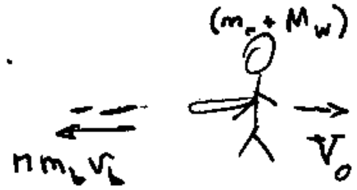
F. Sciulli

I.



firing of rifle

1. Momentum is conserved.
With n = # bullets $= 10$
v_b = velocity of bullet $= 200\ \text{m/s}$
V_0 = velocity of warrior = ?

$$n m_b v_b = (m_r + M_W) V_0$$

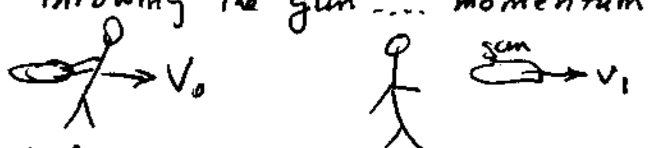
$$V_0 = \frac{n m_b}{m_r + M_W} v_b = \frac{(10)(.05)}{5 + 85}(200\ \text{m/s})$$

$$V_0 = 1.11\ \text{m/s}$$

2. Since $L = V_0 t$, the time for him to stop is

$$t = \frac{L}{V_0} = \frac{10\ \text{m}}{1.11\ \text{m/s}} = 9.0 \text{ seconds}$$

3. Throwing the gun .... momentum conserved again ?



$$(m_r + M_W) V_0 = 0 + m_r V_1$$

Speed of gun necessary for him to stop

$$V_1 = \frac{m_r + M_W}{m_r} V_0 = \frac{90}{5}(1.11)$$

$$V_1 = 20\ \text{m/s}$$

(Clearly if he throws it faster, he will move to left and also be saved.)

The direction of the gun must, of course, be to the right — over the precipice.

4. Let K_{after} = kinetic energy of gun after thrown
K_{before} = kinetic energy of man before gun thrown

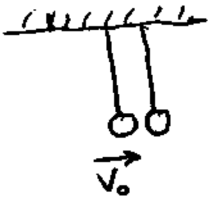
$$R = \frac{K_{after}}{K_{before}} \text{ is the desired ratio}$$

$$R = \frac{\frac{1}{2} m_r V_1^2}{\frac{1}{2}(m_r + M_W) V_0^2} = \frac{\cancel{m_r}}{m_r + M_W}\left(\frac{m_r + M_W}{m_r}\right)^{\cancel{2}}$$

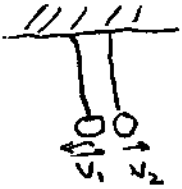
$$R = \frac{m_r + M_W}{m_r} = \frac{90}{5} = 18$$

The extra energy comes from the man's effort (ultimately chemical energy stored in his body)

II.



Just before collision



Just after collision

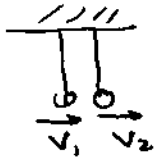
Energy conservation is the relevant principle that applies for parts 1 & 2.

1. $m_1 g h_o = \frac{1}{2} m_1 v_o^2 \qquad v_o = \sqrt{2gh_o} = \sqrt{2(9.8)(1.0)} = 4.43 \text{ m/s}$

2. Similarly, $v_1 = \sqrt{2gh_1} = \sqrt{2(9.8)(0.5)} = 3.13 \text{ m/s}$

$$v_2 = \sqrt{2gh_2} = \sqrt{2(9.8)(2.0)} = 6.26 \text{ m/s}$$

3. For <u>both</u> masses moving to the right

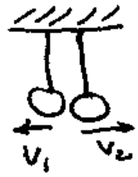


Applying momentum conservation

$$m_1 v_o = m_1 v_1 + m_2 v_2$$

$$m_2 = \frac{v_o - v_1}{v_2} m_1 = \frac{4.43 - 3.13}{6.26}(1.0 \text{ kg}) = 0.207 \text{ kg}$$

4. If m_1 instead moves to the left



Momentum conservation still applies

$$m_1 v_o = -m_1 v_1 + m_2 v_2$$

$$m_2 = \frac{v_o + v_1}{v_2} m_1 = \frac{4.43 + 3.13}{6.26}(1.0 \text{ kg}) = 1.21 \text{ kg}$$

5. The energy before collision is

$$U_o = m_1 g h_o = (1)(g)(1) = g \text{ joules}$$

and after in (4)

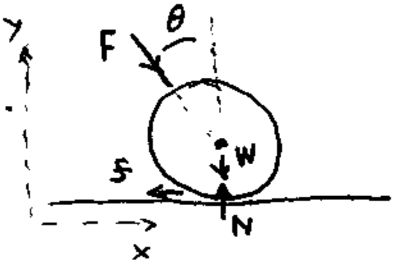
$$U_f = m_1 g h_1 + m_2 g h_2 = g\left[(1)(0.5) + (1.21)(2.0)\right]$$

$$U_f = 2.92\, g$$

$U_f > U_o$ so the collision was exothermic and <u>added</u> energy

[Note that this is not the case in figure (b), where
$U_f = g\left[(1)(0.5) + (0.207)(2.0)\right]$
$U_f = 0.91\, g < U_o$]

III.



To find acceleration, we must first find the net force.

The drawing at the left shows the most general case of forces on the ball

1/ For perfectly smooth contact at the floor, $f = 0$.
Clearly, (a) ~~the~~ net $\Sigma F_x \neq 0$ and the ball translates to the right
(b) there are no torques about the center of mass, so the sphere does not rotate

$$\Sigma F_x = F \sin\theta = M a_1 = \frac{W}{g} a_1 \qquad \therefore\ a_1 = g \frac{F}{W} \sin\theta$$

Using the givens $a_1 = (9.8)(2) \sin 60^\circ$
$a_1 = 17.0\ m/s^2$

2 In this case $f \neq 0$

ⓐ $\Sigma F_x = F \sin\theta - f = \frac{W}{g} a_2$

ⓑ $\Sigma F_y = N - W - F\cos\theta = 0$

ⓒ $\underset{c.m.}{\Sigma \tau} = I\alpha = fR$
$\Downarrow$
ⓓ $f = \frac{1}{R} I \frac{a_2}{R} = \frac{I}{R^2} a_2$

Substituting $F \sin\theta = a_2\left(\frac{I}{R^2} + \frac{W}{g}\right) = \frac{a_2}{g} W \left(1 + \frac{I}{MR^2}\right)$

Note
sphere $I = \frac{2}{5} MR^2$

$$a_2 = g \frac{F}{W} \frac{\sin\theta}{1 + I/MR^2} = (9.8)(2) \frac{\sin 60^\circ}{1 + 2/5} = 12.1\ m/s^2$$

3/ Putting into ⓓ $f = \frac{I}{MR^2} \frac{W}{g} a_2 \qquad f = \frac{I}{MR^2} \cancel{\frac{W}{g}} \cancel{g} \frac{F}{\cancel{W}} \frac{\sin\theta}{1 + I/MR^2} = \frac{F \sin\theta}{\frac{MR^2}{I} + 1}$

From ⓑ $N = W + F\cos\theta = F\left(\frac{W}{F} + \cos\theta\right)$

$$\mu_c = \frac{f}{N} = \frac{\frac{\cancel{F} \sin\theta}{MR^2/I + 1}}{\cancel{F}(W/F + \cos\theta)} = \frac{\sin\theta}{\left(1 + \frac{MR^2}{I}\right)\left(\cos\theta + \frac{W}{F}\right)} = \frac{\sin 60^\circ}{\left(1 + \frac{5}{2}\right)\left(\cos 60^\circ + \frac{1}{2}\right)} = 0.247$$

For $\mu_s < \mu_c = 0.247$, the ball will slip!