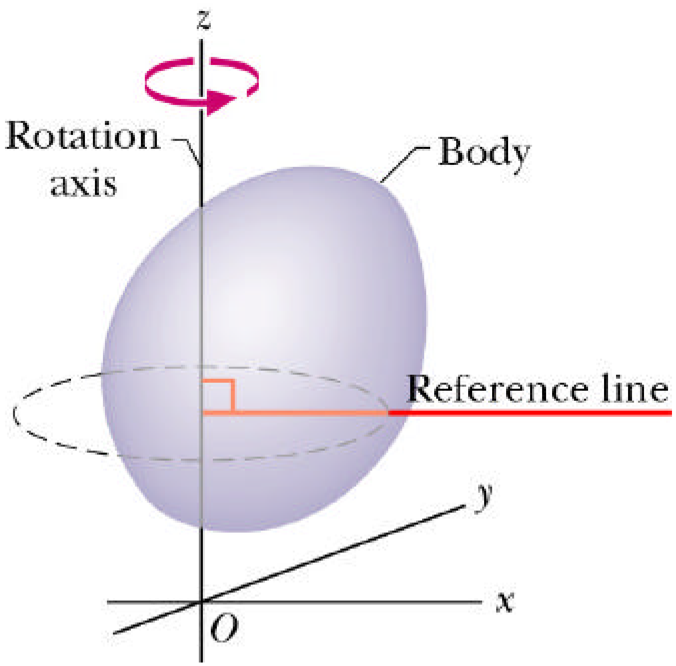
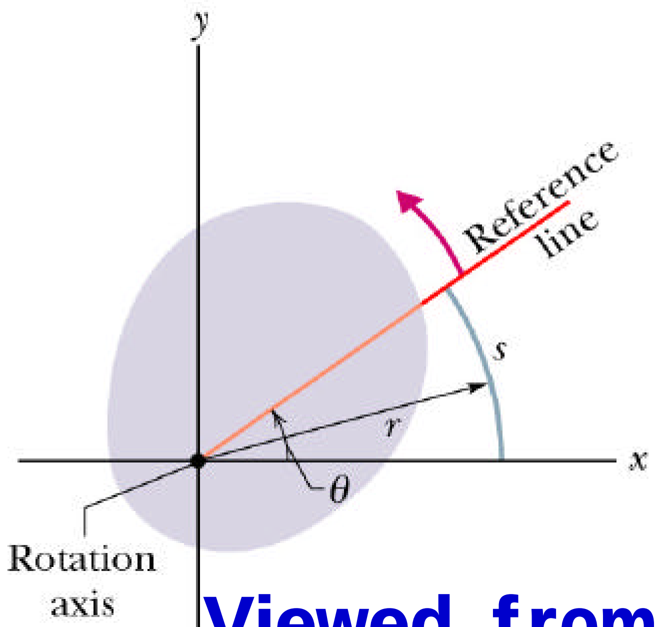
Today - Lecture 13

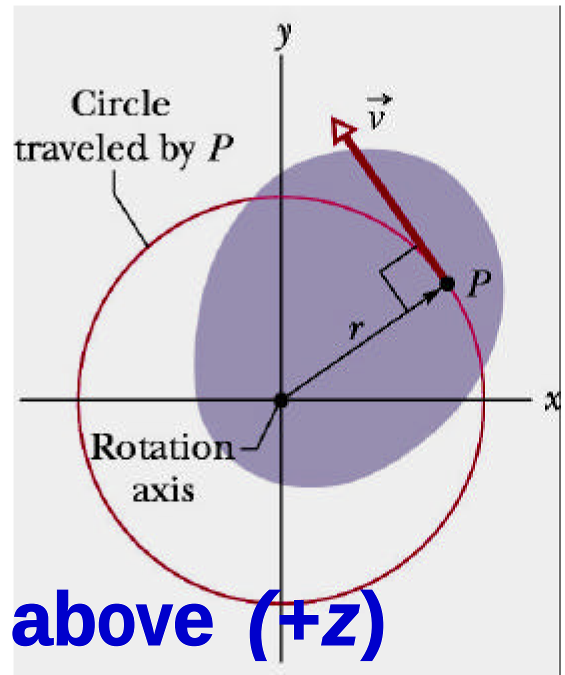
- Today's lecture … continue with rotations, torque, …
- Note that chapters 11, 12, 13 all involve rotations

review

Rotations
Chapters 11 & 12







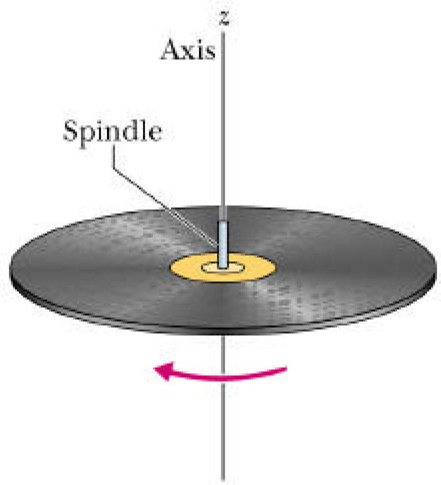
Viewed from above (+z)

- **Rotational, or angular velocity, gives tangential velocity**

$$\omega \equiv \frac{d\theta}{dt} \qquad v = \frac{ds}{dt} = r\,\frac{d\theta}{dt}$$

$$v = r\omega$$

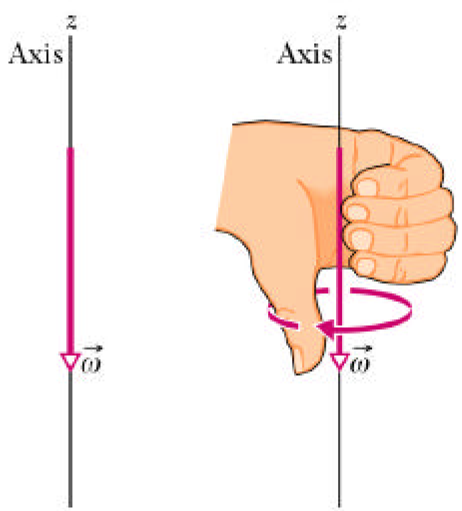
Vector angular velocity



(a)

Define vector ang. velocity

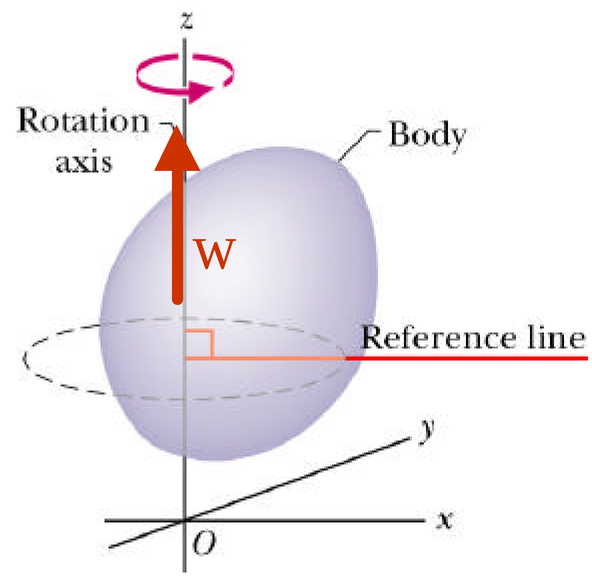
$$\vec{w} \equiv \begin{cases} \text{magnitude } w = \frac{dq}{dt} \\ \text{direction along rotation axis} \\ \text{sense using right-hand rule} \end{cases}$$



(b) (c)

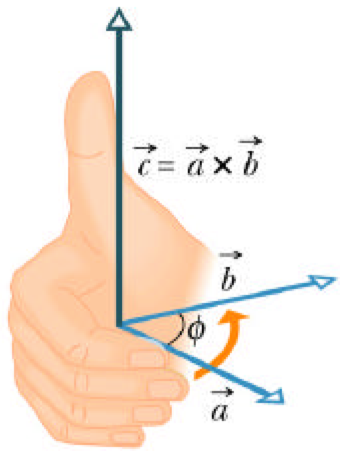
Examples

- Record at left has w along –z axis
- body to right has w along +z axis

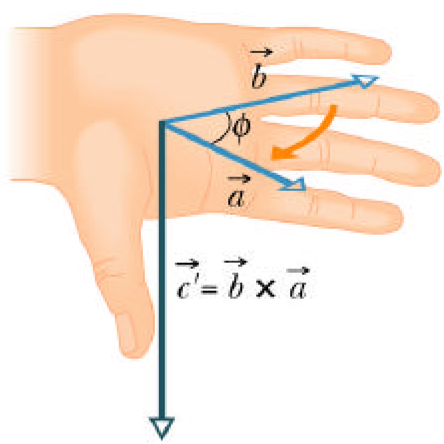


review

Vector Product (ch 3)



(a)



(b)

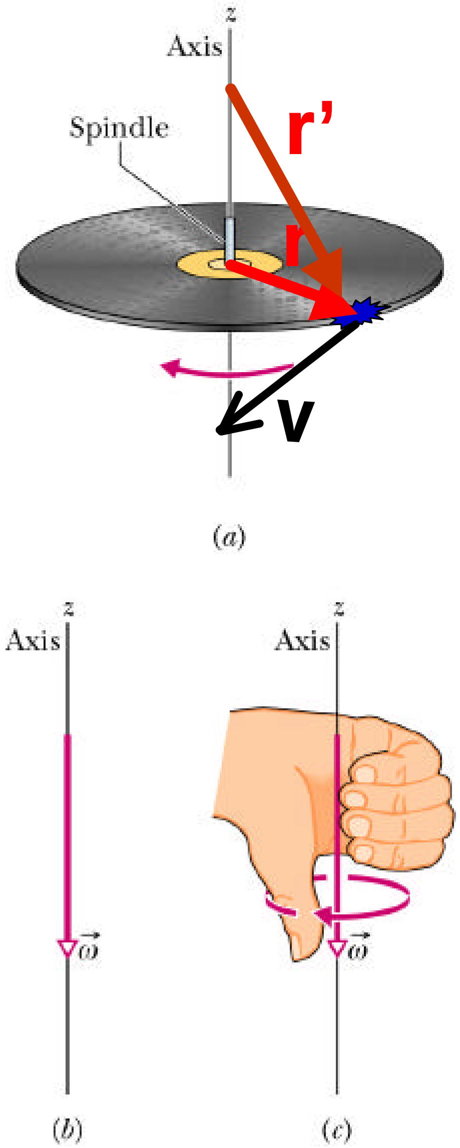
- Convention for ω recalls the definition of the vector product (not a coincidence)
- The vector product of chapter 3 is a convenient way to use the vector angular velocity (along with other rotation quantities)

$$\vec{c} = \vec{a} \times \vec{b}$$

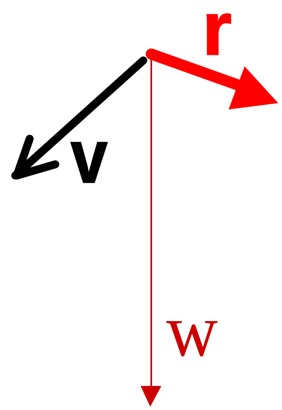
$$= -\vec{b} \times \vec{a}$$

$$c = ab \sin \phi$$

Vector product gives tangential velocity



$$v = r w$$



$$\vec{v} = \vec{w} \times \vec{r}$$

Note rel part of *r* is the ^ distance to the axis of rotation

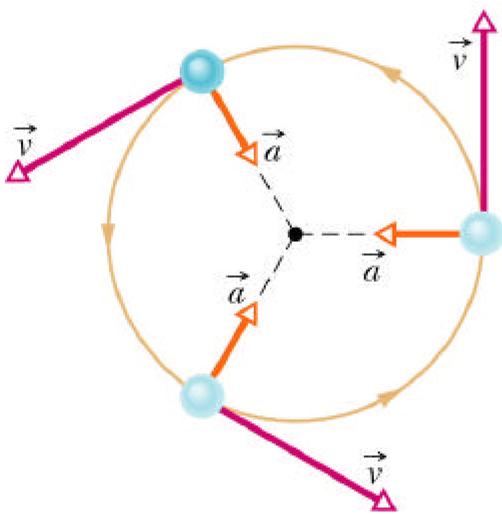
$$\vec{v} = \vec{w} \times \vec{r}\,'$$

- Already seen the tangential speed in terms of dist from axis and ang velocity
- Vector or cross-product gives direction of *v* ^ *w* & *r*

True for <u>any</u> rotating vector

One Step Further

$$\vec{v} = \vec{w} \times \vec{r}$$

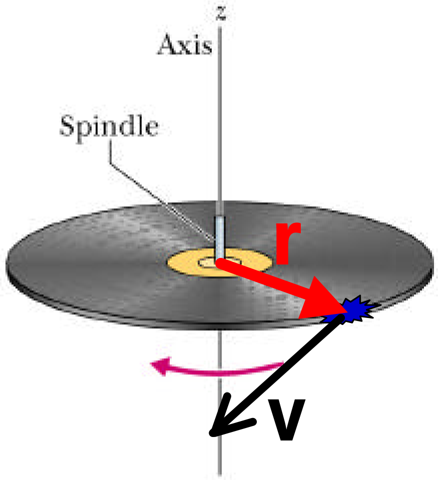


- Any rotating vector of fixed length simple time rate of change
  - Was true for radius vector
  - Also true for velocity vector
- $a_c = v^2/r$
- Directed inward
- The velocity vector
  - Has fixed magnitude
  - Rotates with angular velocity, w

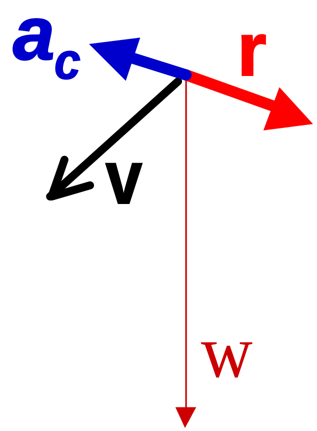
For $\vec{c}$ any vector of const. length rotating $\vec{w}$, then

$$\frac{d\vec{c}}{dt} = \vec{w} \times \vec{c}$$

Centripetal Acceleration
(revisited as a Vector)



(a)

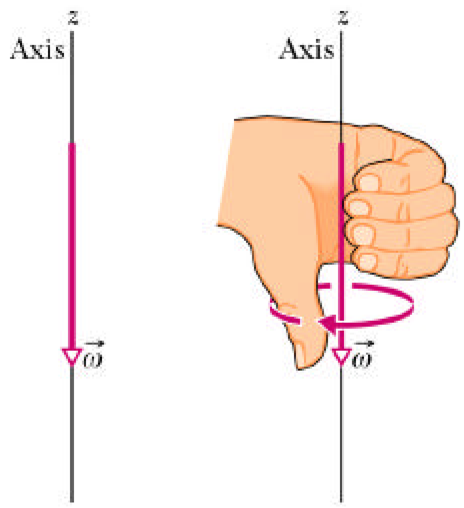


$$\vec{v} = \vec{w} \times \vec{r}$$

$$\vec{a}_c = \vec{w} \times \vec{v}$$

$$= \vec{w} \times (\vec{w} \times \vec{r})$$

$$\vec{a}_c = -w^2 \vec{r}$$



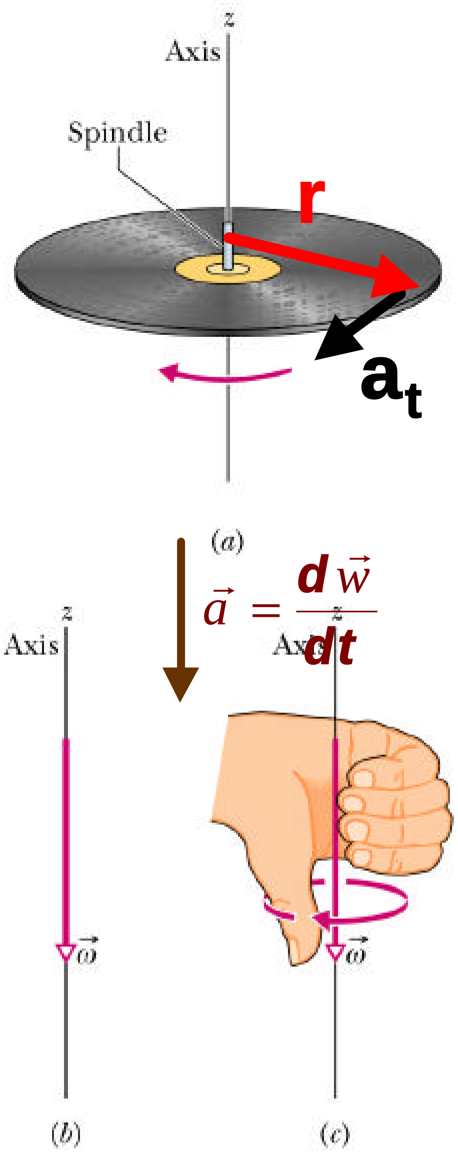
(b) (c)

- Rate of change of position around axis given by cross-product
- Also, rate of change of velocity around axis of rotation given by same cross-product

Vector Identity (appx E)

$$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$$

Angular acceleration



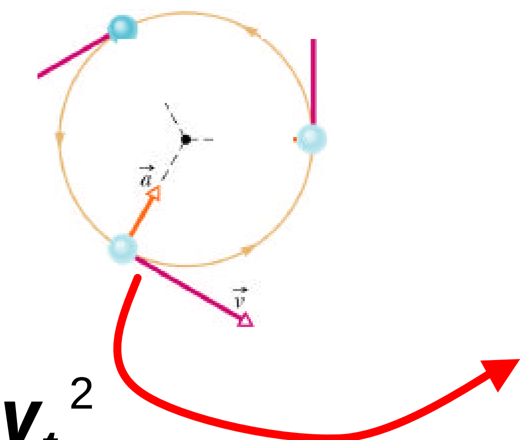
- If ang velocity, w, changes, there is an angular acceleration, a .
- Point at radius, r, has <u>linear acceleration</u>, $a_t = r a$, tangent to circle at r.

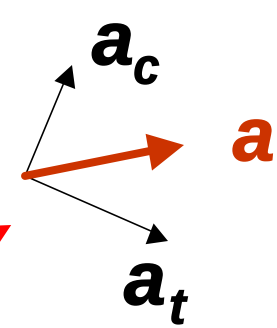
$$\vec{a} \equiv \frac{d\vec{w}}{dt} \qquad \vec{a}_t = \vec{a} \times \vec{r}$$

angular acceleration.MOV

Angular and Centripetal Acceleration

- Relationships for <u>constant</u> angular acceleration similar to those of constant linear acceleration
- See table 11-1
- Note that pt moving on circle, in general, will have <u>two</u> components of acceleration
- The two add vectorially to give total acceleration vector





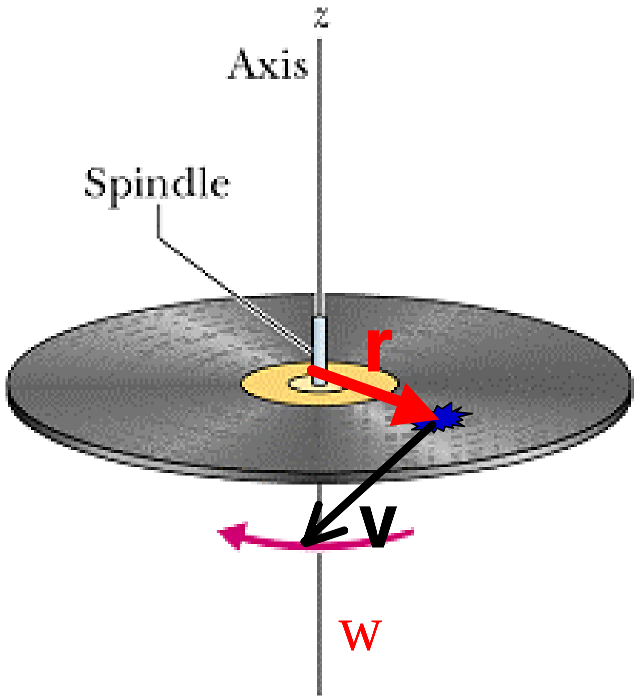
$$a_c = a_r = \frac{v_t^2}{r}$$

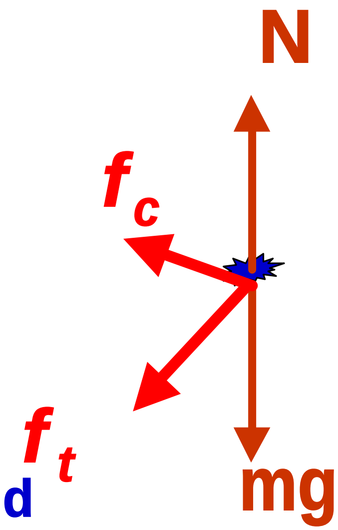
pointing to rotation axis

$$a_t = \frac{dv_t}{dt} = r\alpha$$

along direction of rotation

Hint: HW problem 11-32





$$v = r\omega$$

$$|a_c| = \frac{v^2}{r} = r\omega^2$$

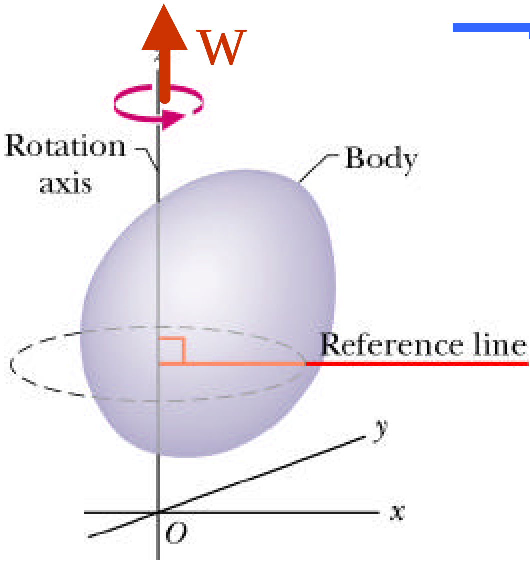
$$|a_t| = r\alpha$$

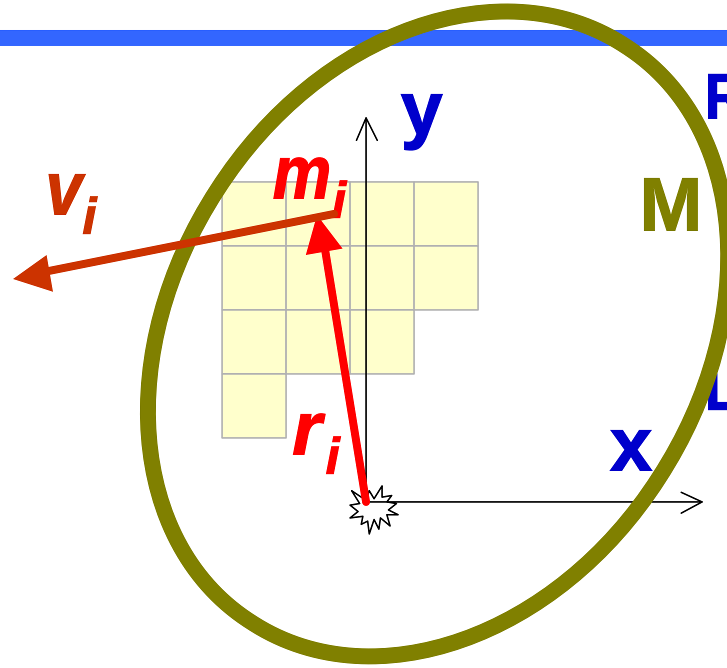
$$\alpha = \frac{\omega - \omega_0}{\Delta t}$$

- Watermelon seed on record player
  - Starts at rest $\omega_0=0$
  - Reaches 33 1/3 rpm
- Convert to standard units
- Two components (at right angles) of acceleration
- Find net acceleration … net friction force causes it

- Coefficient of static friction must be big enough to withstand this <u>total force</u>

Kinetic Energy of Rotation: Moment of Inertia





Rotation of a rigid body about an axis implies Kinetic Energy...

Look down from +z axis at mass pieces in body

energy in rotational motion.MOV

$$K = \sum_i \tfrac{1}{2} m_i v_i^2$$

$$= \tfrac{1}{2} \sum_i m_i r_i^2 w^2$$

$$K = \tfrac{1}{2} I w^2$$

$$I \equiv \sum_i m_i r_i^2 \quad \text{discrete } m_i$$

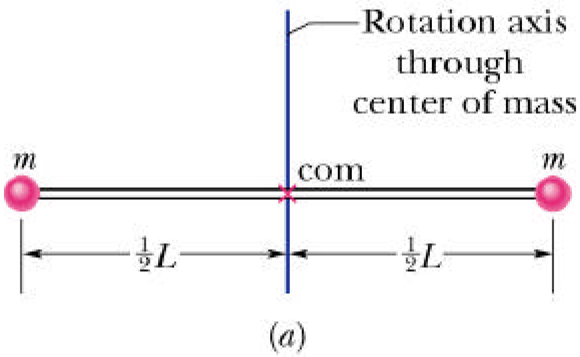
$$I \equiv \int_M r^2 dm \quad \text{continuous } m$$

Point masses: sample prob. 11-5

- Rigid body of two masses (m) connected by rigid massless bar.
- Find I for two axes

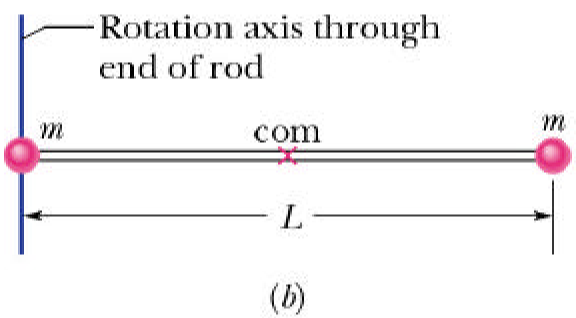
$$I \equiv \sum_i m_i r_i^2 \quad \text{discrete } m_i$$

$$I \equiv \int_M r^2 dm \quad \text{continuous } m$$



$$I_{com} = m(\tfrac{1}{2}L)^2 + m(\tfrac{1}{2}L)^2$$

$$I_{com} = \tfrac{1}{2}mL^2$$

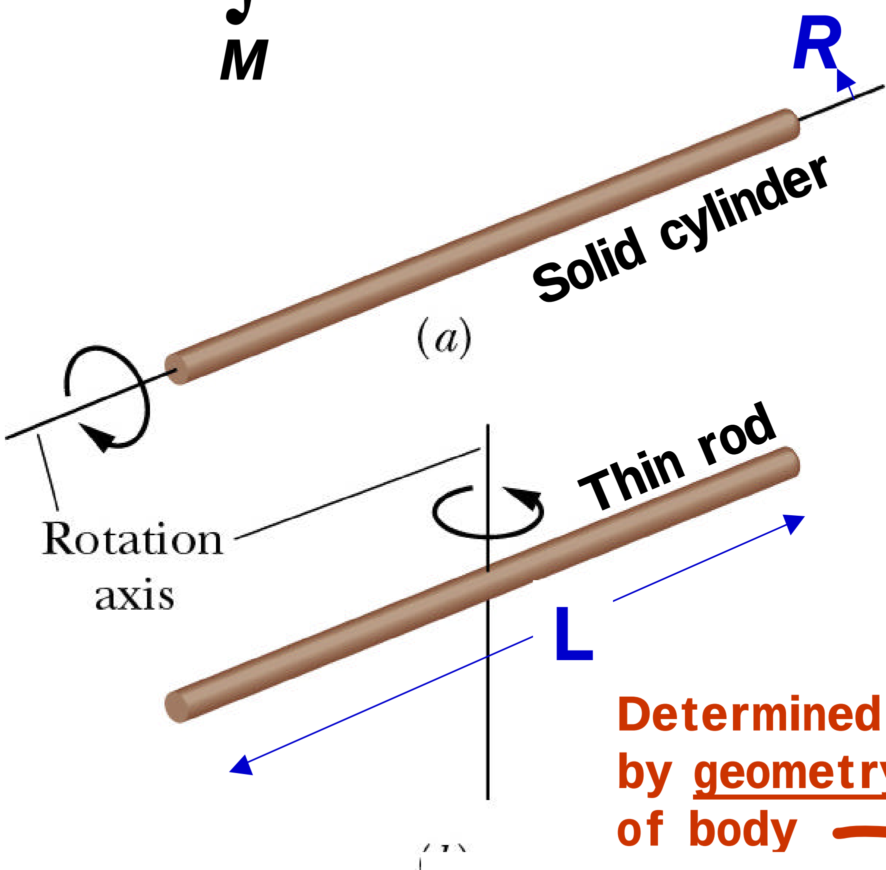


$$I_{end} = m(0)^2 + m(L)^2$$

$$I_{end} = mL^2$$

Moments of Inertia …
continuous rigid bodies

$$I \equiv \int_M r^2 dm$$



- **Rod of mass, M, radius, *R*, and length, *L*   *R*<<*L***
- **Moments of inertia about two axes, (a) and (b)**

$$I_a = \tfrac{1}{2} MR^2$$

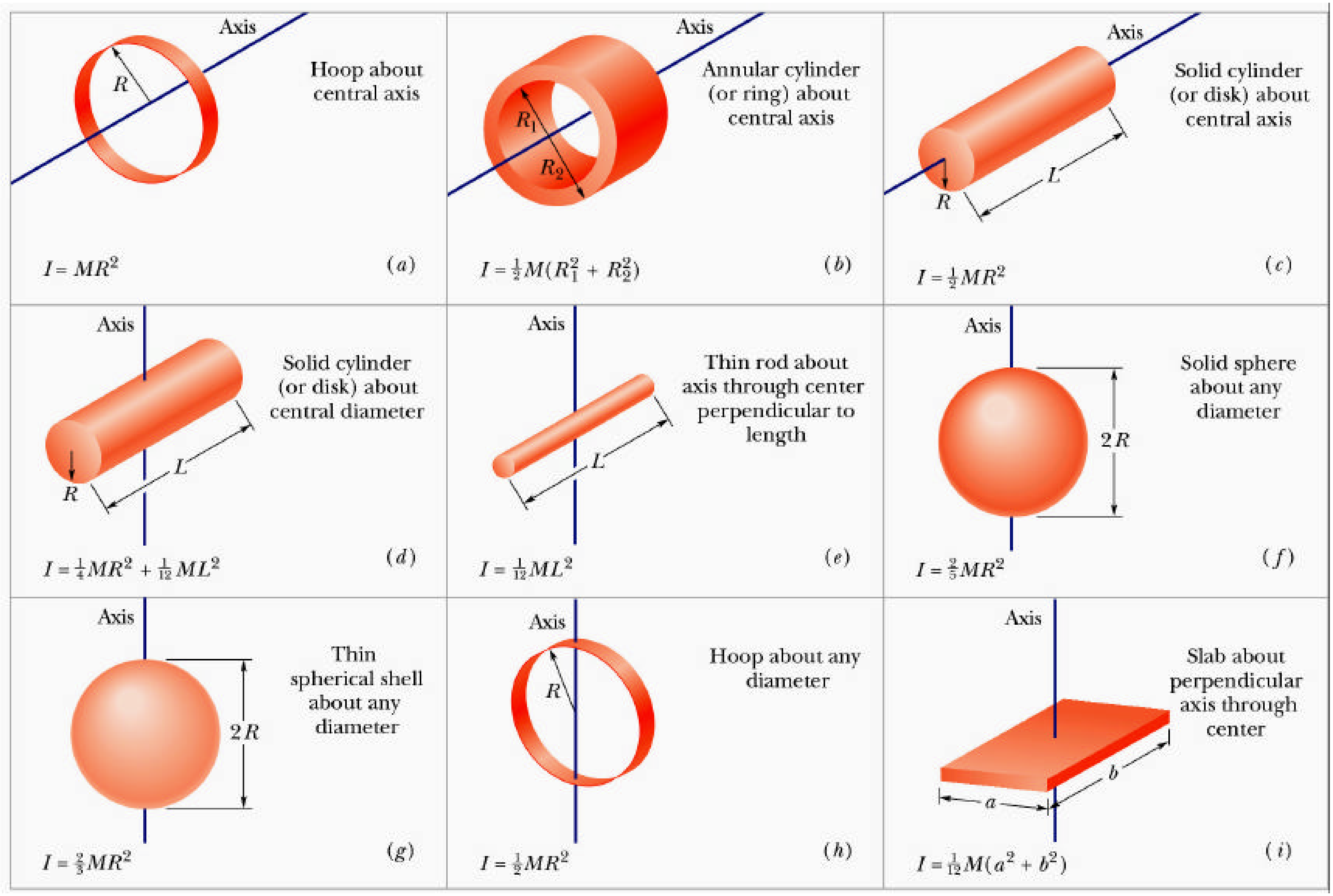
$$I_b = \tfrac{1}{12} ML^2$$

Determined by <u>geometry</u> of body

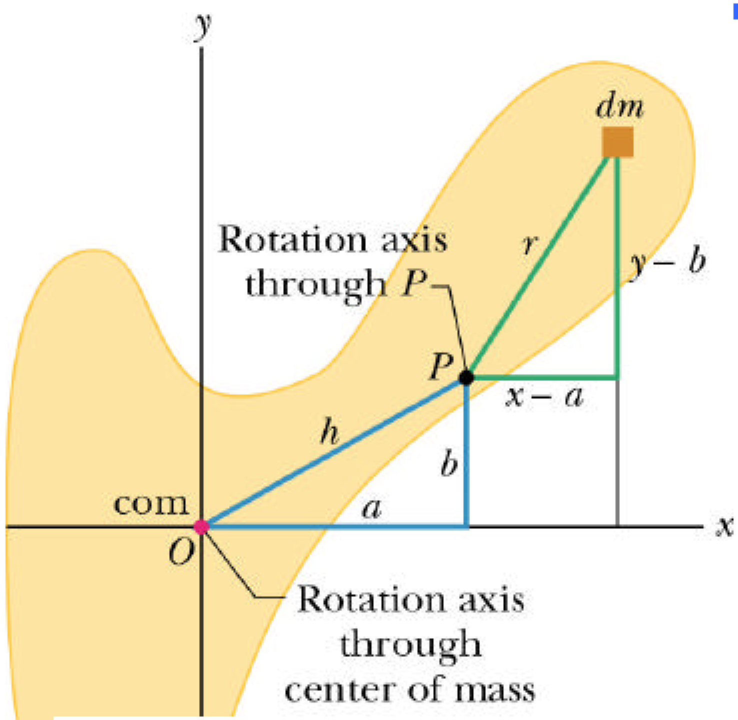
Characteristic <u>dimension</u> of body about rotation axis

$$I \equiv \int_M r^2 dm$$

Typical Shapes – table 11-2



Parallel Axis Theorem



- **Simple technique for finding *I* about a different axis (when know it about axis through the cntr of mss)**
- **Pick origin at c of m with *z*-axis along w**
- **So $x_{com} = y_{com} = 0$**

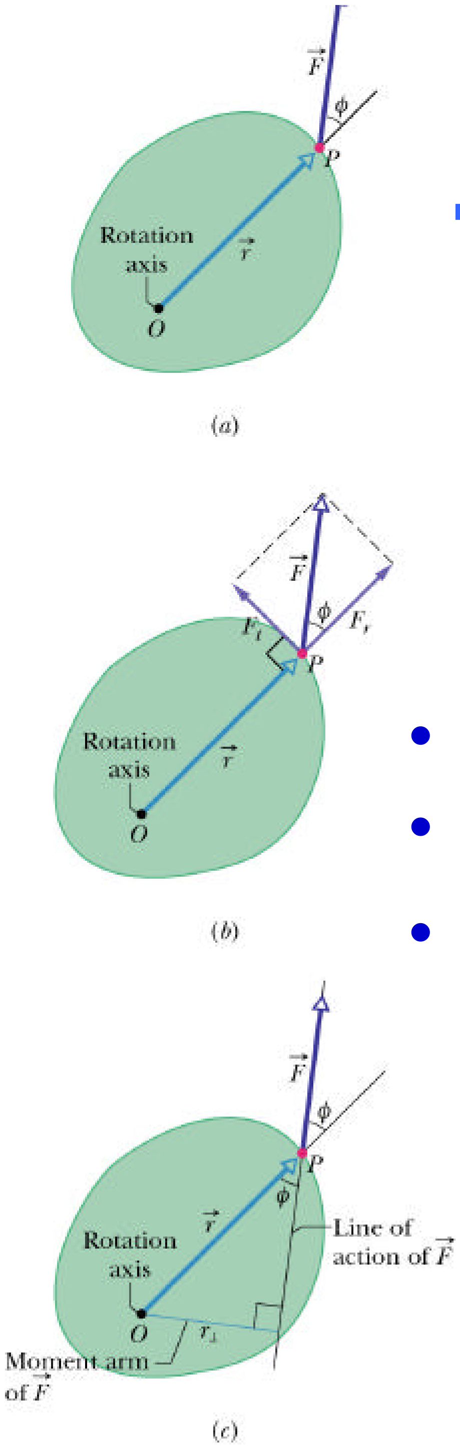
$$I_P = \int r^2 dm = \int \left\{ (x - a)^2 + (y - b)^2 \right\} dm$$

$$x^2 - 2ax + a^2 + y^2 - 2by + b^2$$

$$= \int (x^2 + y^2) dm - 2a \int x dm - 2b \int y dm + \int h^2 dm$$

$$I_P = I_{com} + Mh^2$$

parallel axis theorem.MOV



Torque: Can change Rotational Kinetic Energy

Definition of Torque

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = rF \sin\phi \quad \text{in direction} \perp \; rF \text{ plane}$$

$$\tau = r_{\perp}F = rF_t$$

- **Torque produces Angular Acceleration**
- **Body's rotational energy changes as result**
- **Work done by torque**

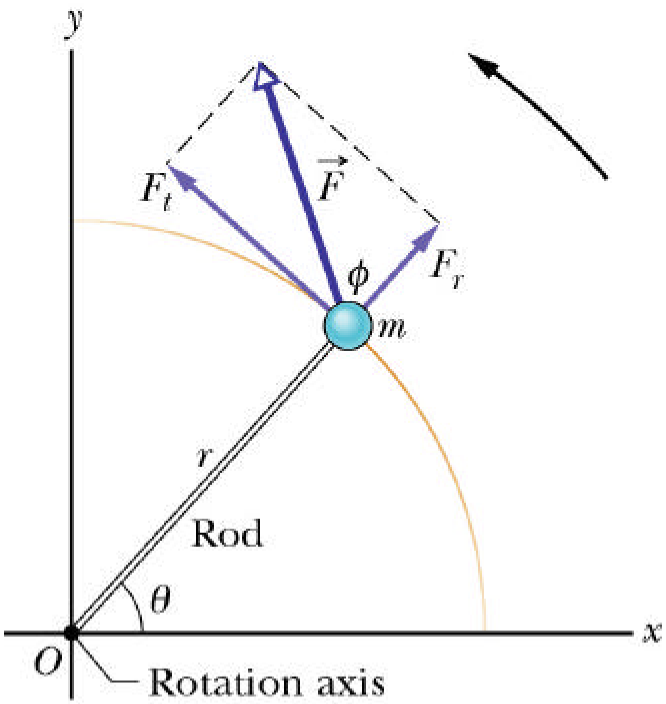
$$\textbf{Work} = F_t s = Fr\theta$$

$$= \tau\theta$$

work from torque.MOV

$$\textbf{Power} = \tau \frac{d\theta}{dt} = \tau\omega$$

Utility of Torque- rotation about fixed axis



$\boldsymbol{F_r}$ component can't move $\boldsymbol{m}$

$$\boldsymbol{F_t} = \boldsymbol{ma_t} = \boldsymbol{mr}\,a$$

$$t = \boldsymbol{rF_t} = \boldsymbol{rmr}\,a$$

$$t = \boldsymbol{I}\,a \quad \text{with} \quad \boldsymbol{I} = \boldsymbol{mr}^2$$

Note directions of $\vec{t}$ and $\vec{a}$

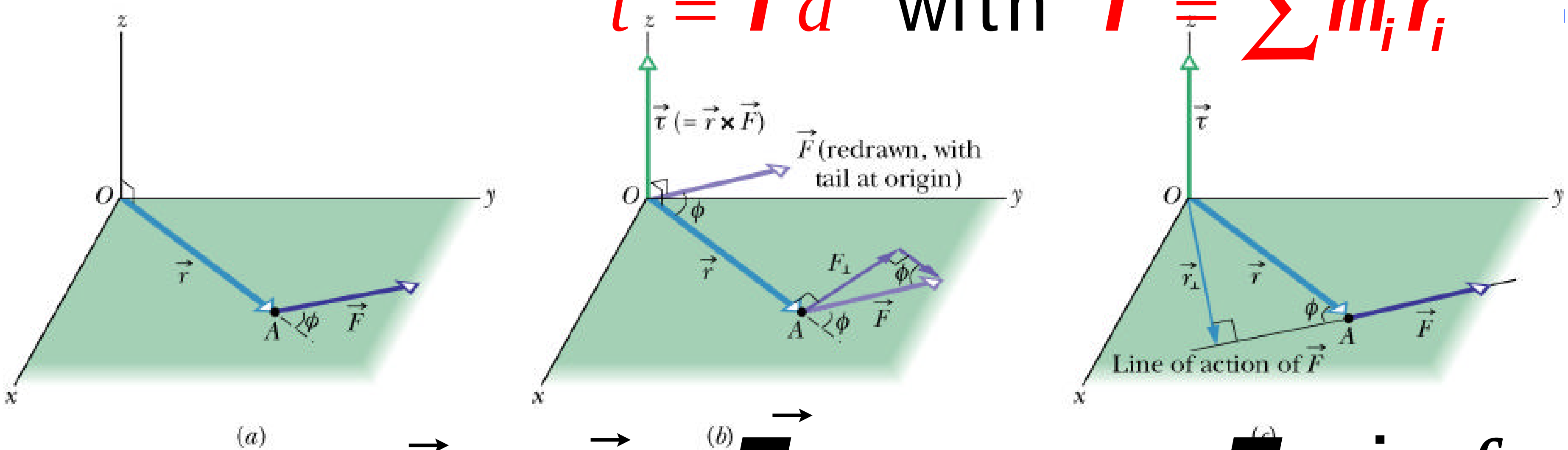
$$\vec{t} = \boldsymbol{I}\,\vec{a} \quad \text{with} \quad \boldsymbol{I} = \sum \boldsymbol{m_i r_i}^2$$

- **Massless rod of length, *r*, with pt-mass, *m*, at end**
- **apply force, *F*, as shown**

- **When net torque on body is zero, there can be no angular acceleration**
  - **If w=0, and no torque, w stays that way**

Specifying Torque

$$\vec{\tau} = I\,\vec{\alpha} \quad \text{with} \quad I = \sum m_i r_i^2$$



$$\vec{\tau} = \vec{r} \times \vec{F} \qquad \tau = rF\sin\phi$$

- **Note the equivalence of the two perspectives**
  - **distance times transverse force component**
  - **lever, or moment, arm times force**
  - **Importance of all seen in pix**

platform- torque.MOV

I larger, *a* smaller

$$\vec{t} = \boldsymbol{I}\,\vec{a} \quad \text{with} \quad \boldsymbol{I} = \sum m_i r_i^{\,2}$$

- **Note that this form is similar to Newton's 2nd Law: *F=ma***
  - **Motion driver (force) produces linear acceleration … smaller if inertia (mass) larger**
- **Torque drives angular acceleration**
  - ***I*, property of the body, is a measure of rotational inertia**

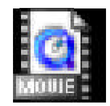
platform- I w bodies1-SNDRS.MOV

platform- I w bodies2-SNDRS.MOV

Torques do Work and Make Energy

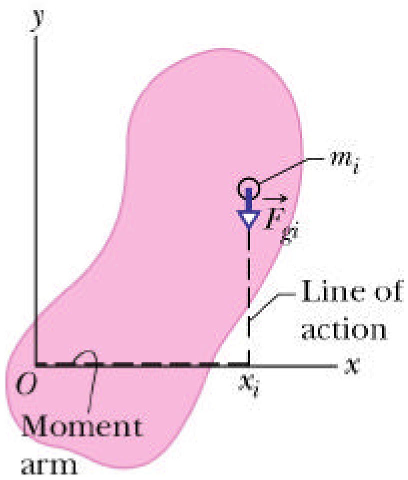
- **Rotational Mechanical Energy**
- **Kinetic Energy result of angular acceleration or Work done**
- **Energy rot pix**
- **Analogy between energies**
  - translational
  - rotational

$$\textit{Work} = F_t s = Fr\theta$$
$$= \tau\theta$$
$$\textit{Power} = \tau\omega$$

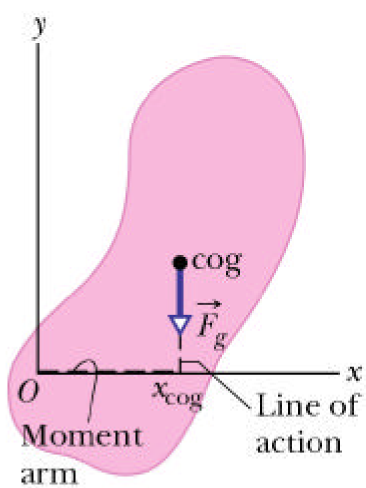


energy-power in rotational motion2.MOV

Center of Gravity

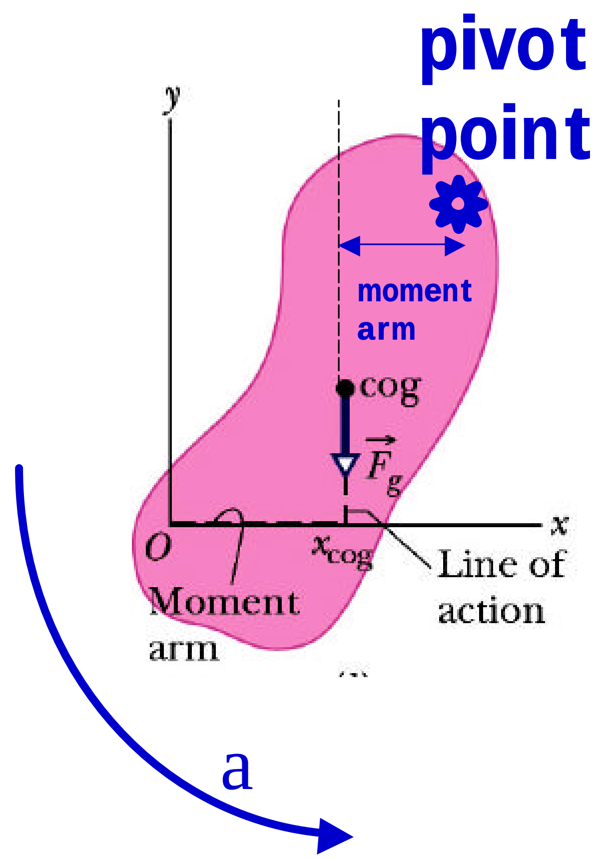


(a)

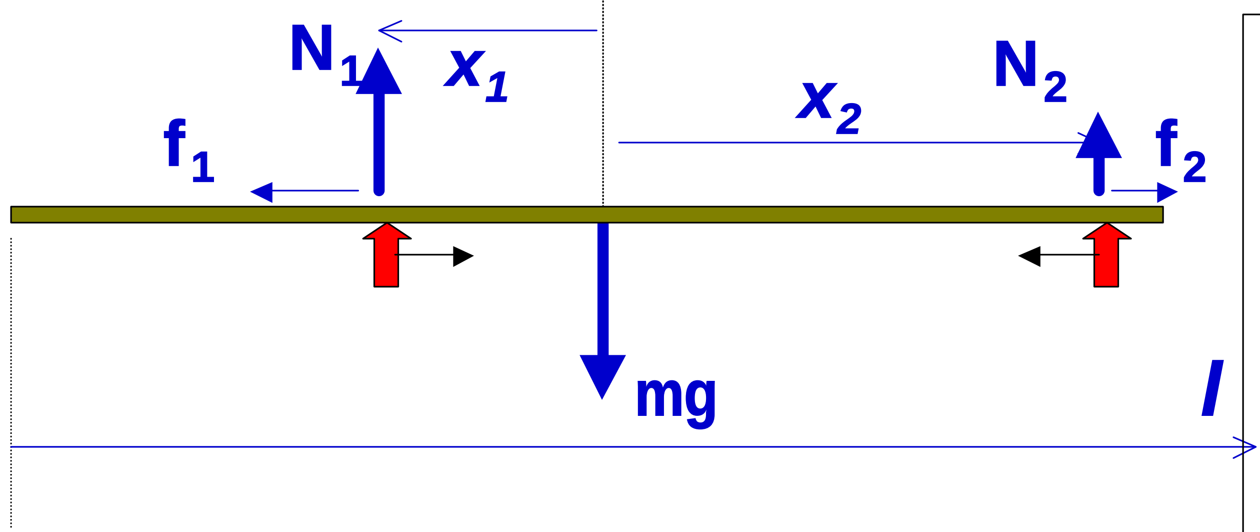


(b)

- **Rigid body feels gravity as if all the body mass concentrated at center of mass**
  - **See proof in text (sec 13-3)**
- **Consequence**
  - **Pivot rigid body and cog must lie <u>below</u> it**
- **Demo**
- **Pivot body at cog and stable (no torque)**



Finding C of M Empirically



- Allow rod to slide across hands
- Works because the forces and torques on the rod must sum to zero

Why does this work??

$$\vec{r}_{com} = \frac{1}{M}\int \vec{r}\, dm$$

1*D* -- like stick

$$m \equiv \frac{dm}{dl} = \frac{M}{L}$$

$$l_{com} = \frac{1}{M}\int_0^L l\, m\, dl = \frac{1}{L}\int_0^L l\, dl = \frac{L}{2}$$

Conclude

- Remember that HW 6 on chapters 10-11 due on Wednesday
- Read through chapters 12-13 … will finish in the next two lectures