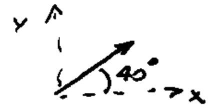
Solutions – Midterm I sample



I. 1. $\vec{v}_0 = 10\,\text{m/s}\cos 40°\,\hat{\imath} + 10\,\text{m/s}\,\sin 40°\,\hat{\jmath}$

$\Rightarrow$ $v_{x0} = 7.66$ m/s $\quad v_{y0} = 6.42$ m/s

2. Subsequent motion (taking origin where rock starts)

$v_x = v_{x0} \qquad v_y = v_{y0} - gt$

$x = v_{x0} t \qquad y = v_{y0} t - \frac{1}{2} g t^2$

Maximum height where $v_y = 0$ or $t_m = \frac{v_{y0}}{g} = \frac{6.42}{9.8} = 0.66$ s

Height there $y_m = v_{y0} t_m - \frac{1}{2} g t_m^2 = v_{y0}\frac{v_{y0}}{g} - \frac{1}{2} g \left(\frac{v_{y0}}{g}\right)^2$

$\Rightarrow$ $y_m = \frac{1}{2}\frac{v_{y0}^2}{g} = \frac{1}{2}\frac{(6.42)^2}{9.8} = 2.10$ m

3. Time for rock to hit the ground (t_g) occurs when $y = y_g = -100$ m

$y_g = v_{y0} t_g - \frac{1}{2} g t_g^2$

$-100\,\text{m} = 6.42\, t_g - \frac{1}{2}(9.8)\, t_g^2$

$4.9\, t_g^2 - 6.42\, t_g - 100 = 0$

$t_g = \frac{6.42 \pm \sqrt{(6.42)^2 + 4(100)(4.9)}}{2(4.9)}$

Only +ve root meaningful

$t_g = 5.22$ sec. $\Leftarrow$

4. $v_x = v_{x0} = 7.66$ m/s $\Leftarrow$

$v_y = v_{y0} - g t_g = 6.42 - 9.8(5.22)$
$= -44.7$ $\Leftarrow$

5. The arm throwing the rock started with $v = 0$ and ended with $\vec{v}_0$ over $\Delta \ell = 1.2$ meter

Hence $\vec{v}_{av} = \frac{\vec{v}_0 + 0}{2} = \frac{7.66}{2}\hat{\imath} + \frac{6.42}{2}\hat{\jmath} = 3.83\hat{\imath} + 3.21\hat{\jmath}$ m/s

$|\vec{v}_{av}| = \frac{1}{2}(10\,\text{m/s}) = 5$ m/s

So it took a time

$\Delta t = \frac{\Delta \ell}{v_{av}} = \frac{1.2\,\text{m}}{5\,\text{m/s}} = 0.24$ sec.

Average acceleration was $\vec{a}_{av} = \frac{\vec{v}_0 - 0}{\Delta t} = \frac{7.66\hat{\imath} + 6.42\hat{\jmath}}{0.24}$ m/s²

This has magnitude $|\vec{a}_{av}| = \frac{v_0}{\Delta t} = \frac{10\,\text{m/s}}{.24\,\text{s}} = 41.7$ m/s² $\Leftarrow$

and direction same as $\vec{v}_0 \Rightarrow 40°$ to horiz.

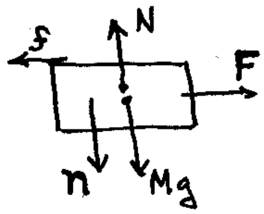
Force in same direction, with magnitude $F = ma = (0.5)(41.7) = 20.8$ N $\Leftarrow$

II.

1. For m and M stuck together, they act as single mass of mass (m+M)

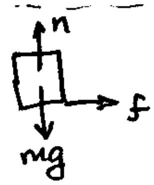
Hence $a = \frac{F}{m+M} = \frac{50\ N}{13\ kg} = 3.85\ m/s^2$

2. on M



F = external force (given)
N = normal force pushing up by ground
Mg = weight of M
f = friction force of m on M
n = normal force of m on M

on m



n = normal force of M on m
f = friction force of M on m
mg = weight of m

3. $\left\{\begin{matrix} n = mg \\ N = mg + n \end{matrix}\right\}$ If both accelerate with a, then $\left\{ f = ma \qquad F - f = Ma \right\}$ give acc. as in part (1)

If f small enough, this will occur. They only slide if $f \geq \mu_s n = \mu_s mg$

This corresponds to $\mu_s \leq \frac{ma}{mg} = \frac{a}{g} = \frac{3.85}{9.8} = .392$

Hence, the two masses will travel together if $\mu_s \geq .392$

4. If the two masses slide (with friction), Newton's second law requires

$$f = ma_m \qquad F - f = Ma_M$$

Given fact $a_m = \frac{1}{2}a_M$, and using $f = \mu_k n = \mu_k mg$ gives

$$\mu_k \cancel{m} g = \cancel{m}\left(\tfrac{1}{2}a_M\right) \qquad F - \mu_k mg = Ma_M$$

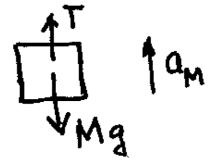
Substituting $F = \mu_k mg + 2M\mu_k g$

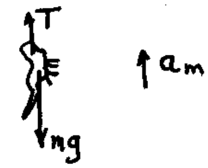
$$\mu_k (m+2M) g = F \Rightarrow \mu_k = \frac{F}{(m+2M)g} = \frac{50}{(23)(9.8)} = 0.222$$

and $a_M = \frac{2F}{(m+2M)} = \frac{2(50)}{(23)}$

or $a_M = 4.35\ m/s^2$

III. Forces on package (M) and monkey (m)





1. Writing F=ma for package

$$T - Mg = Ma_M$$

$$\Rightarrow \quad T = M(g + a_M) = 10(9.8 + 1.0) = 108\ N \Leftarrow$$

2. For monkey, net force is

$$\Rightarrow \quad F_m = T - mg = 108 - (8)(9.8) = 29.6\ N$$

Using Newton's second law: $F_m = ma_m$

$$\Rightarrow \quad a_m = \frac{F_m}{m} = \frac{29.6\ N}{8\ kg} = 3.70\ m/s^2$$

3. Monkey moves up, in $t = 0.5s$, a distance

$$\Rightarrow \quad y_m = \tfrac{1}{2}a_m t^2 = \tfrac{1}{2}(3.70)(0.5)^2 = 0.46 \text{ meters}$$

4. When, after 0.5s, the monkey holds on to the rope (stops climbing) he moves up with the rope. Since $M > m$, the system is such that M accelerates down and m up with the <u>same</u> acceleration.

The monkey hence continues up and moves a subsequent distance just equal to the distance the <u>package</u> had moved up in part (3). That distance is

$$Y_M = \tfrac{1}{2}a_M t^2$$

$$Y_M = \tfrac{1}{2}(1.0\ m/s^2)(0.5\ sec)^2 = 0.125 \text{ meters}$$

Hence, total distance monkey has moved up when the package hits the ground is

$$\Rightarrow \quad Y_{total} = |Y_m| + |Y_M| = 0.46 + 0.125 = 0.585 \text{ meters}$$