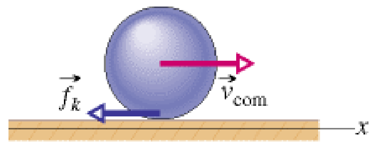
PHYSICS 1401 (1) homework solutions

12-14 A bowler throws a bowling ball of radius R = 11 cm along a lane. The ball slides on the lane, with initial speed $v_{com,0}$=8.5 m/s and initial angular speed ω_0=0. The coefficient of kinetic friction between the ball and the lane is 0.21. The kinetic frictional force f_k acting on the ball (Fig. 12-34) causes a linear acceleration of the ball while producing a torque that causes an angular acceleration of the ball. When speed v_{com} has decreased enough and angular speed ω has increased enough, the ball stops sliding and then rolls smoothly.

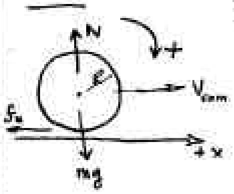
(a) What then is v_{com} in terms of w?

During the sliding, what are the ball's

(b) linear acceleration and
(c) angular acceleration?
(d) How long does the ball slide?
(e) How far does the ball slide?
(f) What is the speed of the ball when smooth rolling begins?



12-14 First, sketch the ball, with forces acting on it, and direction conventions



$v_0 = 8.5$ m/s
$R = 0.11$ m
$\mu_k = 0.21$

Initially, ball is travelling with

$$V_{com} = V_0$$

and not rotating. Hence, ball must be sliding along surface. This motion implies a friction force, f_k, at the point of contact. Since f_k creates a torque (the only one) acting on the ball (about the c.o.m.), then the ball must rotate.

As the torque operates, the ball begins to rotate, or ω goes from 0 to a finite value.

Notice that velocity of the pt. in contact, V_{bottom}, is

$$V_{bottom} = V_{com} - R\omega$$

So, when $\omega = 0$ $\quad V_{bottom} = V_{com}$

(a) As ω increases, $V_{bottom} < V_{com}$ until $V_{bottom} = 0$, when the ball begins rolling: $V_{com,roll} = R\omega_{roll}$

(b) During that period, τ is constant

$$\tau = f_k R = I\alpha$$

The friction force, $f_k = \mu_k N = \mu_k mg$ also causes the c.of.m. to slow down

$$-f_k = m a_{com} \Rightarrow a_{com} = -\frac{\mu_k mg}{m} = -\mu_k g$$

$$a_{com} = -(0.21)(9.8) = -2.06 \text{ m/s}^2$$

(c) $$\alpha = \frac{\mu_k mgR}{I} = \mu_k \frac{\cancel{m} g \cancel{R}}{\frac{2}{5}\cancel{m}R^{\cancel{2}}} = \frac{5}{2}\frac{\mu_k g}{R} = \frac{5}{2}\frac{(0.21)(9.8)}{0.11} = 46.8 \text{ rad/s}^2$$

(d) $V_{com,roll} = V_0 + a_{com} t_r \Rightarrow$

and

$$\omega_{roll} = \alpha t_r \Rightarrow \frac{V_{com,roll}}{\omega_{roll}} = \frac{V_0 + a_{com} t_r}{\alpha t} = R \quad \left(\text{from part (a)}\right)$$

$$V_0 = t(-a_{com} + R\alpha) \Rightarrow t_r = \frac{V_0}{-a_{com} + R\alpha} = \frac{8.5}{+2.06 + (.11)(46.8)}$$

$$t_r = 1.18 \text{ seconds}$$

(e) $x = V_0 t_r + \frac{1}{2} a_{com} t_r^2 = \left[(8.5) - \frac{1}{2}(2.06)(1.18)\right](1.18) = 8.60$ m

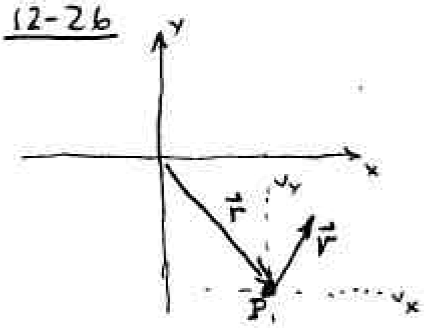
(f) $V_{roll} = V_0 + a_{com} t_r = 8.5 - (2.06)(1.18) = 6.07$ m/s

12-26 A 2.0 kg particle-like object moves in a plane with velocity components v_x = 30 m/s and v_y = 60 m/s as it passes through the point with (x, y) coordinates of (3.0, -4.0) m. Just then, what is its angular momentum relative to

(a) the origin and

(b) the point (-2.0, -2.0) m?

12-26



$\vec{r} = 3.0\,\hat{\imath} - 4.0\,\hat{\jmath}$ meters to P from origin

$\vec{v} = 30.\,\hat{\imath} + 60.\,\hat{\jmath}$ m/s

Angular momentum $\vec{\ell}$ is

$$\vec{\ell} = \vec{r} \times \vec{p} = m\,(\vec{r} \times \vec{v})$$

Since $\vec{r}$ and $\vec{v}$ are in the x-y plane, then $\vec{\ell}$ is in the z-direction:

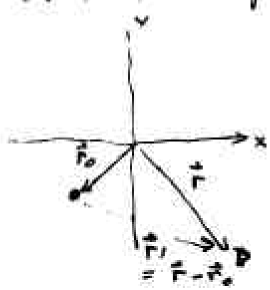
$$\vec{\ell} = \ell_z \hat{k}$$

(a) About the origin

$$\ell_z = m\,(x v_y - y v_x) = m\left[(3.0)(60.) + (4.0)(30.)\right] \ \frac{\text{kg-m}^2}{\text{s}}$$

$$\ell_z = 2.0\left[180. + 120.\right] = 600 \ \text{kg-m}^2/\text{s}$$

(b) About the point, $\vec{r}_o = -2.0\,\hat{\imath} - 2.0\,\hat{\jmath}$



The vector from this point to the point P is

$$\vec{r}' = \vec{r} - \vec{r}_o = (3.0 - [-2.0])\hat{\imath} + [-4.0 - (-2.0)]\hat{\jmath}$$

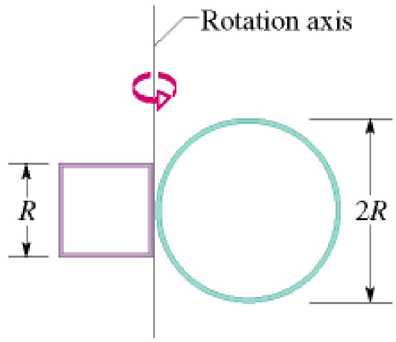
$$\vec{r}' = 5.0\,\hat{\imath} - 2.0\,\hat{\jmath}$$

So, now

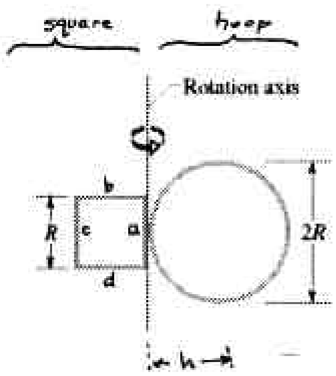
$$\vec{\ell}' = 2.0\left[(5.0)(60.) + (2.0)(30.)\right]$$

$$\vec{\ell}' = 2.0\left[360\right] = 720. \ \text{kg-m}^2/\text{s}$$

12-38 Figure 12-39 shows a rigid structure consisting of a circular hoop of radius R and mass m, and a square made of four thin bars, each of length R and mass m. The rigid structure rotates at a constant speed about a vertical axis, with a period of rotation of 2.5 s. Assuming R = 0.50 m and m = 2.0 kg, calculate (a) the structure's rotational inertia about the axis of rotation and (b) its angular momentum about that axis.



12-38



rods each mass = m
hoop has mass = m

(a) The complete structure is a rigid body, consisting of two parts:

square ⇒ has 4 bars
hoop ⇒ circle

joined at the rotation axis.

We calculate the total moment of inertia from the individual ones:

$$I_{hoop} = I_{c.of m.} + mh^2$$

is the mom. of inertia of the hoop about the rotation axis and $I_{c.of m.}$, about the c. of mass. Here $h = R$ is the distance from the c. of m. to the rotation axis. Hence

$$I_{hoop} = \tfrac{1}{2}mR^2 + mR^2 = \tfrac{3}{2}mR^2$$

For the square, it has 4 rods, so $I_{square} = I_a + I_b + I_c + I_d$ for each piece labelled as in the figure

Since each is very thin, $I_a = 0$

$$I_b = I_d = I_{\substack{com\\rod}} + m\left(\tfrac{1}{2}R\right)^2 \quad \text{by parallel axis theorem}$$

$$I_b = I_d = \tfrac{1}{12}mR^2 + \tfrac{1}{4}mR^2 = \tfrac{1}{3}mR^2$$

and $\quad I_c = mR^2 \quad$ (since all its mass is distance R from rotation axis)

Hence, $\quad I_{square} = 0 + \tfrac{1}{3}mR^2 + \tfrac{1}{3}mR^2 + mR^2$

$$I_{square} = \tfrac{5}{3}mR^2$$

For entire structure $\quad I = I_{square} + I_{hoop} = \tfrac{5}{3}mR^2 + \tfrac{3}{2}mR^2$

$$I = \tfrac{19}{6}mR^2 = \left(\tfrac{19}{6}\right)(2.0\,\text{kg})(0.50\,\text{m})^2 = 1.64\,\text{kg}\cdot\text{m}^2$$

(b) For period, $T = 2.5\,\text{s}$, $\quad \omega = \frac{2\pi}{T} = \frac{2\pi}{2.5} = 2.51\,\text{rad/s}$

$$L = I\omega = (1.64)(2.51)$$

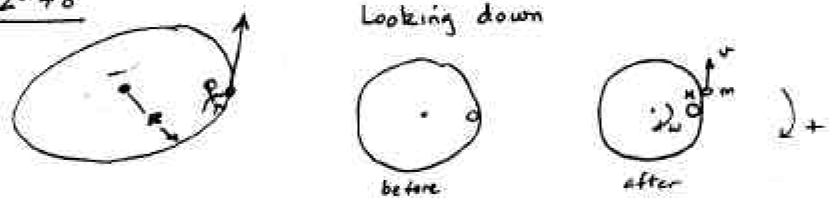
$$L = 4.12\,\text{kg}\cdot\text{m}^2/\text{s}$$

PHYSICS 1401 (1) homework solutions

12-48 A girl of mass M stands on the rim of a frictionless merry-go-round of radius R and rotational inertia I that is not moving. She throws a rock of mass m horizontally in a direction that is tangent to the outer edge of the merry-go-round. The speed of the rock, relative to the ground, is v. Afterward, what are

(a) the angular speed of the merry-go-round and
(b) the linear speed of the girl?

12-48



(a) Angular momentum before = 0

Since merry-go-round is frictionless, there are no external torques, so angular momentum is conserved.

Define clockwise as +ve.

$$0 = I\omega + MR^2\omega - mRv$$

ang mom of m-go-round; ang mom of girl; angular momentum of rock

So $\omega (I + MR^2) = mRv \Rightarrow \omega = \dfrac{mRv}{I + MR^2}$

(b) The linear speed of the girl is

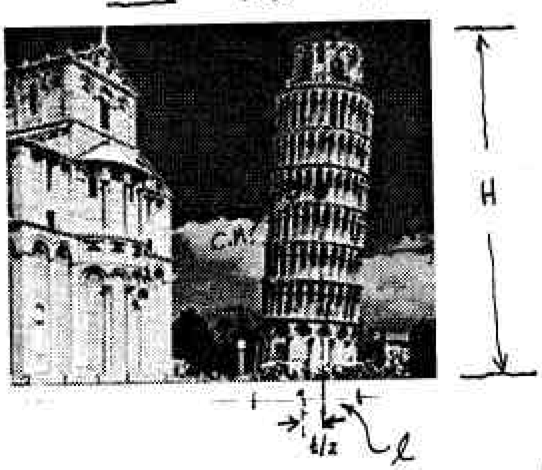
$$v_g = R\omega = \frac{mR^2 v}{I + MR^2}$$

13-2 The leaning Tower of Pisa (Fig. 13-22) is 55 m high and 7.0 m in diameter. The top of the tower is displaced 4.5 m from the vertical. Treat the tower as a uniform, circular cylinder.

(a) What additional displacement, measured at the top, would bring the tower to the verge of toppling?

(b) What angle would the tower then make with the vertical?

13-2



If a point on the top has moved a distance, $t = 4.5$ m from the "normal" location, then the C.M. (halfway up) has moved

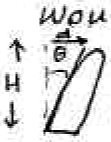
$\frac{t}{2} = 2.25$ m from above the base center

If the base has diameter $d = 7.0$ m, then this point is now still inside the base by an amount $\ell = \frac{d}{2} - \frac{t}{2} = 3.5 - 2.25$

$\ell = 1.25$ m

(a) For the C.M. to move outside the base (so the tower would topple), the top must move a distance

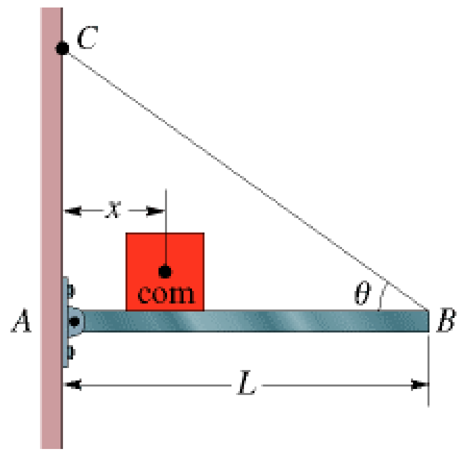
$2\ell = 2(1.25) = 2.5$ m more!



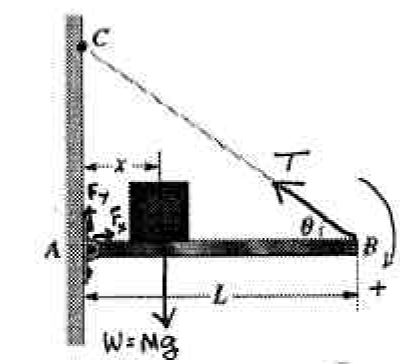
(b) $\tan\theta = \frac{d}{H} = \frac{7.}{55.} = .127$ $\theta = 7.25°$

13-28 In Fig. 13-39 , a thin horizontal bar AB of negligible weight and length L is hinged to a vertical wall at A and supported at B by a thin wire BC that makes an angle q with the horizontal. A load of weight W can be moved anywhere along the bar; its position is defined by the distance x from the wall to its center of mass. As a function of x, find

(a) the tension in the wire, and
(b) horizontal and
(c) vertical components of the force on the bar from the hinge at A.



13-28



The only external forces on the system of the bar + load are
a) tension in wire at B ($\vec{T}$)
(b) weight of Load, W (down)
(c) Force (in unknown direction) at hinge at A

To make life simpler, take torques about A. (Then forces at A don't appear, since they have lever arm = 0.)

(a) For clockwise +ve,

$$\sum \text{torque}_{\text{about } A} = Wx - (T\sin\theta)L = 0$$

$$T = \frac{Wx}{L\sin\theta}$$

(b) Use Newton's 2nd Law to get Force components at A

$$F_x = T\cos\theta = \frac{Wx}{L\tan\theta}$$

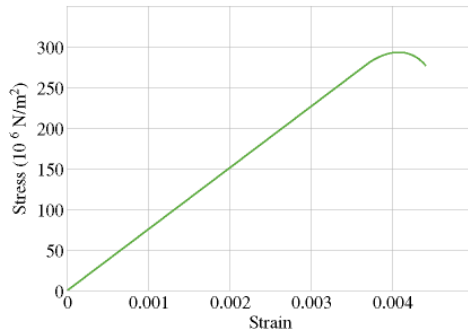
(c)

$$F_y = W - T\sin\theta = W - \frac{Wx}{L\cancel{\sin\theta}}\cancel{\sin\theta}$$

$$F_y = W\left[1 - \frac{x}{L}\right]$$

13-36 Figure 13-44 shows the stress–strain curve for quartzite. What are

(a) the Young's modulus and
(b) the approximate yield strength for this material?



<u>13-36</u>

(a) $Y = \frac{\text{stress}}{\text{strain}}$ (in linear region)

$Y = \frac{250 \times 10^6 \text{ N/m}^2}{.0033}$

$Y = 7.6 \times 10^{10} \text{ N/m}^2$

(b) The curve deviates significantly from linear when $S \approx 280 \times 10^6 \text{ N/m}^2$

so yield pt $\approx 2.8 \times 10^8 \text{ N/m}^2$