Today

- Homework 4 due (usual box)
- Center of Mass
- Momentum

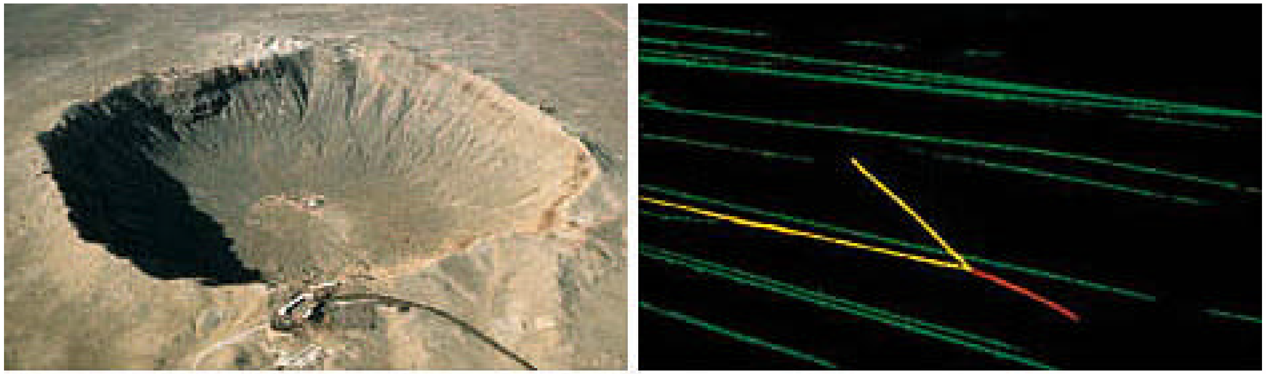
review

Conservation of Energy

- Generalization of Work-Energy Theorem
- Says that for any isolated system, the total energy is conserved (same at all times)
- In general (any system), work done equals the change in energy
- Isolated (closed) system: no work done on it or by it
  - Account for all forms of energy … ENERGY CONSERVED in such a system
- Forms of Energy
  - mechanical: kinetic and potential (orderly …)
  - thermal: heat (measure by body's temperature)
  - chemical: *e.g.* elements in isolation when brought together can liberate energy (eg thermal energy from exothermic chemical reactions)
    - Note that chemical energy (electromagnetic in origin) is stored energy (so potential function can be defined). Some fraction liberated as light and heat. But it can be used *(eg* electric batteries and muscle control) for mechanical work.
  - others:
    - sound, light, ….

We will get to issue of COLLISIONS

Collisions classified: elastic vs inelastic



(a) (b)

Highly inelastic: Result from collision of meteor with ground (Arizona) 1220m high and 200 m deep

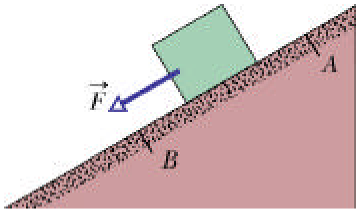
Highly elastic: Path of alpha particle (yellow) colliding with Nitrogen nucleus

Momentum discussed in chapters 9 -10

review

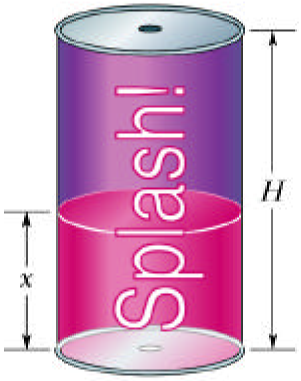
New Stuff - Ch 9-10

- New principle: conservation of momentum
- So far, we have dealt with simple systems
  - point-like objects
  - symmetric objects



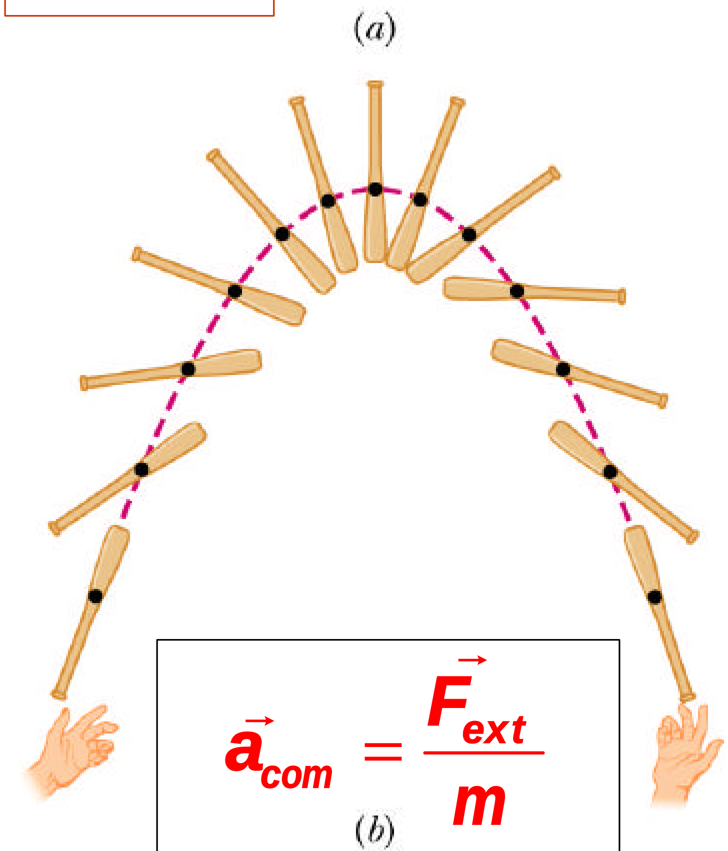
- Now we learn how Newton's Laws apply to complicated systems
  - Example of issues --- *F=ma* --- but for what point?
  - Assert the answer: Center of Mass!
  - Prove later and get more!!

Complications because most masses are not simple



Extended body: Center of mass obeys *F=ma*

review



$$\vec{a}_{com} = \frac{\vec{F}_{ext}}{m}$$

Above case:

$$\vec{a}_{com} = \frac{m\vec{g}}{m} = \vec{g}$$

Demos

- wrench (no gravity)
- planar shape (gravity)

DEMO

- pix of bat in text

- First specify the <u>system</u> under discussion and know its mass (*m*)
- Determine (calculate) the center of mass (*com*) within the body
- Determine the total <u>external</u> force, F_{ext}, on the body

Center of Mass Defined

Weighted average of position:

the weight is the mass

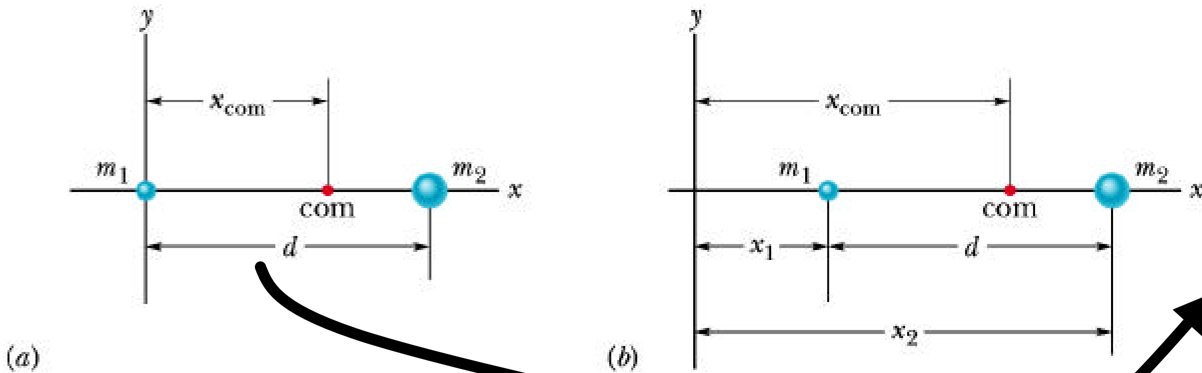
$$x_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i$$

$$y_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i$$

$$z_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i$$

$$\vec{r}_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i$$

Calculate for these two cases



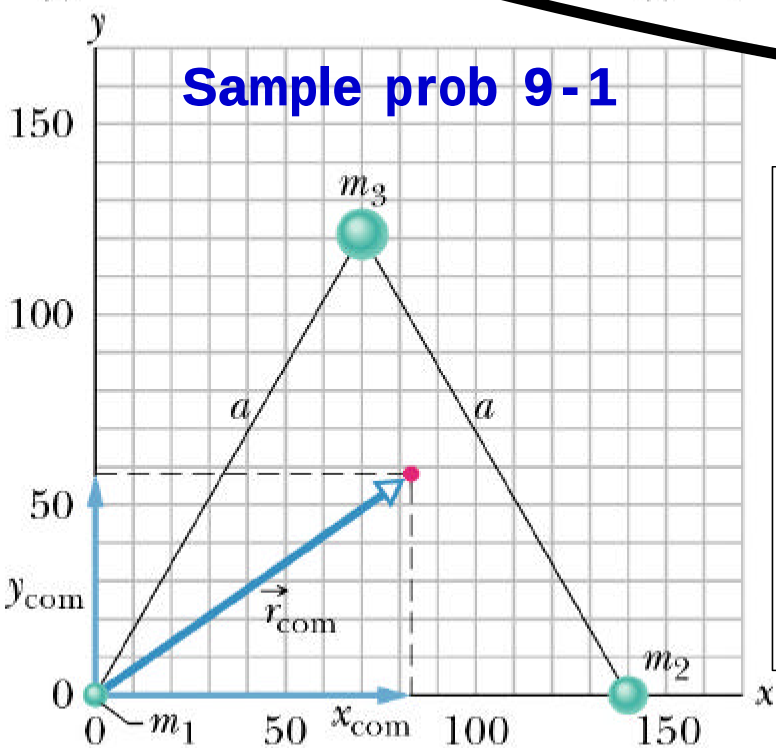
$$x_{com} = \frac{1}{m_1+m_2}\sum_{i=1}^{n} m_i x_i$$

$$x_{com} = \frac{1}{m_1+m_2}\left[m_1(0) + m_2 d\right]$$

$$x_{com} = \frac{m_2 d}{m_1+m_2}$$

$$x_{com} = x_1 + \frac{m_2 d}{m_1+m_2}$$

Sample prob 9-1

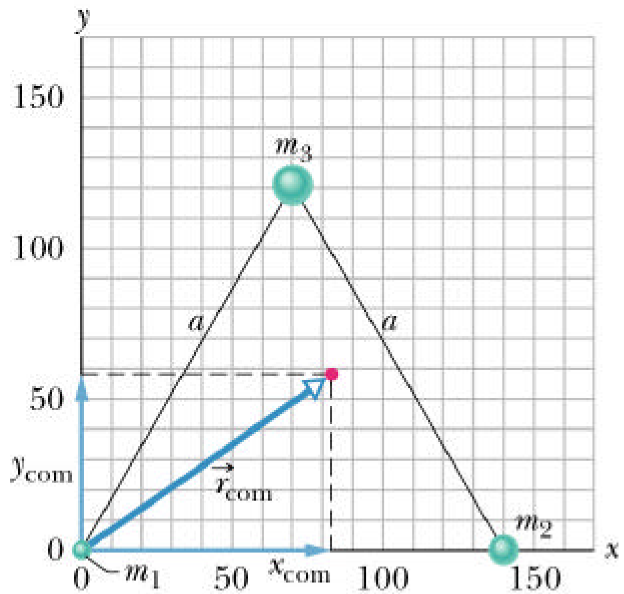


$$x_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i = \frac{1}{M}\left[140 m_2 + 70 m_3\right]$$

$$y_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i = \frac{1}{M}\left[120 m_3\right]$$

$$M = m_1 + m_2 + m_3$$

Center of Mass Motion



$$\vec{r}_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i$$

$$\frac{d}{dt}\vec{r}_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i \frac{d}{dt}\vec{r}_i$$

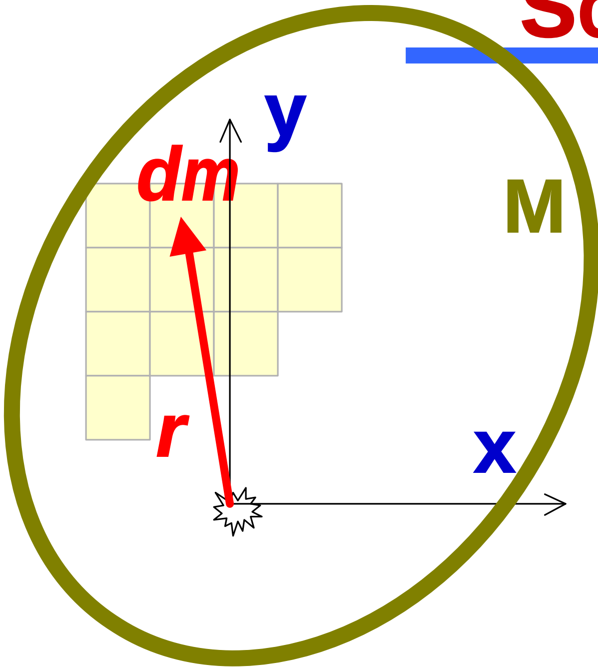
$$\vec{v}_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{v}_i$$

$$\frac{d}{dt}\vec{v}_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i \frac{d}{dt}\vec{v}_i$$

$$\vec{a}_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{a}_i$$

- **CM is weighted average**
  - ◆CM has average position
  - ◆CM has average velocity
  - ◆CM has average acceleration

Continuous Bodies and Some Math Notation

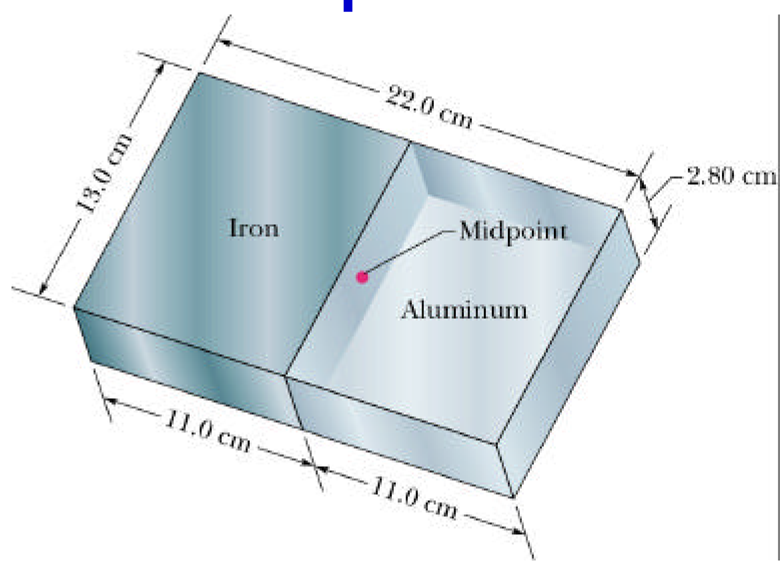


$$\vec{r}_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i$$

For solids:

$$\vec{r}_{com} = \frac{1}{M}\int \vec{r}\, dm$$

Works for complicated objects like prob 9-6



3D

$$r \equiv \frac{dm}{dV} \qquad M = \int r\, dV$$

$$\vec{r}_{com} = \frac{1}{V}\int \vec{r}\, dV$$

Calculating Center of Mass for Continuous and Uniform Bodies

$$\vec{r}_{com} = \frac{1}{M}\int \vec{r}\,dm$$

- **Apply to bodies as appropriate**

2*D* -- like plate

s = mass area density

$$s \equiv \frac{dm}{dA} \qquad M = \int s\,dA$$

$$\vec{r}_{com} = \tfrac{1}{A}\int \vec{r}\,dA$$

3*D* -- like cube

r = volume mass density

$$r \equiv \frac{dm}{dV} \qquad M = \int r\,dV$$

$$\vec{r}_{com} = \tfrac{1}{V}\int \vec{r}\,dV$$

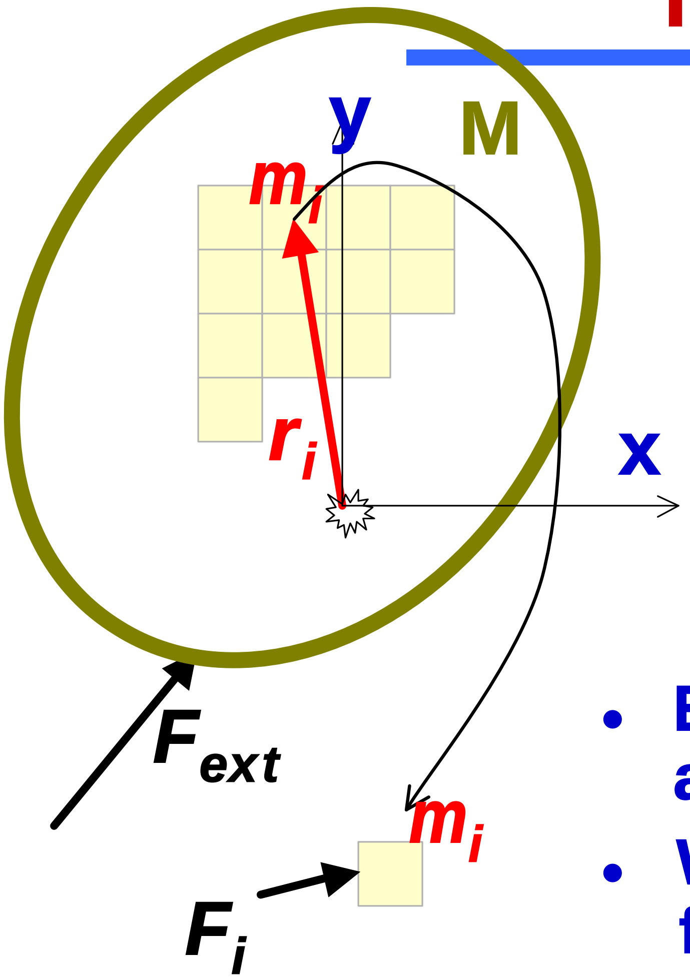
1*D* -- like stick

µ = length mass density

$$m \equiv \frac{dm}{dl} \qquad M = \int m\,dl$$

$$\vec{r}_{com} = \tfrac{1}{L}\int \vec{r}\,dl$$

Proof: C of M is the point for which *F=ma* … part 1



$$\vec{F}_1 = \vec{f}_1^{\,ext} + \vec{f}_{21} + \vec{f}_{31} + \ldots = m_1\vec{a}_1$$

$$\vec{F}_2 = \vec{f}_2^{\,ext} + \vec{f}_{12} + \vec{f}_{32} + \ldots = m_2\vec{a}_2$$

....

$$\sum \vec{F}_i = \sum m_i \vec{a}_i$$

$$\vec{F}_i = m_i \vec{a}_i$$

- Each F_i will have an external piece and an internal piece
- When sum over body, the internal forces must cancel (eg $f_{12}=-f_{21}$)
  - action = - reaction
  - LHS is total external force
- RHS is sum we already saw!

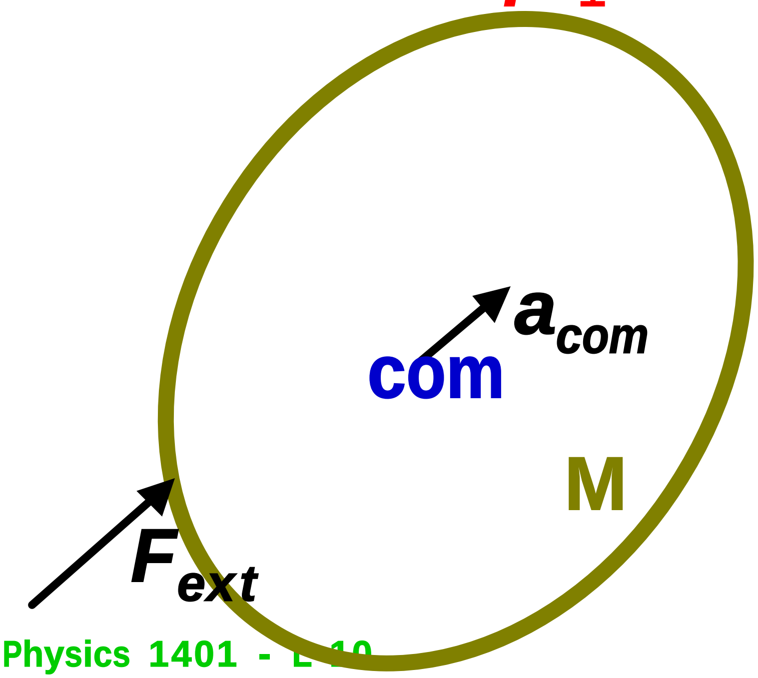
Proof: C of M is the point for which *F=ma* … part 2

$$\sum \vec{F}_i = \vec{F}_{ext}$$

Internal forces cancel

$$\vec{a}_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{a}_i$$

From definition of center of mass



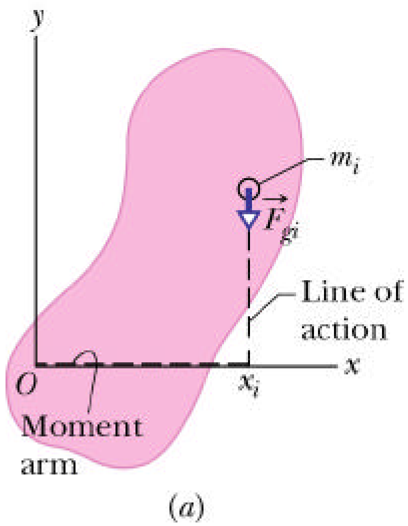
$$\sum \vec{F}_i = \sum m_i \vec{a}_i \quad \text{implies}$$

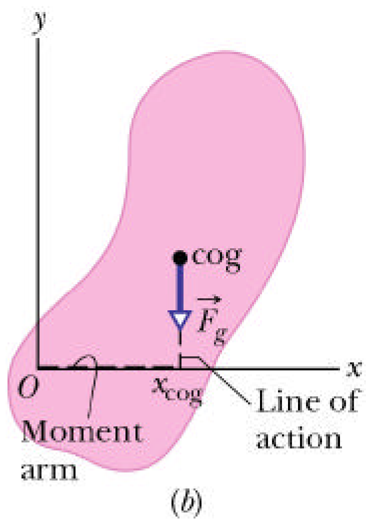
$$\vec{F}_{ext} = M\vec{a}_{com}$$

There may also be <u>rotation</u> … later!

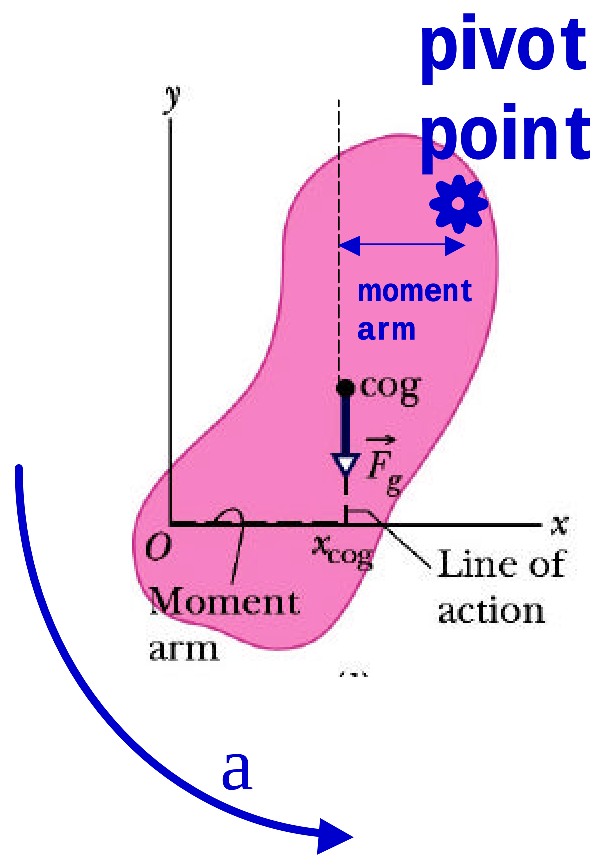
cm motion for body is F=ma.MOV

Center of Gravity = Center of Mass



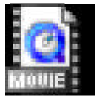


- Rigid body feels gravity as if all the body mass concentrated at center of mass
  - See proof in text (sec 13-3)
- Consequence
  - Pivot rigid body and cog must lie <u>below</u> it
- Demo
- Pivot body at cog and stable (no torque)

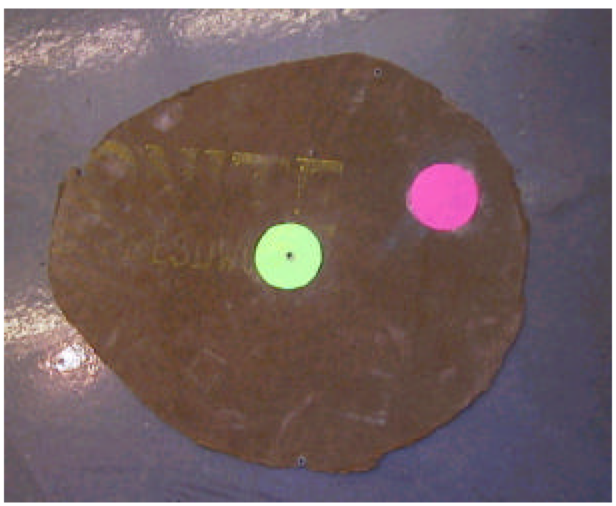


Center of Mass obeys F=ma

- **Examples**
  - Glider w pendulum
  - Motion of irregular shape



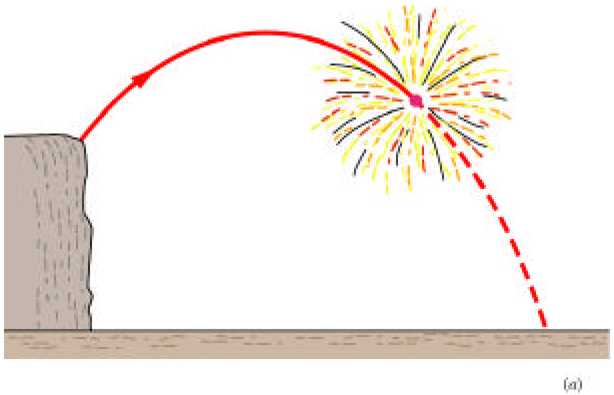
cm fixed of pendulum on glider.MOV

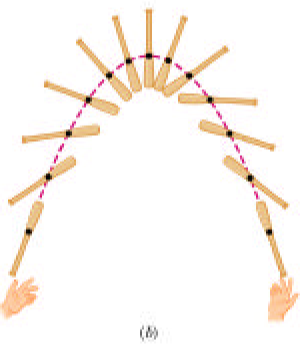


Generality of conclusion

The proof is valid whether the system of particles

- can move relative to each other
- are fixed relative to each other (rigid body)
- something in between





secrets of the ballerina

- C of M <u>must</u> travel in a parabola while she is in free flight
- Keeps head level (floats) by raising legs

Momentum

Define a new quantity

p ≡ momentum (vector)

For a point particle

$$\vec{p}_i \equiv m_i \vec{v}_i$$

$$\vec{P} \equiv \sum_{i=1}^{n} m_i \vec{v}_i$$

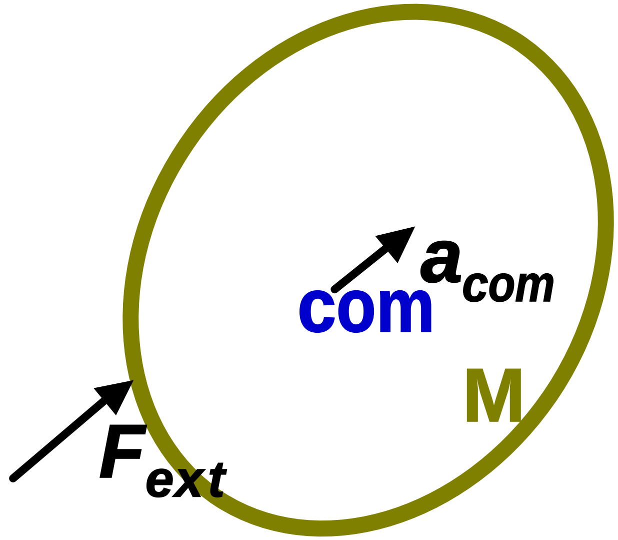
- **If this point mass is but part of a larger system, the total momentum of the system must be sum of pieces**

$$\vec{v}_{com} \equiv \frac{1}{M} \sum_{i=1}^{n} m_i \vec{v}_i$$

- **To what point in the body does this total momentum refer?**

$$\vec{P} = M \vec{v}_{com}$$

Momentum of system



System: bunch of particles or rigid body…

$$\vec{F}_{ext} = M\vec{a}_{com}$$

$$\vec{F}_{ext} = \frac{d}{dt}(M\vec{v}_{com})$$

$$\vec{P}_{com} = M\vec{v}_{com} = \sum_{i=1}^{n} m_i \vec{v}_i$$

$$\vec{F}_{ext} = \frac{d\vec{P}_{com}}{dt}$$

Newton's Second Law applies to complex system of particles

Total external force

= total body mass times acceleration of center-of-mass

= time rate of change of body (com) momentum

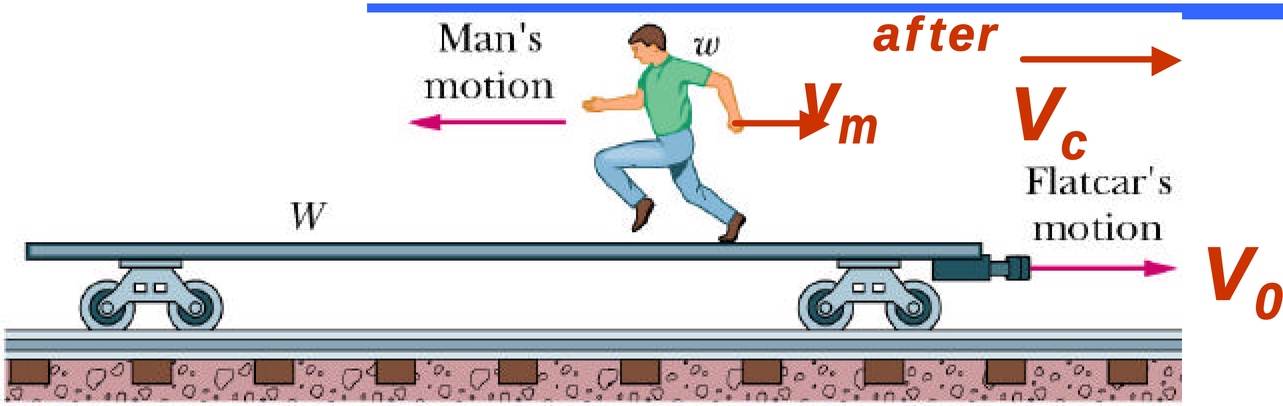
Momentum and Newton's 2nd Law

$$\vec{F}_{ext} = \frac{d\vec{P}_{com}}{dt}$$

$$\vec{F} = \frac{d\vec{P}}{dt}$$

We will see that this way of writing Newton's Second Law is <u>more general</u> and more useful that the old way (F=ma)

Prob 9-32: Momentum Conservation applied



- Initially man (weight *w*) is standing on the car (weight *W*) as it travels to the right, so both <u>man and car</u> travel to right with speed V_0
- At a certain time, the man begins to run to the left with a speed (relative to the car) V_{rel} as shown
- How much does he increase flatcar speed $\Delta V = V_c - V_0$ to the right?
- Note that, for man+car system, <u>no external horizontal force</u>

$$\Delta V = \frac{w}{W + w} V_{rel}$$

Conservation Rules

- Important part of physics knowledge
- As always, subject to check by experiment!
- So far, two
  - Energy
  - Momentum
- To come … many more
  - Angular momentum
  - Electric Charge
  - Other kinds of charge …
  - …

cultural

Conservation Rules mean more

- Obvious utility in doing problems (*e.g.* energy)
- More conservation rules:
  - Momentum, angular momentum, elect. chg.,…
- 20th Century Understanding says each implies
  - deeper statements about nature
- Energy Conservation Û Symmetry in Time
- Classical Physics culmination was the discovery that light is electromagnetic waves characterized by frequency (or wavelength)
  - Frequency µ 1/Wave period (time)
- Quantum Mechanics, at most elemental level, states that light is carried by photons of specific energy related to wave frequency

Illustration: Time and Energy Connected

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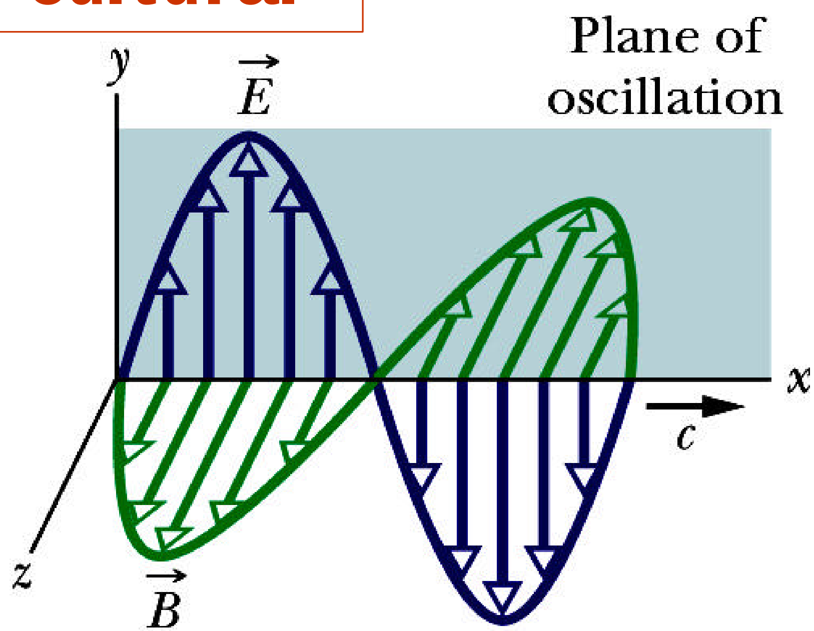

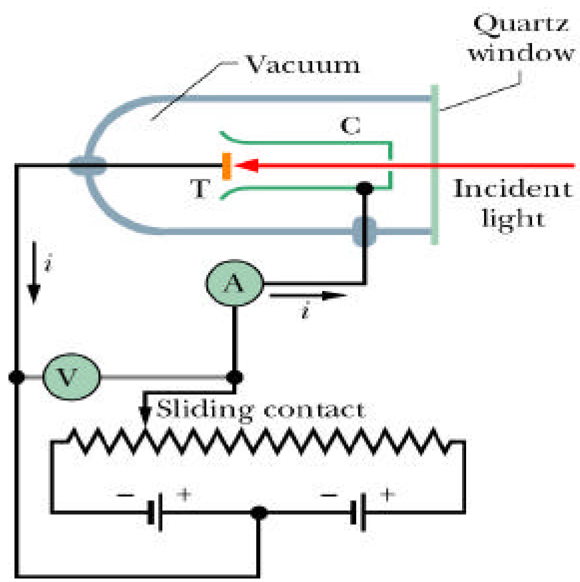

Maxwell's Equations:

Light consists of oscillating (in time) of electric and magnetic fields char by *f*, *T*

Einstein's Photoelectric Effect: Light is photons

Each photon has energy equal to *hf=h/T*

20th Century Physics--intimate connections

(Energy,t) (Momentum,x) (Ang Mom,q)

Conservation Laws and Symmetry Principles

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Conserved Physical Quantity	Symmetry of Nature…Laws of Physics do not change with…
Energy	Time
Momentum	Position
Angular Momentum	Angle (Orientation)

Reminders

- Homework 4 due (see box)
- Become familiar with use of energy and energy conservation
- Read and understand chapters 9 & 10
- Next time: rockets and collisions