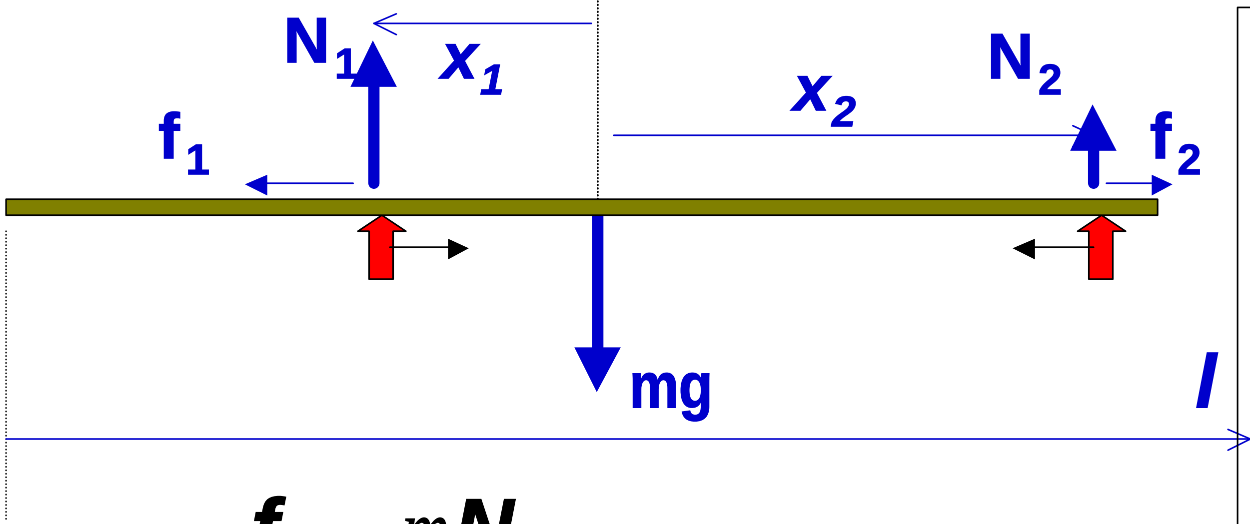
# Lectures

- **Today: Rolling and Angular Momentum in ch 12**
  - Homework 6 due
- **Next time:**
  - Complete angular momentum (chapter 12) and begin equilibrium (chapter 13)
- **By Monday, will post at website**
  - Sample midterm II and solution
- **Midterm II – two weeks from today (11/5)**
  - Covers especially chapters 7 - 13

# Finding C of M Empirically




- Allow rod to slide across hands
- Works because the forces and torques on the rod must sum to zero

$$f = mN$$

$$N_1 + N_2 = mg$$

$$x_1 N_1 = x_2 N_2$$

$$x_2 > x_1$$

$$N_1 > N_2$$

$$f_1 > f_2$$

So slips at 2

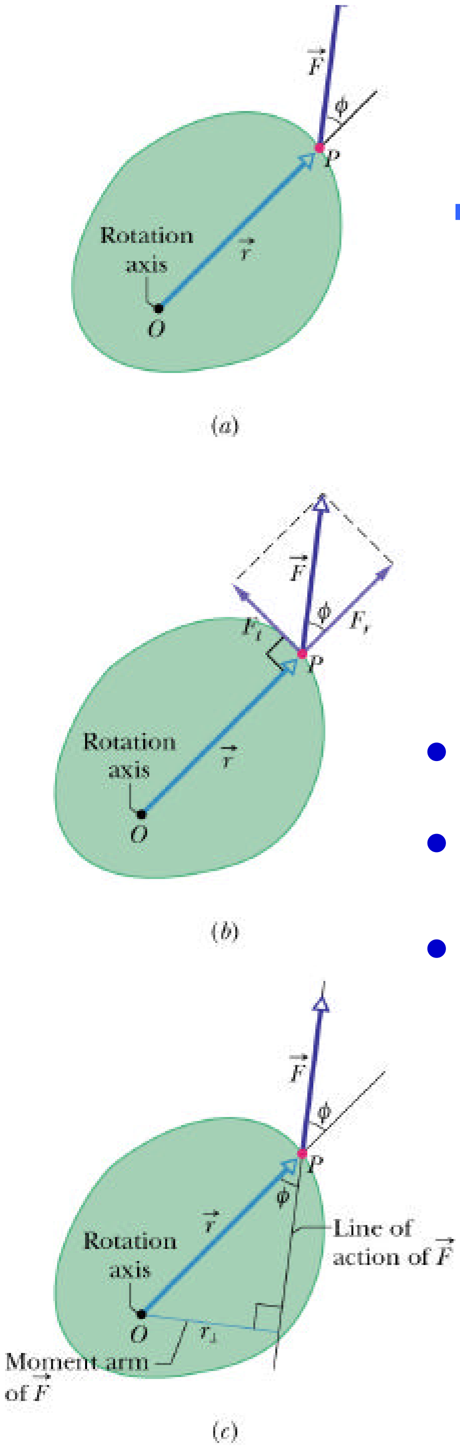
(and vice versa)

$$\vec{r}_{com} = \tfrac{1}{M}\int \vec{r}\, dm$$

1*D* -- like stick

$$m \equiv \frac{dm}{dl} = \frac{M}{L}$$

$$l_{com} = \tfrac{1}{M}\int_0^L l\, m\, dl = \tfrac{1}{L}\int_0^L l\, dl = \frac{L}{2}$$




# Review: Torque-Producing Rotational Kinetic Energy

## Definition of Torque

$$\vec{t} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}}$$

$$t = \boldsymbol{rF} \sin f \quad \text{in direction} \perp \boldsymbol{rF} \text{ plane}$$

$$t = \boldsymbol{r}_{\perp}\boldsymbol{F} = \boldsymbol{rF_t}$$

- **Torque produces Angular Acceleration**
- **Body's rotational energy results**
- **Work done by torque**

$$\boldsymbol{Work} = \boldsymbol{F_t s} = \boldsymbol{Fr}q$$

$$= tq$$

$$\boldsymbol{Power} = tw$$

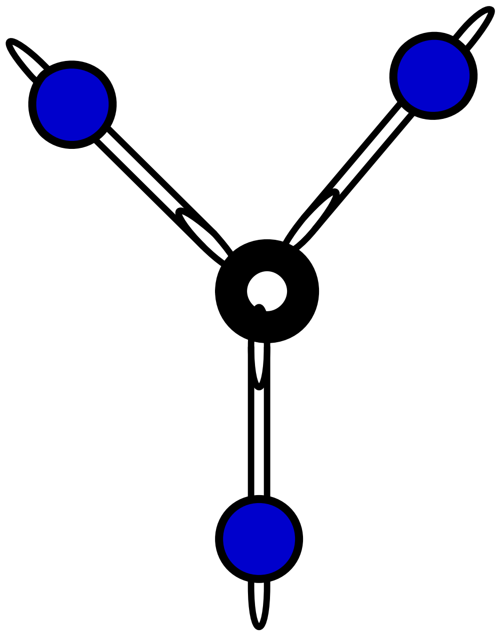
review

# $I$ larger, $\alpha$ smaller

$$\vec{\tau} = I\,\vec{\alpha} \quad \text{with} \quad I = \sum m_i r_i^2$$

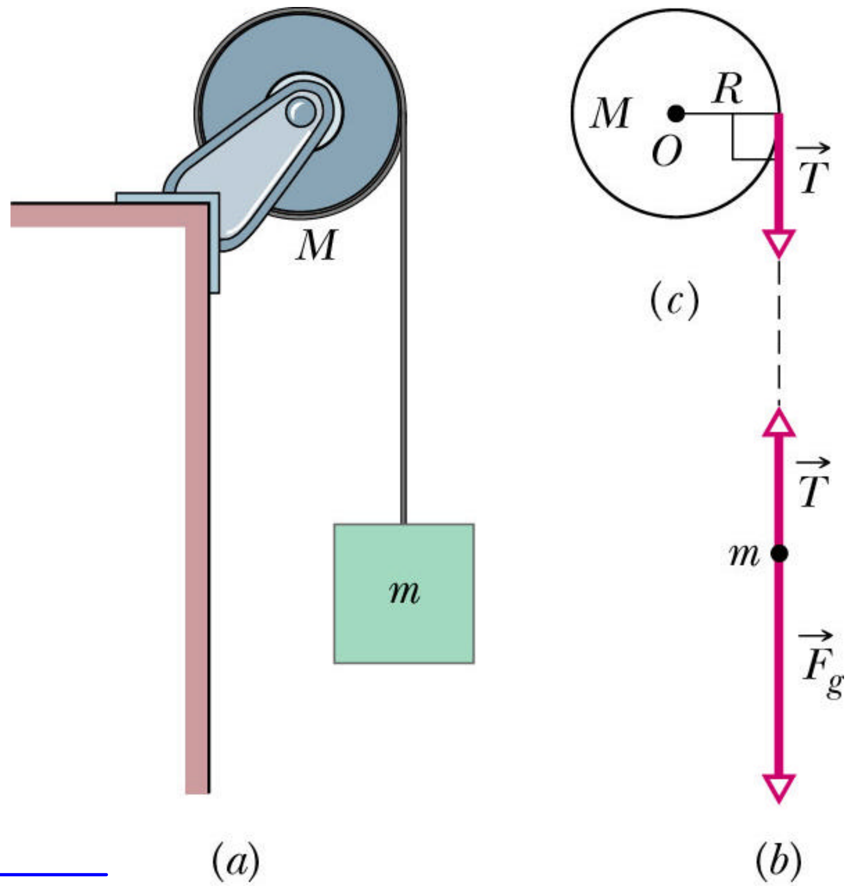
- Note that this form is similar to Newton's 2nd Law: *F=ma*
  - Motion driver (force) produces linear acceleration … smaller if inertia (mass) larger
- Torque drives angular acceleration
  - *I*, property of the body, is a measure of rotational inertia

- DEMO: further masses are from rotation axis
  - Larger I
  - Smaller $\alpha$



# Weight on Wheel
# Sample Prob 11-7

- **Note solution method**
- **Relies on understanding relation between translation of *m* and rotation of *M***



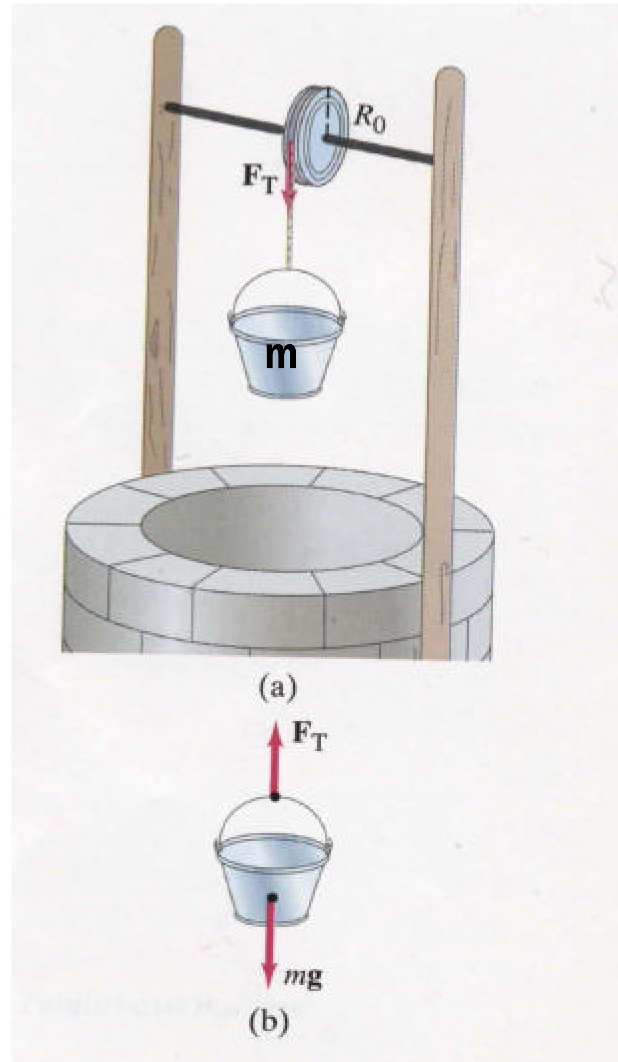

**Answer:**

$$a = g\frac{m}{m + \frac{I}{R^2}} = g\frac{m}{m + \frac{1}{2}M}$$

# Problem

- **Problem of pulley and well**
- **How much potential energy lost when bucket falls 1 meter?**
- **What is *v* after 1 meter?**
- **What is kinetic energy then?**

For m=1.53kg, M=4.0kg

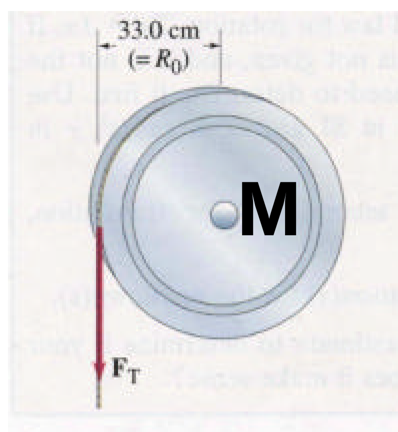



$$U = mgy = 15.0 \text{ Joules}$$

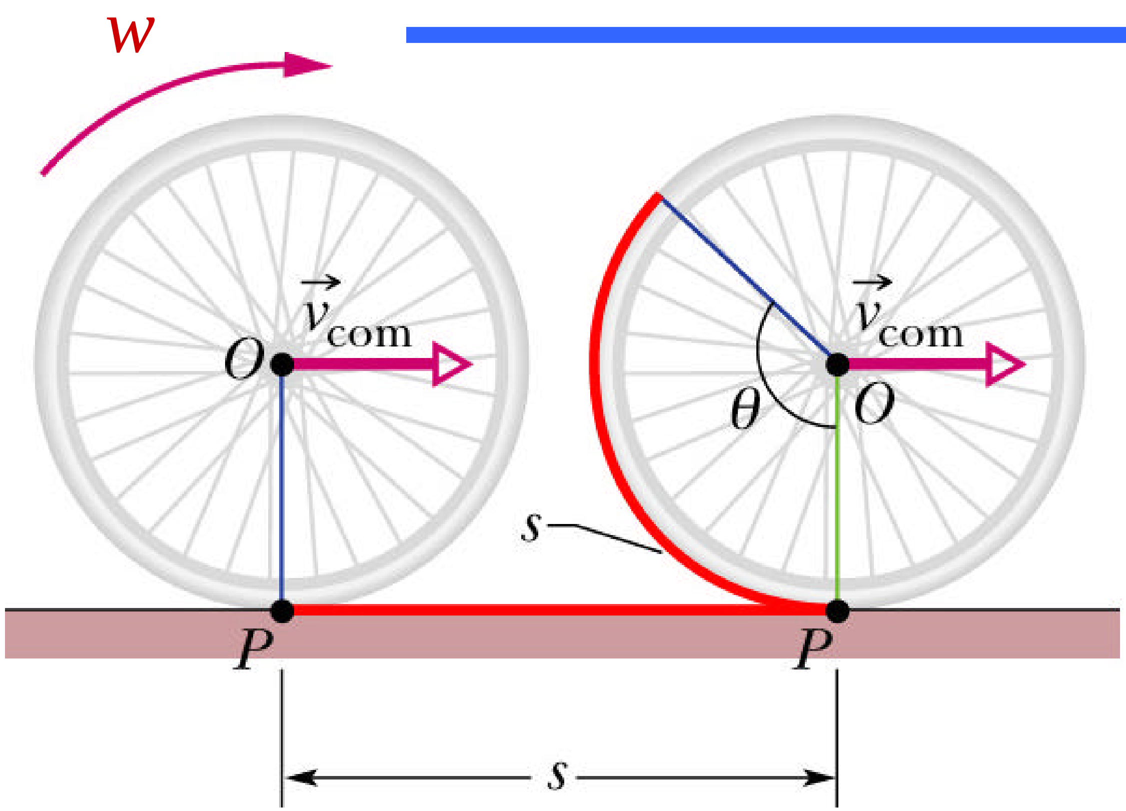
$$a = g\frac{m}{m + \frac{1}{2}M} = 0.433g$$

$$v_{1\,m} = \sqrt{2gy} = 2.91 \text{ m/s}^2$$

$$K = \tfrac{1}{2}(m + \tfrac{1}{2}M)v^2 = 15.0 \text{ Joules}$$



# Rolling, rolling, rolling




$$v = \frac{ds}{dt} \qquad s = R\theta$$

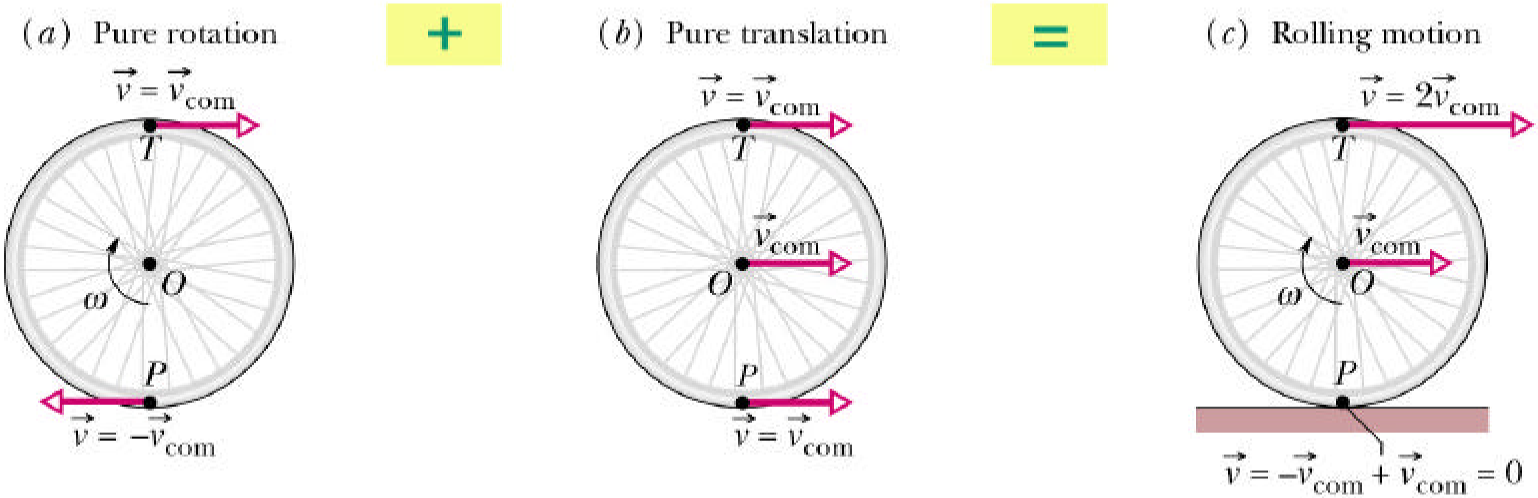
$$\omega = \frac{d\theta}{dt} \qquad v_{com} = R\omega$$

- Wheel com moves distance, *s*, while wheel rotates angle, θ.
- Tells us immediately the velocity of com

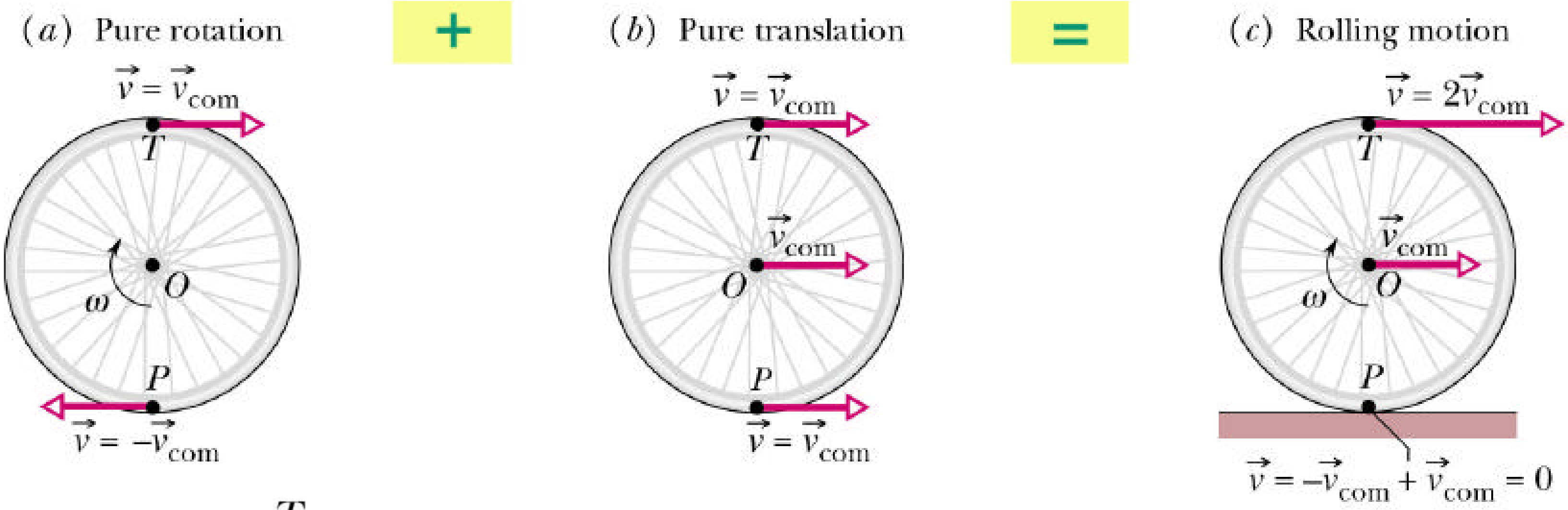
# Critical element of rolling

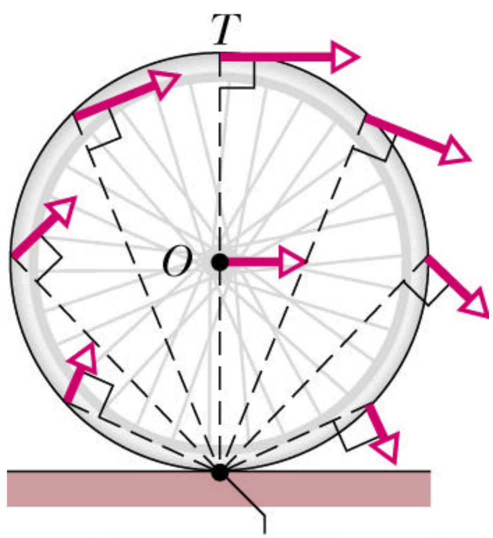
- **The point at contact with the ground is at rest!**
  - Otherwise, that point would slip
  - Implies that static friction is relevant when done properly

$$v_{com} = R\omega$$



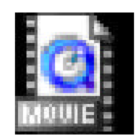
# Can view as rotation about pt of contact



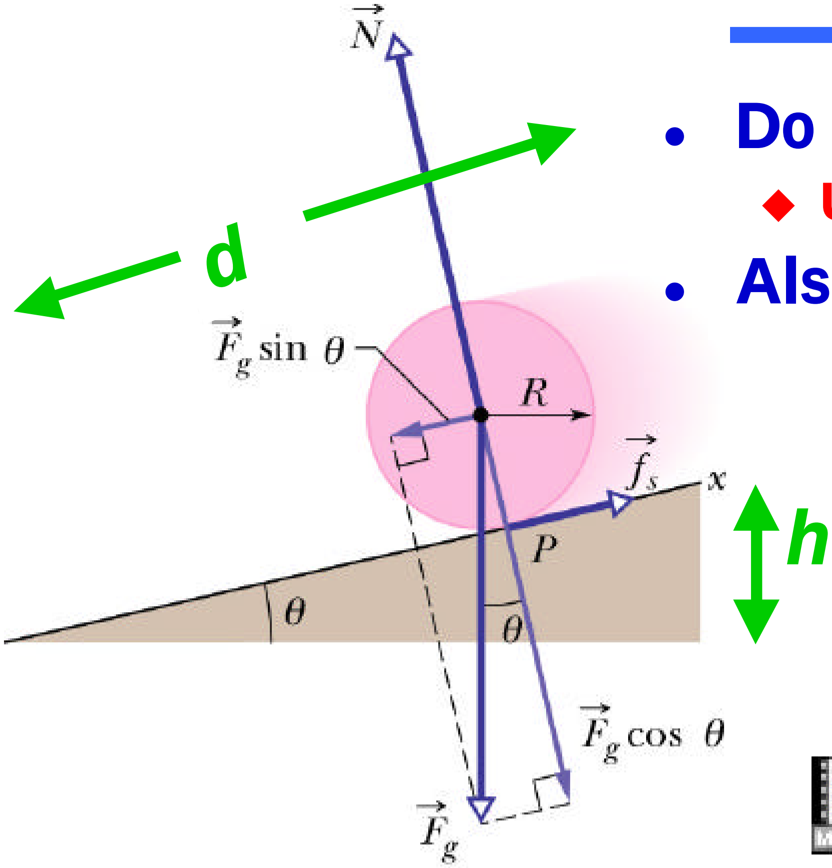

Rotation axis at *P*

$$v_{com} = R\omega$$




points on rolling wheel.MOV

# Rolling Ball (or cylinder …)




- **Do with forces (as text: eqn 12-10)**
  - **Use const acc equation to give speed**
- **Also do by energy (flick –49s)**

$$a_{com} = \frac{g \sin\theta}{1 + \frac{I}{MR^2}} \quad \text{down the ramp}$$

$$v_{com}^2 = 2ad$$

$$v_{com} = \sqrt{\frac{2gh}{1 + \frac{I}{MR^2}}}$$

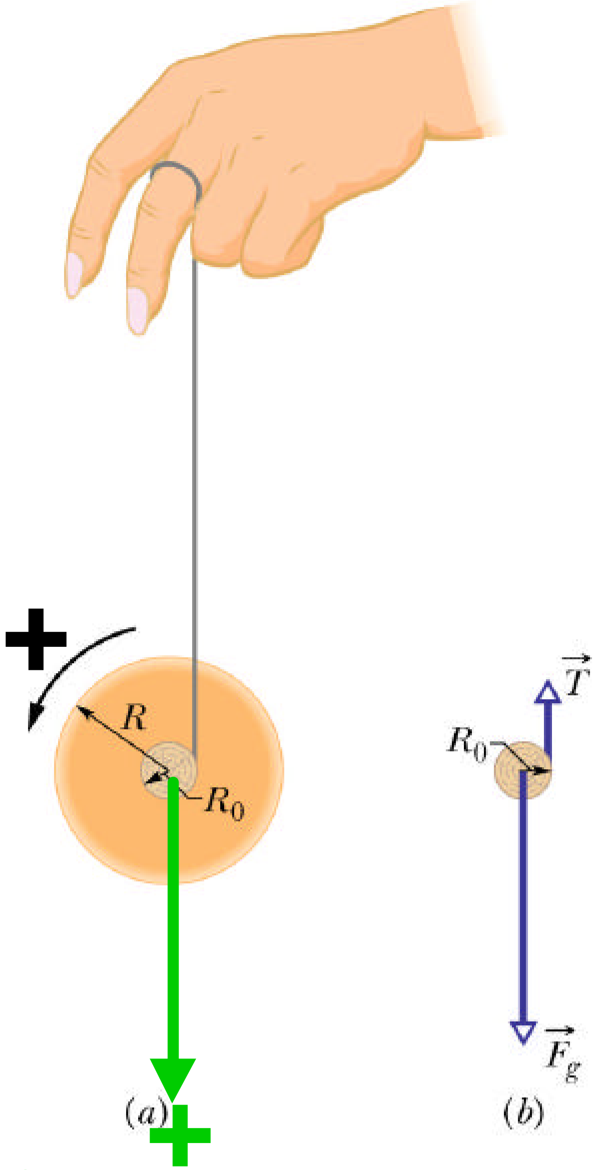
balls rolling down plane.MOV

**Determined by <u>geometry</u> of body**

$$I_{uniform\ can} = \tfrac{1}{2} MR^2$$

**So v at bottom depends only on <u>shape and mass distribution</u> of rolling body**

# Yo-yo: Rolls down String
# See section 12-4

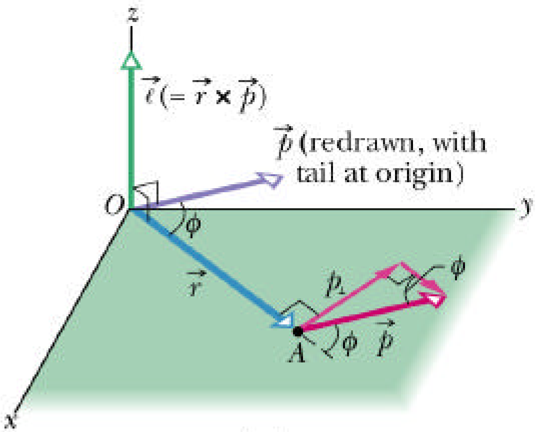



$$a = \frac{g}{1 + \frac{I_{com}}{MR_0^2}}$$

- **First define consistent conventions**
  - Sense of +ve rotation
  - Sense of +ve translation
- **Torques about c of m**
- **Forces**
- **Solve**

# New concept: Angular Momentum of a Particle Mass

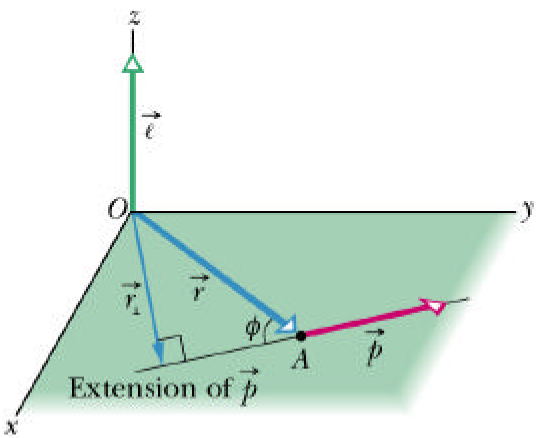
$$\vec{p} = m\vec{v}$$



(a)

$$\vec{l} \equiv \vec{r} \times \vec{p} \quad \text{so} \quad l = rp \sin \phi$$

angular momentum vector.MOV



(b)

**Angular momentum arises in many cases**

- **single particle as shown**
- **rotating rigid body (later)**

$$\vec{\tau} = \frac{d\vec{l}}{dt}$$

## Note that right hand side is also $I\alpha$

# Prove Torque-Ang Mom Relation

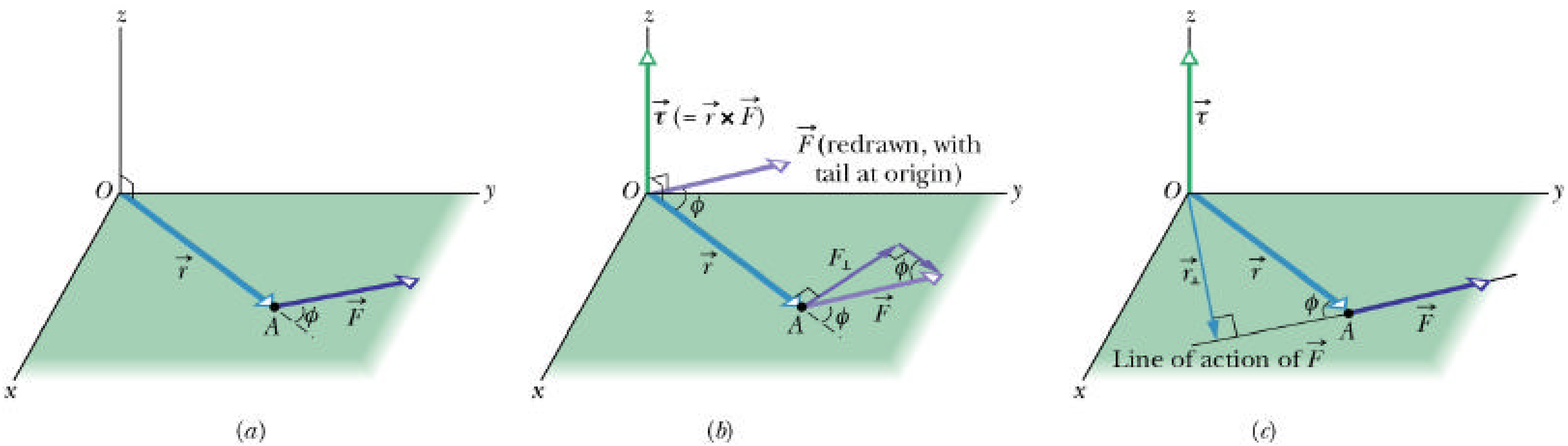
$$\frac{d\vec{l}}{dt} = \frac{d}{dt}(\vec{r} \times m\vec{v})$$

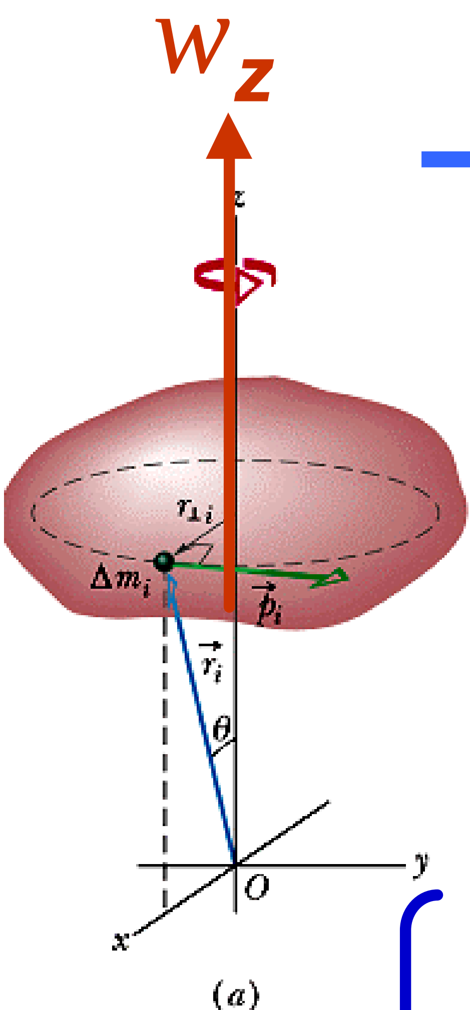
$$\frac{d\vec{l}}{dt} = m\left[\vec{r} \times \frac{d\vec{v}}{dt} + \vec{v} \times \frac{d\vec{r}}{dt}\right]$$

(annotations: $\frac{d\vec{v}}{dt}$ = **a**; $\frac{d\vec{r}}{dt}$ = **v**; the term $\vec{v} \times \frac{d\vec{r}}{dt}$ is struck out)

$$\frac{d\vec{l}}{dt} = \vec{r} \times m\vec{a} = \vec{r} \times \vec{F} = \vec{\tau}$$

- Here F is the total net force acting on m
- $\tau$ is the net torque






# Body Rotating about a Fixed Axis

$$L_z = I w_z$$

For body rotating about fixed axis

$$\vec{L} = I \vec{w}$$

proof

$$L_z = \sum I_{iz} = \sum r_{\perp i}(\Delta m_i)(w_z r_{\perp i})$$

$$L_z = w_z \sum r_{\perp i}^2 \Delta m_i$$

$$L_z = I_z w_z$$

# Conservation of Angular Momentum

$$\vec{\tau}_{\substack{ext\\net}} = \frac{d\vec{L}}{dt}$$

If $\vec{\tau}_{\substack{ext\\net}} = 0$,

$$\frac{d\vec{L}}{dt} = 0$$

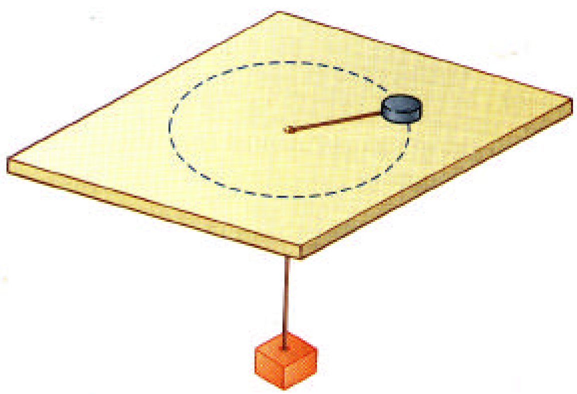
$\vec{L}$ is constant

$$\vec{L}_f = \vec{L}_i$$

- Important Conservation Rule
  - if isolated system (no ext. torque)
- Reflects symmetry of Universe' physical laws wrt orientation in space
- Many examples of its application
  - Text
  - Lecture

# Demos of Ang Mom Conservation

- **Rotation with weights (demo)**
  - Spin with weights outstretched
  - Establish angular velocity
  - pull weights in (reduce moment of inertia)
  - change angular velocity
- **Puck on air table with force along axis**

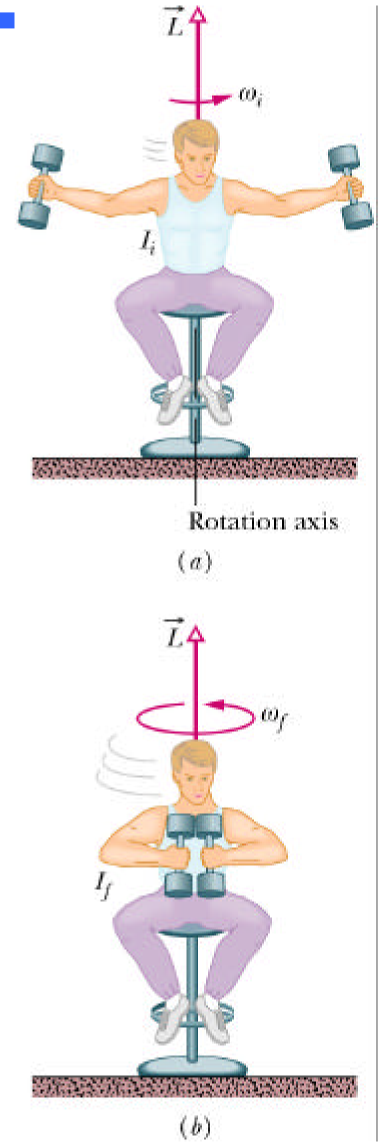


cons ang mom puck astronaut.MOV

$$L_f = L_i$$

$$I_f w_f = I_i w_i$$

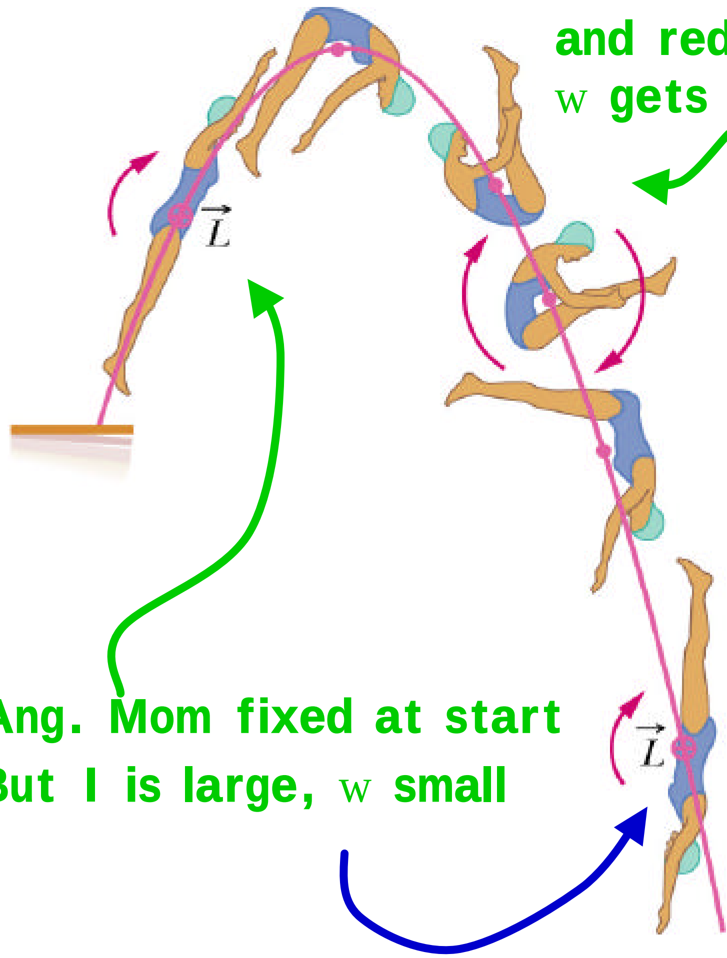
$$w_f = \frac{I_i}{I_f} w_i$$



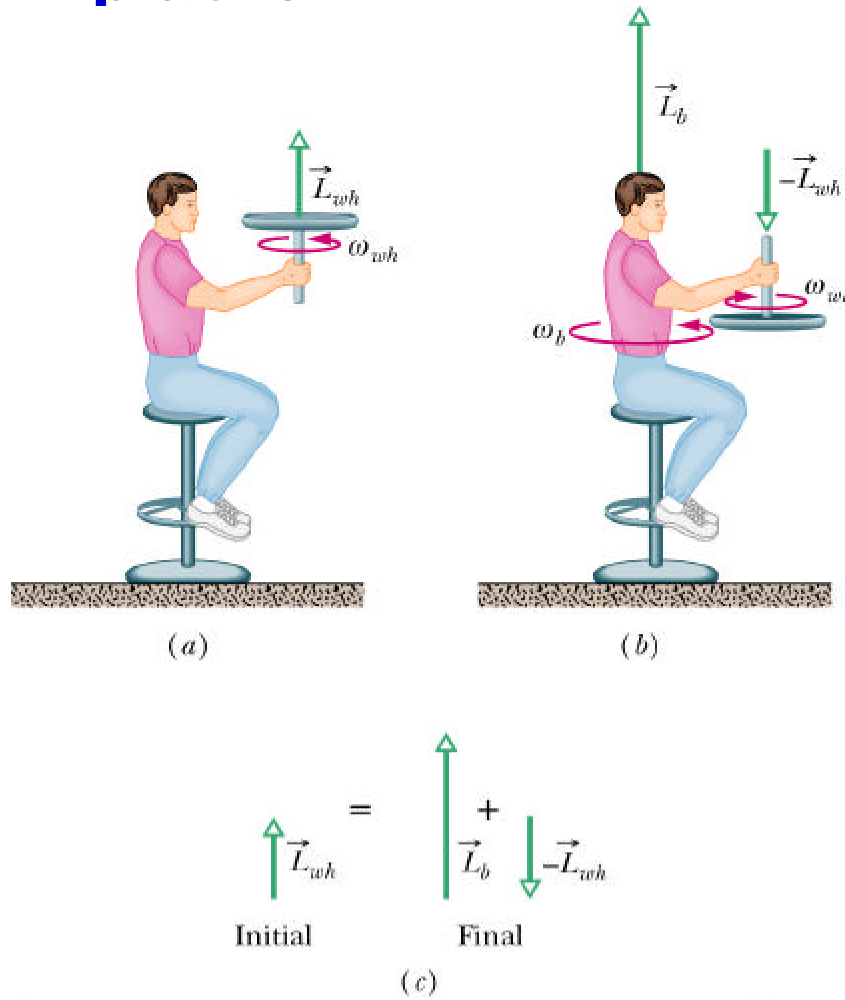
# Examples from text

- **Demos with rotating platform**

**Diver tucks in and reduces I, w gets large**




**Ang. Mom fixed at start**
**But I is large, w small**



# Keep in Mind

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- Next time:
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- By Monday, will post at website
  - Sample midterm II and solution
- Midterm II – two weeks from today
  - Covers especially chapters 7 - 13