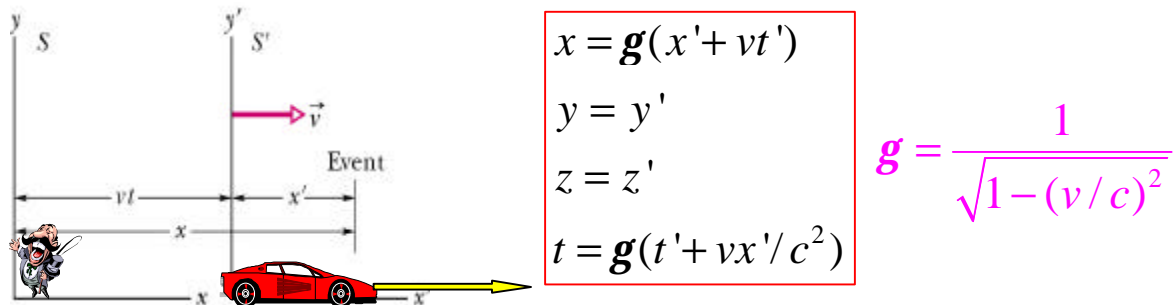


Optional Exercises Accompanying Lectures

www.nevis.columbia.edu/~sciulli/HSTlectures/notes-probs.pdf

F. Sciulli

June 2001



Left picture: Jane pilots a very fast car, at rest in the primed coordinate system, but moving with speed v (close to the speed of light) relative to Joe. . Joe is standing on the ground at the side of the road at rest in the unprimed system.

Right picture: The Lorentz transformation relating position and time between the two coordinate systems.

The first two questions are general about the transformations. The next two are specifically about Joe and Jane.

- 1) Solve the equations for x' and t' in terms of x and t . [You should find that these are the same as those above, except for the sign in front of v . Understand why?]
- 2) Show from the transformation equations directly that the sum here

$$x^2 + y^2 + z^2 - c^2t^2 = \text{invariant}$$

$$= (x')^2 + (y')^2 + (z')^2 - c^2(t')^2$$

is invariant. [Note that the combination involving momenta and energy must, therefore, also be invariant. Understand why?]

- 3) The car turns on its headlights at the instant it passes Joe. Clearly (since this is a light beam), the car sees the light travelling along the x' axis according to the equation, $x' = ct'$. Solve the algebra to find how fast Joe sees the light travelling! [You should get $x = ct$. Understand why?]

$$\begin{aligned}
 p_x &= \mathbf{g}(p'_x + vE'/c^2) \\
 p_y &= p'_y \\
 p_z &= p'_z \\
 E &= \mathbf{g}(E' + vp'_x)
 \end{aligned}$$

Transformation of momenta seen by Joe versus momenta seen by the car.

- 4) The car has mass m . So Jane sees the car to have $p'=0$ and $E'=mc^2$. Using these (primed) values in the expression below, for Jane,

$$E'^2 = p'^2 c^2 + m^2 c^4$$

Find, from the transformation equations, the momentum and energy of the car as seen by Joe. Verify that he gets

$$E^2 = p^2 c^2 + m^2 c^4$$

For the following we use the units,

$$\begin{array}{ccc}
 \boxed{E^2 = p^2 c^2 + m^2 c^4} & & \\
 \uparrow & \uparrow & \uparrow \\
 \text{eV} & \text{eV}/c & \text{eV}/c^2 \\
 \text{MeV} & \text{MeV}/c & \text{MeV}/c^2
 \end{array}$$

In other words, in these units momentum, for example, is measured in the energy units, eV, for $c=1$. So that this simplification permits the equation to be written:

$$E^2 = p^2 + m^2$$

Note that, using these units, the value of γ is

$$\mathbf{g} = \frac{E}{m}$$

- 5) The protons circulating in the Tevatron have energy $E_b=800$ GeV. Find the speed of one of these protons, whose mass energy equals roughly 1 GeV. (Better still, find the difference between the proton speed and that of light.)
- 6) A proton extracted from the Tevatron, with energy $E_b=800$ GeV, hits another proton which is stationary ($E_t = m = 1$ GeV). Using the fact that $E \gg m$ (so $E \sim p$), find the energy available for making new mass (like pions). [Hint: find the difference between the total energy squared and the total momentum squared. This is the center of momentum energy (or energy available to make mass) squared.]
- 7) Show for this general case that the magnitude of the energy available to make mass is $\sqrt{2mE_b + m^2}$. An alternative way to make new particles is to collide two high energy beams going in opposite directions. What is the mass available in this case? [Hint: For this "colliding beam" case, the total momentum of the two particle system is zero.] Compare numerically how much particle mass can be produced for the "fixed target" case versus the colliding beam case.
- 8) Go through the arguments in the lecture regarding longevity of the pions made from the target. These were the absolute necessity of

$$\Delta t = \gamma \Delta t'$$

$$L = \frac{L_0}{\gamma}$$

taking into account the time dilation and Lorentz contraction (depending on point of view). You will find the appropriate pages at www.nevis.columbia.edu/~sciulli/HSTlectures/ under the names [lec1-p36.PDF](#), [lec1-p37.PDF](#), and [lec1-p38.PDF](#).....

For a list of references used for the pictorial material in the lectures, see the file [references.PDF](#) at the same website cited on page 1.