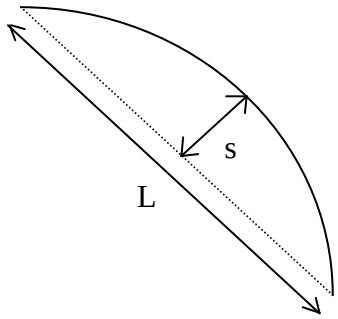
Physics G6050 – Assignment 2

Due: Wednesday October 11, 2006

---

1. Suppose a muon of momentum p travels in an arc of a circle of radius R within a region of constant magnetic field B.

(a) The sagitta (s) is defined as the maximum distance between the arc and the straight line (ie. chord) drawn through the endpoints of the muon's path. The chord, of length L, is shown as the dotted line in the figure below, in which the sagitta is also labeled.



Find an expression for the sagitta in terms of R and L in the limit where R >> L.

(b) Assume that the muon's position is measured at the beginning, middle and end points along this track, and that the transverse coordinates of the position of the muon can be measured with a resolution (standard deviation) of 100 microns at each position. To what percentage accuracy ($\Delta p/p$) can the momentum of the muon be measured? (Hint: This is most easily analyzed by calculating the relative error in the sagitta.)

(c) Calculate the errors on s and p which arise due to multiple scattering. For simplicity, assume (incorrectly) that the measurement errors for the three measurements of the muon position are uncorrelated. Note that the multiple scattering error is independent of p.

(d) As a numerical example, consider L = 1 m, B = 2 T, and that the muon is traveling through the solid iron of a magnet. Calculate the individual contributions to, as well as the total, momentum resolution, for the cases of p = 1, 10, and 100 GeV.

2. Suppose a Time-of-Flight (ToF) particle identification system has a flight length of 1.2 m and a time resolution of 150 ps. Up to what momentum could charged kaons be separated from charged pions with at least three standard deviations?

3. The total pp cross-section for 100 GeV incident protons is 40 mb.

(a) The total proton-nucleus cross-section is $A^{2/3}$ times that of pp. Why?

(b) Given this, calculate the interaction length λ for proton interactions in iron.

(c) How many interactions per second would occur if 10^7 protons/s struck an iron target of thickness 0.1 λ?

(d) The total νp cross-section for a 100 GeV neutrino is 6.8×10^{-37} cm^2. What is the interaction length in iron? What is the probability for such a neutrino to interact as it passes through the Earth? (Hint: The total neutrino-nucleus cross-section is A times that of νp. Why?)

4. Consider the predictions of the simple cascade model for describing the electromagnetic cascade initiated by an electron impinging a crystal of barium-fluoride (BaF_2) which has density $\rho = 4.89$ g/cm^2, radiation length $X_0 = 2.05$ cm and critical energy $E_{crit} = 10$ MeV.

(a) What is the average depth (in cm and in X_0) of the shower resulting from a 50 GeV electron? How does the depth vary with incident electron energy?

(b) The energy resolution is related to the fluctuations in the total number N_{tot} of electrons produced in the shower which produce scintillation light. Using the cascade model, derive a relationship between N_{tot} and the incident electron energy.

(c) Based on this result, estimate the electromagnetic energy resolution $\Delta E/E$ for a BaF_2 detector for 50 GeV electrons.

(d) The electrons produced in the shower lead to the production of scintillation photons in the detector, with about 7000 photons produced per MeV of ionization. Estimate the contribution to the energy resolution which results from the fluctuations in the number of photons detected by the PMT, assuming a quantum efficiency of 20%.

5. Read the accompanying paper about the discovery of the Ω^- baryon. Explain how it was determined experimentally that this new particle had strangeness -3.