PHYSICS 1401 (1) homework solutions

20-26 Determine the average value of the translational kinetic energy of the molecules of an ideal gas at

(a) 0.00°C and
(b) 100°C.

What is the translational kinetic energy per mole of an ideal gas at

(c) 0.00°C and
(d) 100°C?

20-26 Average kinetic energy of translation of a molecule

$$K_{avg} = \frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} kT$$

(a) $K_{avg} = \frac{3}{2}(1.38 \times 10^{-23}\ J/K)(273\ K) = 5.65 \times 10^{-21}$ Joules

(b) At 100°C

$K_{avg} = \frac{3}{2}(1.38 \times 10^{-23})(373\ K) = 7.72 \times 10^{-21}$ Joules

(c) A mole of the gas has $K_{mole} = N_A K_{avg}$ of energy

$K_{mole} = (6.02 \times 10^{23})(5.65 \times 10^{-21}) = 3.40 \times 10^{3}$ Joules

(d) $K_{mole} = (6.02 \times 10^{23})(7.72 \times 10^{-21}) = 4.65 \times 10^{3}$ Joules

20-38

(a) Ten particles are moving with the following speeds: four at 200 m/s, two at 500m/s, and four at 600 m/s. Calculate their average and root-mean-square speeds. Is $v_{rms} > v_{avg}$?

(b) Make up your own speed distribution for the 10 particles and show that v_{rms} ? v_{avg} for your distribution.

(c) Under what condition (if any) does $v_{rms} = v_{avg}$?

20-38

(a) The average and rms speeds are as follows:

$$v_{avg} = \frac{1}{N}\sum_{i=1}^{N} v_i = \frac{1}{10}[4(200\,\text{m/s}) + 2(500\,\text{m/s}) + 4(600\,\text{m/s})] = 420\,\text{m/s},$$

$$v_{rms} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} v_i^2} = \sqrt{\frac{1}{10}[4(200\,\text{m/s})^2 + 2(500\,\text{m/s})^2 + 4(600\,\text{m/s})^2]} = 458\,\text{m/s}.$$

From these results, we see that $v_{rms} > v_{avg}$.

(b) One may check the validity of the inequality $v_{rms} \geq v_{avg}$ for any speed distribution. For example, we consider a set of ten particles divided into two groups of five particles each, with the first group of particles moving at speed v_1 and the second group at v_2 where both v_1 and v_2 are positive-valued (by the definition of speed). In this case, $v_{avg} = (v_1 + v_2)/2$ and

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2}{2}}.$$

To show this must be greater than (or equal to) v_{avg} we examine the difference in the squares of the quantities:

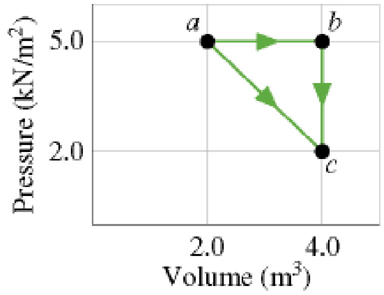
$$\begin{aligned} v_{rms}^2 - v_{avg}^2 &= \frac{v_1^2 + v_2^2}{2} - \frac{1}{4}\left(v_1^2 + v_2^2 + 2v_1v_2\right) \\ &= \frac{v_1^2 + v_2^2 - 2v_1v_2}{4} \\ &= \frac{1}{4}(v_1 - v_2)^2 \quad \geq 0 \end{aligned}$$

which demonstrates that $v_{rms} \geq v_{avg}$ in this situation.

(c) As one can infer from our manipulation in the previous part, we will obtain $v_{rms} = v_{avg}$ if all speeds are the same (if $v_1 = v_2$ in the previous part).

20-48 One mole of an ideal diatomic gas goes from a to c along the diagonal path in Fig. 20-23 . During the transition,

(a) what is the change in internal energy of the gas, and
(b) how much energy is added to the gas as heat?
(c) How much heat is required if the gas goes from a to c along the indirect path abc?



20-48

(a) $E_{int} = \frac{5}{2} nRT$ for diatomic gas

$E_{int} = \frac{5}{2} pV$

$\Delta E_{a\to c} = \frac{5}{2}\left[p_c V_c - p_a V_a\right] = \frac{5}{2}\left[(2.0)(4.0) - (5.0)(2.0)\right] \times 10^3$

$\Delta E_{a\to c} = -5000$ Joules (ie. decreases)

(b) Work is done by the gas as it expands

$W_{ac} = \int_a^c p\,dV$ Note that $p \propto V$ $a \to c$

so $W_{ac} = p_{av}\,\Delta V = \left(\frac{p_a + p_c}{2}\right)(V_c - V_a)$

$W_{ac} = \frac{1}{2}\left[(2.0 + 5.0)\times 10^3\right]\left[4.0 - 2.0\right]$

$W_{ac} = 7000$ Joules

Hence, since $\Delta E_{a\to c} = Q_{ac} - W_{ac}$

$\therefore Q_{ac} = \Delta E_{a\to c} + W_{ac}$

$Q_{ac} = -5000 + 7000 = +2000$ Joules

heat added to gas $a \to c$ direct

(c) $\Delta E_{a\to c} = -5000$ Joule as in part (a) since the change in E_{int} is independent of path

$W_{a\to b\to c} = W_{ab} + W_{bc}$

$= p_a(V_b - V_a) + 0 = (5.0\times 10^3)(4.0 - 2.0) = 10000$ Joule

$Q_{a\to b\to c} = \Delta E + W_{a\to b\to c}$

$Q = -5000 + 10000 = +5000$ Joules

21-7

(a) What is the entropy change of a 12.0 g ice cube that melts completely in a bucket of water whose temperature is just above the freezing point of water?

(b) What is the entropy change of a 5.00 g spoonful of water that evaporates completely on a hot plate whose temperature is slightly above the boiling point of water?

21-7

(a) Since $\Delta S = \frac{Q}{T}$ for a reversible process

Here $Q = Lm$ where L = heat of fusion

So $$\Delta S = \frac{(333\ \mathrm{J/g})(12.0\ \mathrm{g})}{273\ \mathrm{K}} = 14.6\ \mathrm{J/K}$$

(b) Same reasoning as in (a), but here $L = 2256\ \mathrm{J/g}$ and $T = 373\ \mathrm{K}$ with $m = 5.0\ \mathrm{g}$

$$\Delta S = \frac{(2256\ \mathrm{J/g})(5.0\ \mathrm{g})}{373\ \mathrm{K}} = 30.2\ \mathrm{J/K}$$

21-16 An 8.0 g ice cube at -10°C is put into a Thermos flask containing 100 cm^3 of water at 20°C. By how much has the entropy of the cube–water system changed when a final equilibrium state is reached? The specific heat of ice is 2220 J/kg·K.

21-16

Thermos flask = well insulated = adiabatic (no heat enters or leaves)

What happens? .... All come to same temperature!

1. Heat enters water from
   a. ice warming to 0°C
   b. ice melts to water
   c. ice water warms to final temperature (T_f)
2. Water cools to final temperature (T_f)

$c_w = 4190$ J/kg·K
$c_i = 2220$ J/kg·K
$L_f = 333000$ J/kg
$m_i = 8$ g
$m_w = 100$ g

$$Q_{1a} + Q_{1b} + Q_{1c} + Q_2 = 0$$

$$c_i m_i\left[0-(-10)\right] + L_F m_i + c_w m_i (T_f - 0) + c_w m_w (T_f - 20) = 0$$

Note: Since $T_f < 20$ C, this term is negative

$$T_f\left[c_w m_w + c_w m_i\right] = c_w m_w (20) - L_F m_i - c_i m_i (10)$$

$$T_F = \frac{m_w c_w (20) - m_i (L_f + 10 c_i)}{m_w c_w + m_i c_w}$$

$$T_F = \frac{(100)(4190)(20) - (8)\left[333000 + (10)(2220)\right]}{(100)(4190) + (8)(4190)}$$

$$T_F = 12.24°C$$

Each of these heat flows implies entropy change.

$\begin{cases} \text{temp change} & \Delta S_{12} = mc\ln\left(\frac{T_2}{T_1}\right) \\ \text{phase change} & \Delta S = \frac{L_F m}{T} \end{cases}$

$$\Delta S = \Delta S_{1a} + \Delta S_{1b} + \Delta S_{1c} + \Delta S_2 \quad (\text{using Kelvin} = \text{Cent} + 273.15)$$

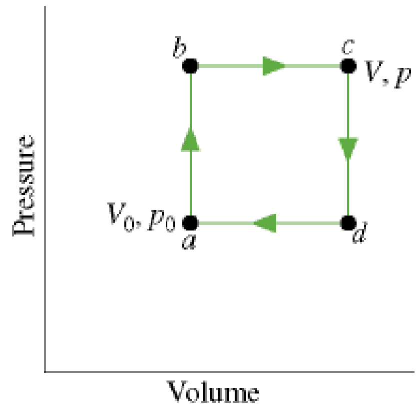
$$\Delta S = m_i c_i \ln\frac{273.15}{263.15} + \frac{L_F m_i}{273.15} + m_i c_w \ln\frac{285.39}{273.15} + m_w c_w \ln\frac{285.39}{293.15}$$

$$\Delta S = (.008)(2220)(.0373) + \frac{333000(.008)}{273.15} + (.008)(4190)(.0438) + (.100)(4190)(-.0276)$$

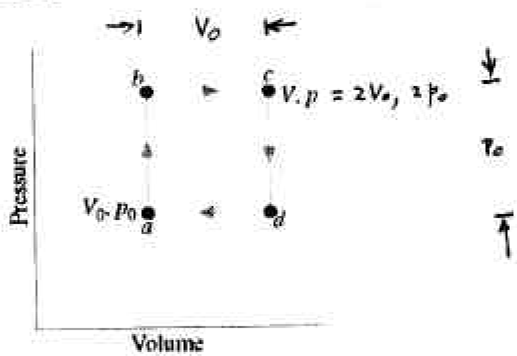
$$\Delta S = 0.6624 + 9.753 + 1.469 - 11.241 = +.644 \text{ J/K}$$

21-29 One mole of an ideal monatomic gas is taken through the cycle shown in Fig. 21-27 . Assume that p = $2p_0$, V = 2V0, p_0 = 1.01 10^5 Pa, and V_0 = 0.0225 m^3. Calculate

(a) the work done during the cycle,
(b) the energy added as heat during stroke abc, and
(c) the efficiency of the cycle.
(d) What is the efficiency of a Carnot engine operating between the highest and lowest temperatures that occur in the cycle? How does this compare to the efficiency calculated in (c)?



21-29



a) W = area of rectangle

$W = (p - p_0)(V - V_0)$

$W = p_0 V_0$

$W = (1.01 \times 10^5 \text{ Pa})(.0225 \text{ m}^3)$

W = 2273 Joules

(b) Total heat added $Q_{abc} = Q_{ab} + Q_{bc}$

1. ab is iso-volume: $Q_{ab} = n C_v (T_b - T_a)$
2. bc is iso baric: $Q_{bc} = n C_p (T_c - T_b)$

$\boxed{\text{monatomic: } C_v = \frac{3}{2}R \quad C_p = \frac{5}{2}R}$

Since $pV = nRT$, then doubling pressure (volume fixed) must double Temp: $T_b = 2T_a$ a→b

, and doubling volume (pressure fixed) as in b→c must double temp again $T_c = 2T_b = 4T_a$

$$Q_{abc} = nRT_a\left[\frac{3}{2}\left(\frac{T_b}{T_a} - 1\right) + \frac{5}{2}\left(\frac{T_c}{T_a} - \frac{T_b}{T_a}\right)\right]$$

$$= p_0 V_0 \left[\frac{3}{2}(2-1) + \frac{5}{2}(4-2)\right]$$

$$Q_{abc} = W\left[\frac{3}{2} + 5\right] = \frac{13}{2}W = \frac{13}{2}(2273) = 14\,800 \text{ Joules}$$

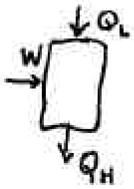
(c) $\epsilon = \frac{W}{Q_{abc}} = \frac{W}{\frac{13}{2}W} = \frac{2}{13} = 0.154$

(d) $\epsilon_{Carnot} = 1 - \frac{T_a}{T_c} = 1 - \frac{1}{4} = 0.75$ (much larger than this case)

21-40 The motor in a refrigerator has a power of 200 W. If the freezing compartment is at 270 K and the outside air is at 300 K, and assuming the efficiency of a Carnot refrigerator, what is the maximum amount of energy that can be extracted as heat from the freezing compartment in 10.0 min?

21-40

The heat extracted per unit work for a cooling unit is easily obtained from schematic at left.



where Q_L removed from low-temp. reservoir
Q_H added to high-temp. reservoir
by adding W

For no change in entropy in full cycle $\Delta S = 0$ so

$$\frac{Q_L}{T_L} = \frac{Q_H}{T_H}$$

$$Q_H = Q_L\left(\frac{T_H}{T_L}\right)$$

So,

$$Q_L + W = Q_H$$

$$W = Q_H - Q_L$$

$$W = Q_L\left[\frac{T_H}{T_L} - 1\right]$$

$$\frac{Q_L}{W} = \frac{1}{T_H/T_L - 1} = \frac{T_L}{T_H - T_L}$$

The work done by the motor in 10 minutes is

$$W = 200\ \frac{J}{s} \times 10\ \cancel{min} \times \frac{60\ \cancel{s}}{1\ \cancel{min}} = 1.2 \times 10^5\ \text{Joules}$$

So

$$Q_L = W\ \frac{T_L}{T_H - T_L} = \left(1.2\times10^5\ J\right)\left(\frac{270\ \cancel{K}}{300-270\ \cancel{K}}\right)$$

$$Q_L = 1.08\times10^6\ \text{Joules}$$