m} - 3.0\ \mathrm{m}}{343\ rmm/s}\\ &= 0.27\ \mathrm{s}\end{aligned}$$

5)

a. The wavelength is

$$\begin{aligned}\lambda &= \frac{v_s}{f}\\ &= \frac{343\ \mathrm{m/s}}{35.0\mathrm{x}10^3\ \mathrm{s^{-1}}}\\ &= 9.80\mathrm{x}10^{-3}\ \mathrm{m}\end{aligned}$$

b. The frequency change is the same for both bats since the problem is symmetric between the bats.

$$\begin{aligned} f' &= \frac{1 + u_o/v_s}{1 - u_s/v_s} f \\ &= \frac{1 + 3.2 \text{ m/s}/343 \text{ m/s}}{1 + 3.2 \text{ m/s}/343 \text{ m/s}} (35.0 \text{ kHz}) \\ &= 35.6 \text{ kHz} \end{aligned}$$