SOLUTIONS - FINAL PHYS 1401 - SAMPLE EXAM

I. 1. After collision, ball at goal line with velocity = center of mass vel (V_c)

$$(m_1+m_2)V_c = m_1 V_1 - m_2 V_2$$

$$V_c = \frac{m_1V_1 - m_2V_2}{m_1+m_2} = \frac{(75)}{(150)}(3.75 - 4.10) = \frac{1}{2}(-0.35) = -0.175 \text{ m/s}$$

Horizontal position ball ends up is

$$x_c = x_0 + V_c t \quad \text{where} \quad \frac{1}{2}gt^2 = y \Rightarrow t = \sqrt{\frac{2y}{g}} \qquad (t = 0.5 \text{ sec})$$

and $x_0 = 0$

$$x_c = V_c\sqrt{\frac{2y}{g}} = -0.175\sqrt{\frac{2(1.2)}{9.8}} = -0.087 \text{ m}$$

or 8.7 cm to <u>left</u> of goal line

(He doesn't make the goal!)

2. $K_o = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}(75)(\overline{3.75}^2 + \overline{4.10}^2) = 1158$ Joules

$K_f = \frac{1}{2}(m_1+m_2)V_c^2 = \frac{1}{2}(150)(\overline{0.175}^2) = 2.3$ Joule

$$\frac{K_f}{K_i} = 2.3/1158 = 2 \times 10^{-3}$$

3. Total initial energy is $K_0 = 1158$ Joules (kinetic)

(potential) $U = (m_1+m_2)g y_{cm} = (150)(9.8)(1.2) = 1760$ Joules

or Total Energy of $K_0 + U = 2920$ Joules
which has been converted to heat (+ little sound...)
by the time the play is over.

II. 1. Ball rolls down ramp $\quad U = mgh$

At bottom, kinetic energy is $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

Since it <u>rolls</u> $\omega = \frac{v}{R}$. Using $I = I_{sphere} = \frac{2}{5}mR^2$

then $K = \frac{1}{2}mv^2 + \frac{1}{2}(\frac{2}{5}m\cancel{R^2})(\frac{v^2}{\cancel{R^2}}) = \frac{1}{2}(\frac{7}{5})mv^2 = \frac{7}{10}mv^2$

Mech energy conserved (no slipping)

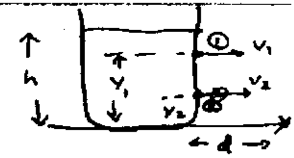
$$\cancel{m}gh = \frac{7}{10}\cancel{m}v^2 \Rightarrow v = \sqrt{\frac{10}{7}gh} = \sqrt{\frac{10}{7}(9.8)(10)} = 3.74 \text{ m/s}$$

2. On the frictionless surface, ball continues to rotate (keeps rotational energy)
So, it reaches a maximum height, h', given by

$$\cancel{m}gh' = \frac{1}{2}\cancel{m}v^2 \qquad h' = \frac{v^2}{2g} = \frac{\overline{3.74}^2}{(2)(9.8)} = 0.71 \text{ meters}$$

This is lower than the initial height because it still
has the rotational kinetic energy it had at the bottom

III. Bernoulli's principle says the exit speed
1.



$$\tfrac{1}{2}\cancel{\rho} v^2 = \cancel{\rho} g(h-y) \quad \text{so} \quad v = \sqrt{2g(h-y)}$$

$\boxed{\begin{array}{l} y_1 = 3.6\,m \\ y_2 = 0.8\,m \end{array}}$

Water subsequently travels horizontally, x

$$x = vt \quad \text{where} \quad y = \tfrac{1}{2}gt^2 \quad \text{so} \quad t = \sqrt{\frac{2y}{g}}$$

$$x = v\sqrt{\frac{2y}{g}} = \sqrt{2\cancel{g}(h-y)\frac{2y}{\cancel{g}}} = 2\sqrt{y(h-y)}$$

For the two jets to hit at the same point requires $x_1 = x_2 = d$

or $\sqrt{y_1(h-y_1)} = \sqrt{y_2(h-y_2)}$

$$h(y_1 - y_2) = y_1^2 - y_2^2 = (y_1 - y_2)(y_1 + y_2)$$

Hence, $\boxed{h = y_1 + y_2}$ is the only possible water level for this situation

$$h = 3.6\,m + 0.8\,m = 4.4 \text{ meters}$$

2. $d = 2\sqrt{y_1(h-y_1)} = 2\sqrt{3.6(0.8)} = 3.4$ meters

3. Water level drops to $h' = 4.4 - 0.5 = 3.9$ meters, then

$$\Delta x = x_2 - x_1 = 2\left[\sqrt{y_2(h'-y_2)} - \sqrt{y_1(h'-y_1)}\right]$$

$$= 2\left[\sqrt{0.8(3.1)} - \sqrt{3.6(0.3)}\right]$$

$$\Delta x = 2\left[1.57 - 1.04\right] = 1.07\ m$$

IV. Gravitational force 1. $F = G\frac{Mm}{R^2}$ On Earth $mg_E = G\frac{Mm}{R_E^2}$ so $g_E = G\frac{M_E}{R_E^2}$

On the asteroid $g_A = G\frac{M_s}{R_A^2} \Rightarrow g_A = g_E\left(\frac{M_A}{M_E}\right)\left(\frac{R_E}{R_A}\right)^2$

$$g_A = 9.8\,m/s^2\left(\frac{3.45\times10^{15}\,kg}{6.0\times10^{24}\,kg}\right)\left(\frac{6.4\times10^6\,m}{12\times10^3\,m}\right)^2 = 1.60\times10^{-3}\,m/s^2$$

2. When the tangential velocity is such that gravity can no longer supply the centripetal force, the rock flies off

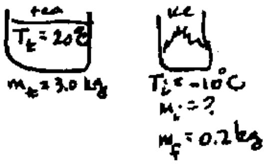
$$\cancel{m}R_A\omega^2 > \cancel{m}g_A \quad \text{or} \quad \omega > \omega_1 = \sqrt{\frac{g_A}{R}} = \sqrt{\frac{1.6\times10^{-3}\,m/s^2}{12\times10^3\,m}} = 3.65\times10^{-4}\ rad/s$$

3. When the surface speed exceeds escape velocity, rock doesn't return
This is the case for $E = U + K = -G\frac{M_A\cancel{m}}{R_A} + \frac{1}{2}\cancel{m}R_A^2\omega_2^2 \geq 0$

$$\text{or} \quad \omega_2^2\frac{R_A^2}{2} \geq G\frac{M_A}{R_A} = g_A R_A$$

$$\text{or} \quad \omega_2 \geq \sqrt{\frac{2g_A}{R_A}} = \sqrt{2}\,\omega_1 = 5.1?\times10^{-4}\ rad/s$$

V. 1. As a mixture at equilibrium, the ice and tea are at the equilibrium temperature of $T_f = 0°C$. In the process, heat flows from the tea to the ice.



$$Q_{tea} = m_t c_w (T_t - T_f)$$

$$Q_{ice} = m_i c_i (T_f - T_i) + \underbrace{L_f (m_i - m_f)}_{\text{heat of fusion for ice that melts}}$$

So for $T_f = 0°C$:

$$m_t c_w T_t = -m_i c_i T_i + L_f (m_i - m_f)$$

$$m_i (L_f - c_i T_i) = m_t c_w T_t + m_f L_f$$

$$m_i = \frac{m_t c_w T_t + m_f L_f}{L_f - c_i T_i} = \frac{(3.0\,\text{kg})(4190\,\frac{J}{kg\,C})(20\,C) + (0.2\,\text{kg})(334 \times 10^3)}{(334 \times 10^3\,\frac{J}{kg}) - (2220\,\frac{J}{kg\,C})(-10\,C)}$$

$$m_i = \frac{2.51 \times 10^5 + 0.67 \times 10^5}{3.56 \times 10^5} = 0.89 \text{ kg of ice to start}$$

→ The mixture ends up at 0°C

2. The rate of heat gain is given by

$$H = \frac{dQ}{dt} = \frac{k}{L} A \Delta T$$

where k = thermal conductivity = 0.025 W/m·°C
L = thickness of container = 1.0 cm = .01 m
A = surface area = 2500 cm^2 × $(10^{-2}\,\text{m/cm})^2$ = 0.25 m^2
ΔT = 20° − 0° = 20 C

$$H = \frac{(0.025)}{.01}(0.25)(20) = 12.5 \text{ Watts}$$

3. This heat goes into melting ice (until it is all gone)

$$Q = mL$$

$$H = \frac{dQ}{dt} = L\frac{dm}{dt} \qquad \frac{dm}{dt} = \frac{1}{L}H = \frac{12.5\text{ Watts}}{334 \times 10^3\text{ J/kg}} = 3.74 \times 10^{-5}\text{ kg/sec}$$

Time it takes for all ice to melt is given by

$$H = L\frac{m_f}{t} \quad \text{so} \quad t = L\frac{m_f}{H} = \frac{m_f}{\frac{dm}{dt}} = \frac{0.2\text{ kg}}{3.74 \times 10^{-5}\text{ kg/s}}$$

$$t = 5344 \text{ sec} \approx 1.5 \text{ hours}$$

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VI. 1. $\Delta E_{int} = Q - W$

During isochoric processes, no work is done.

$$Q = \Delta E_{int} = nC_v(T_H - T_L)$$

2. During isothermal expansion ($\Delta E_{int} = 0$)

$$Q_H = W_H = \int p\,dV = nRT_H \int_{V_a}^{V_b} \frac{dV}{V} = nRT_H \ln \frac{V_b}{V_a}$$

$$Q_H = nRT_H \ln\left(\frac{V_b}{V_a}\right)$$

$$Q_L = nRT_L \ln\left(\frac{V_a}{V_b}\right)$$

3. Over entire cycle, $\Delta E_{int} = 0$ $\quad W = Q_H - Q_L$

$$W = nR(T_H - T_L)\ln\left(\frac{V_b}{V_a}\right)$$

4. Entropy change during $d \to a$ part of the cycle is

$$\Delta S_{da} = \int_d^a \frac{dQ}{T} = nC_v \int_d^a \frac{dT}{T} = nC_v \ln\left(\frac{T_H}{T_L}\right)$$