Sample 2003

Midterm 2 Solutions

Physics 1401(1)

P1

I. 1. No friction ⇒ conservation of energy

- $mgH = \frac{1}{2} m v_B^2$

$v_B = \sqrt{2gH} = \sqrt{2(9.8)(100)}$

$v_B = 44.3$ m/s ideal case

2. For friction $W = \bar{f}_k L_{AB}$ where L = total distance travelled $v_B = 35$ m/s

$W = mgH_A - \frac{1}{2} m v_B^2 = (75)\left[(9.8)(100) - \frac{1}{2}(35)^2\right]$

$W = 27560$ Joules

$\bar{f}_k = \frac{W}{L_{AB}} = \frac{27560\ J}{500\ m} = 55.1\ N$

3. $W_{BC} = \bar{f}_k L_{BC} = \frac{1}{2} m v_B^2 \Rightarrow$ $L_{BC} = \frac{m v_B^2}{2 \bar{f}_k} = \frac{(75)(35)^2}{2(55.1)} = 834$ meters

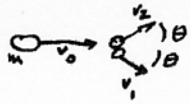
4. Conservation of linear momentum

$m v_B = (2m) v_B'$

$v_B' = \frac{1}{2} v_A = \frac{1}{2}(35) = 17.5$ m/s, and $L_{BC}' \propto (v_B')^2$

so $L_{BC}' = \frac{1}{4}$(of above L_{BC}) = 208 meters

II. 1. $K_i = \frac{1}{2} m v_0^2 = \frac{1}{2}(1000)(100)^2 = \frac{1}{2} \cdot 10^7 = 5 \times 10^6$ joules

2. 

$m_1 = m_2 = \frac{m}{2}$

Cons. of momentum

$m v_0 = m_1 v_1 \cos\theta + m_2 v_2 \cos\theta$ [along x]

$\cancel{m} v_0 = \frac{\cancel{m}}{2} \cos\theta\,(v_1 + v_2)$

[Along y:]

$v_1 \sin\theta = v_2 \sin\theta \quad \therefore \quad v_1 = v_2 \equiv v$

$v = v_0/\cos\theta \Rightarrow v_1 = v_2 = v_0/\cos\theta$

$= 100 \frac{m}{s} / \cos 60^\circ$

$v_1 = v_2 = 200$ m/s ←

3. Energy made in explosion $\equiv Q = K_f - K_i$

$K_f = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2}\frac{m}{2}(2v^2)$

$K_f = \frac{m v_0^2}{2\cos^2\theta}$

$Q/K_i = \frac{K_f - K_i}{K_i} = \frac{\cancel{\frac{1}{2} m v_0^2}\left(\frac{1}{\cos^2\theta} - 1\right)}{\cancel{\frac{1}{2} m v_0^2}} = \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$

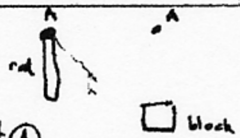
$\frac{Q}{K_i} = \tan^2\theta$ ←

4. For $\theta = 60^\circ$ $Q/K_i = (1.73)^2 = 3$

$Q = 3(5 \times 10^6$ joules$)$

$Q = 1.5 \times 10^7$ joules ←

III.



1. About Ⓐ

$I_{rod} = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2$

$= \left(\frac{1}{12} + \frac{1}{4}\right)ML^2$

$I_{rod} = \frac{1}{3}ML^2$

$I_{block} = ML^2$

$I_{bullet} = mL^2$

$I = I_{rod} + I_{block} + I_{bullet}$

$= \frac{1}{3}ML^2 + ML^2 + mL^2$

$I = \left(\frac{4}{3}M + m\right)L^2$

2. $\ell = pL = mvL$

3. Conservation of angular momentum

$mvL = I\omega \qquad \omega = \frac{mvL}{\left(\frac{4}{3}M+m\right)L^2} = \frac{mv}{\left(\frac{4}{3}M+m\right)L}$

4. $v = L\omega = \frac{mv}{\frac{4}{3}M+m}$

5. $K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{4}{3}M+m\right)\cancel{L^2}\frac{m^2v^2}{\left(\frac{4}{3}M+m\right)^{\cancel{2}}\cancel{L^2}} = \frac{1}{2}\frac{m^2}{\left(\frac{4}{3}M+m\right)}v^2$

This rotational energy must equal increase of potential energy when apparatus is a top of swing. Take h = height of block there.

$U = \overset{(M+m)}{\cancel{M}}gh + Mg\frac{h}{2} = \left(m + \frac{3}{2}M\right)gh$

At top of swing, block and bullet have risen by h, and rod c.m. by a distance $h/2$.

For $U = K$

$\left(m + \frac{3}{2}M\right)gh = \frac{1}{2}\frac{m^2}{\left(\frac{4}{3}M+m\right)}v^2$

$h = \frac{1}{2}\frac{m^2v^2}{\left(m+\frac{3}{2}M\right)\left(\frac{4}{3}M+m\right)g}$