I. \( v_0 = 10 \text{ m/s} \cos 40^\circ \hat{i} + 10 \text{ m/s} \sin 40^\circ \hat{j} \)

\[ v_x = 7.66 \text{ m/s} \quad v_y = 6.42 \text{ m/s} \]

2. Subsequent motion (taking origin where rock starts)

\[ v_x = v_{x0} \quad v_y = v_{y0} - gt \]

\[ x = v_{x0} t \quad y = v_{y0} t - \frac{1}{2} gt^2 \]

Maximum height where \( v_y = 0 \) or \( t_m = \frac{v_{y0}}{g} = \frac{6.42}{9.8} = 0.66 \text{ s} \)

Height there \( y_m = v_{yo} t_m - \frac{1}{2} g t_m^2 = v_{yo} \frac{v_{y0}}{g} - \frac{1}{2} \left( \frac{v_{y0}}{g} \right)^2 \)

\[ y_m = \frac{1}{2} \frac{v_{y0}^2}{g} = \frac{1}{2} \left( \frac{6.42}{9.8} \right)^2 = 2.10 \text{ m} \]

3. Time for rock to hit the ground \((t_g)\) occurs when \( y = y_g = -100 \text{ m} \)

\[ y_g = v_{y0} t_g - \frac{1}{2} gt_g^2 \]

\(-100 = 6.42 t_g - \frac{1}{2} (9.8) t_g^2 \)

\[ 4.9 t_g^2 - 6.42 t_g - 100 = 0 \]

\[ t_g = \frac{6.42 \pm \sqrt{(6.42)^2 + 4(100)(4.9)}}{2(4.9)} \]

Only the root meaningful

\[ t_g = 5.22 \text{ sec} \]

4. \( v_x = v_{x0} = 7.66 \text{ m/s} \)

\[ v_y = v_{y0} - gt_g = 6.42 - 9.8(5.22) \]

\[ = -44.7 \text{ m/s} \]

5. The arm throwing the rock started with \( v = 0 \)

and ended with \( \vec{v}_0 \)

over \( \Delta l = 1.2 \text{ meter} \)

Hence \( \vec{v}_{av} = \frac{\vec{v}_0 + 0}{2} = \frac{7.66 \hat{i} + 6.42 \hat{j}}{2} = 3.83 \hat{i} + 3.21 \hat{j} \text{ m/s} \)

\[ |v_{av}| = \frac{1}{2} (10 \text{ m/s}) = 5 \text{ m/s} \]

So it took a time

\[ \Delta t = \frac{\Delta l}{v_{av}} = \frac{1.2}{5} = 0.24 \text{ sec} \]

Average acceleration was \( \vec{a}_{av} = \frac{\vec{v}_0 - 0}{\Delta t} = \frac{7.66 \hat{i} + 6.42 \hat{j}}{0.24} \text{ m/s}^2 \)

This has magnitude \( |\vec{a}_{av}| = \frac{v_0}{\frac{1}{2} \sqrt{1.24}} = 10 \text{ m/s} \)

and direction same as \( \vec{v}_0 \rightarrow 40^\circ \) to horiz.

Force in same direction, with magnitude \( F = ma = 0.5(41.7) = 20.8 \text{ N} \)
II. 1. For \( m \) and \( M \) stuck together, they act as single mass of mass \((m+M)\)

\[
\text{Hence } a = \frac{F}{m+M} = \frac{50 \text{ N}}{13 \text{ kg}} = 3.85 \text{ m/s}^2
\]

2. \[
\begin{array}{c}
\text{F = external force (given)} \\
\text{N = normal force pushing up by ground} \\
\text{Mg = weight of } M \\
f = friction force of } m \text{ on } M \\
n = normal force of } m \text{ on } M
\end{array}
\]

\[
\begin{array}{c}
n = normal force of } M \text{ on } m \\
f = friction force of } M \text{ on } m \\
mg = weight of } m
\end{array}
\]

3. \[
\begin{align*}
\begin{cases}
\{ n = mg \\
(N = mg + n)
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\{ f = ma \\
F - f = Ma
\end{align*}
\]

If both accelerate with \( a \), then

\[
\text{If } f \text{ small enough, this will occur. They only slide if } f \geq \mu_s n = \mu_s mg
\]

This corresponds to \( \mu_s \leq \frac{ma}{mg} = \frac{a}{g} = \frac{3.85}{9.8} = 0.392 \)

Hence, the two masses will travel together if

\[
\mu_s \geq 0.392
\]

4. If the two masses slide (with friction), Newton's second law requires

\[
f = ma_m \\
F - f = Ma_m
\]

Given that \( a_m = \frac{1}{2} a_m \), and using \( f = \mu_k n = \mu_k mg \)

\[
\begin{align*}
\mu_k mg = \mu_k (\frac{1}{2} a_m) \\
F - \mu_k mg = Ma_m
\end{align*}
\]

Substituting

\[
\begin{align*}
\mu_k (m+2M)g = F \\
\mu_k = \frac{F}{(m+2M)g} = \frac{50}{(23)(9.8)} = 0.222
\end{align*}
\]

and

\[
a_m = \frac{2F}{(m+2M)} = \frac{2(50)}{(23)} \\
\text{or } a_m = 4.35 \text{ m/s}^2
\]
III. Forces on package (M) and monkey (m)

1. Writing $F=ma$ for package
   \[ T - Mg = Ma_M \]
   \[ \Rightarrow T = M(g + a_M) = 10(9.8 + 1.0) = 108 \text{ N} \]

2. For monkey, net force is
   \[ F_m = T - mg = 108 - (8)(9.8) = 29.6 \text{ N} \]
   Using Newton's second law: $F_m = ma_m$
   \[ a_m = \frac{F_m}{m} = \frac{29.6 \text{ N}}{8 \text{ kg}} = 3.70 \text{ m/s}^2 \]

3. Monkey moves up, in $t=0.5\text{s}$, a distance
   \[ \gamma_m = \frac{1}{2} a_m t^2 = \frac{1}{2} (3.70)(0.5)^2 = 0.46 \text{ meters} \]

4. When, after $0.5\text{s}$, the monkey holds on to the rope (stops climbing),
   he moves up with the rope. Since $M>m$, the system is such that $M$ accelerates down and $m$ up with the same acceleration.
   The monkey hence continues up and moves a subsequent distance just equal to the distance the package had moved up in part (3). That distance is
   \[ \gamma_M = \frac{1}{2} a_M t^2 \]
   \[ \gamma_M = \frac{1}{2} (1.0 \text{ m/s}^2)(0.5 \text{ sec})^2 = 0.125 \text{ meters} \]
   Hence, total distance monkey has moved up when the package hits the ground is
   \[ \gamma_{total} = |\gamma_m| + |\gamma_M| = 0.46 + 0.125 = 0.585 \text{ meters} \]