Today – lecture 15

- A few reminders and culture items on angular momentum
- Chapter 13 on equilibrium
  - Mainly do some problems involving requirements for systems to stay in equilibrium at rest (*ie*, don't move)
  - Ideas of forces on materials
    - Tension and Compression
    - Shear
- Remember HW 7 due Wednesday

review

Conservation of Angular Momentum

$$\vec{\tau}_{\substack{ext\\net}} = \frac{d\vec{L}}{dt}$$

If $\vec{\tau}_{\substack{ext\\net}} = 0$,

$$\frac{d\vec{L}}{dt} = 0$$

$\vec{L}$ is constant

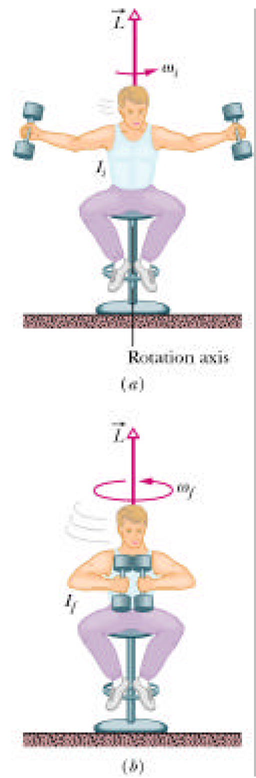
$$\vec{L}_f = \vec{L}_i$$

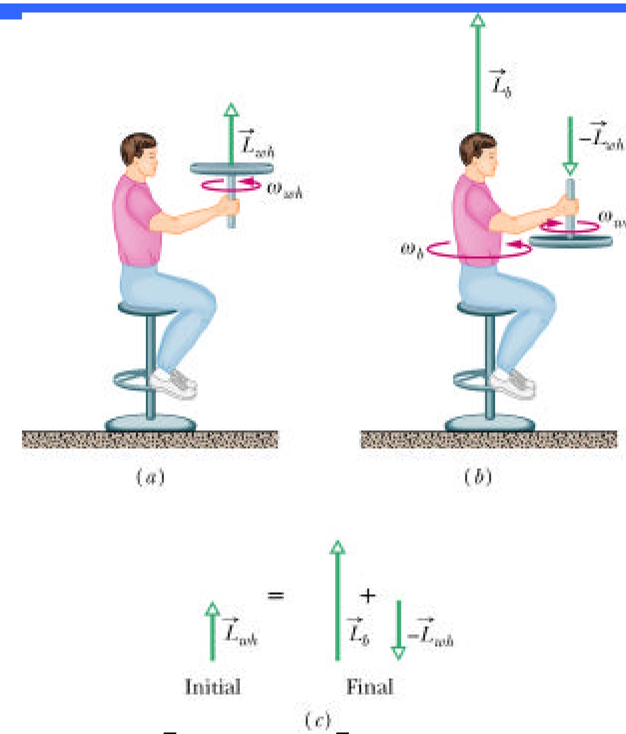
- Important Conservation Rule
  - if isolated system (no ext. torque)
- Reflects symmetry of Universe' physical laws wrt orientation in space
- Many examples of its application
  - Text
  - Lecture

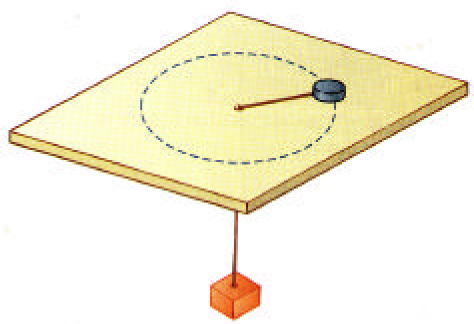
Demos of Ang Mom Conservation

review

- **Rotation with weights (demo)**
  - Spin with weights outstretched
  - Establish angular velocity
  - pull weights in (reduce moment of inertia)
  - change angular velocity
- **Puck on air table with force at axis**







cons ang mom puck astronaut.MOV

$$L_f = L_i$$

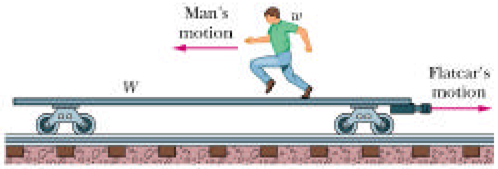
$$I_f w_f = I_i w_i$$

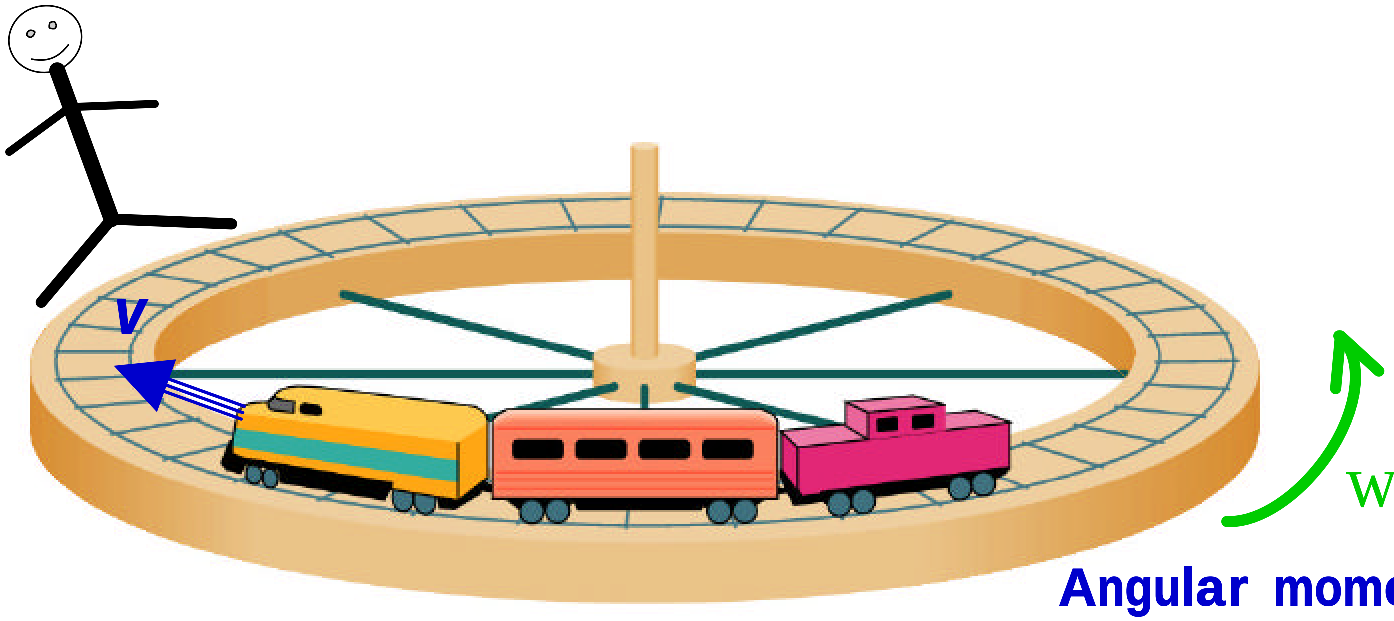
$$w_f = \frac{I_i}{I_f} w_i$$

Prob 12-45 (not assigned)

- Initially all at rest
- Entire track (*M*) free to rotate.
- Treat track as all *M* in hub, radius *R*
- Train (*m*) moves *v* relative to track
- How fast does track rotate?

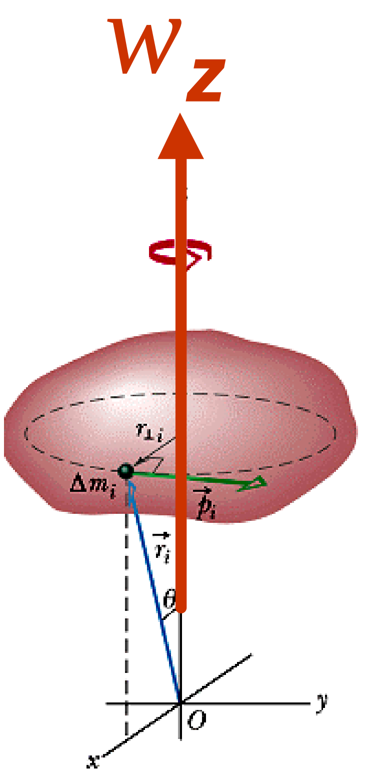
Linear mom conservation





Angular momentum conservation

Why all the complicated algebra (vector products …) ?



Definition of Torque

$$\vec{t} = \vec{r} \times \vec{F}$$

$$\vec{p} = m\vec{v}$$

$$\vec{l} \equiv \vec{r} \times \vec{p}$$

$$\vec{t} = \frac{d\vec{l}}{dt}$$

$$L_z \overset{(a)}{=} I\, w_z$$

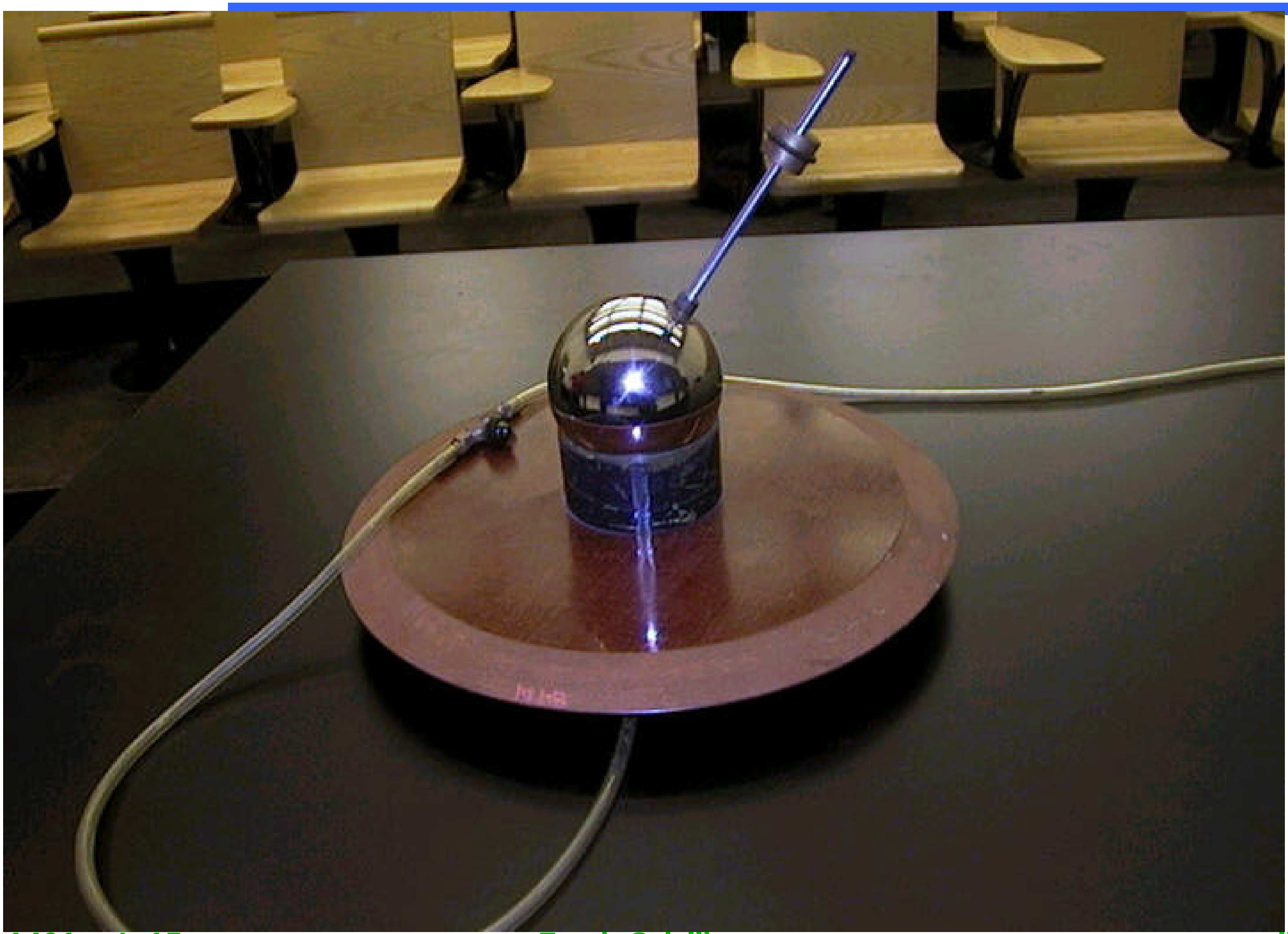
For body rotating about **fixed** axis

$$\vec{L} = I\, \vec{w}$$

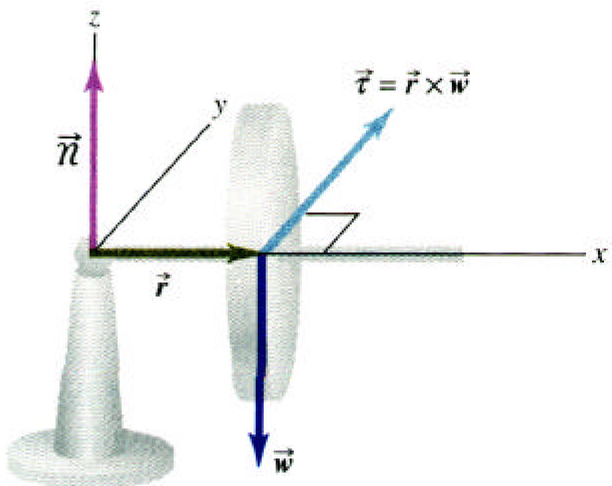
$$t_{com} = I_{com}\, a$$

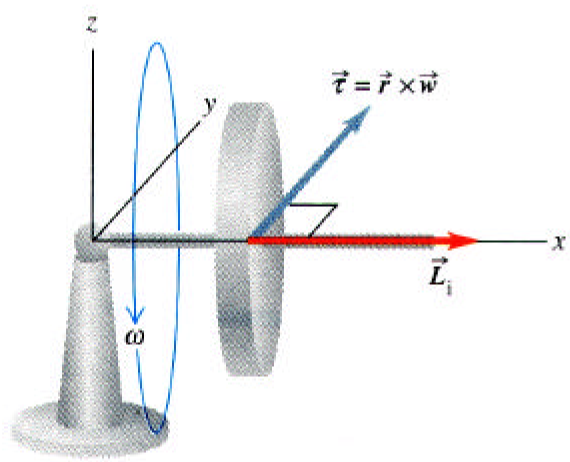
- We don't really need to use cross-products here … more examples in a few minutes
- But important for more complicated problems involving rotations about moving axes

Gyroscope DEMO (culture)



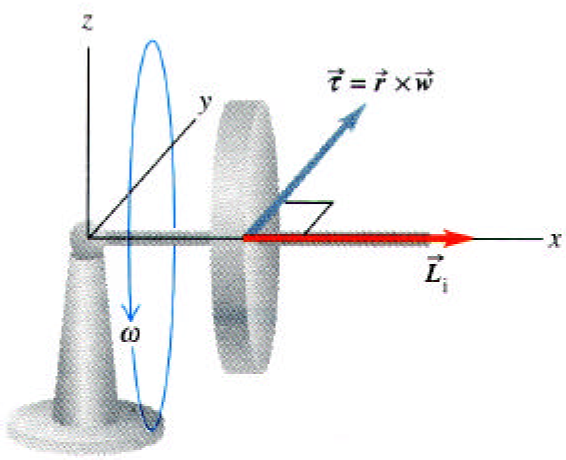
Advanced Concepts in Angular Momentum (Cultural)

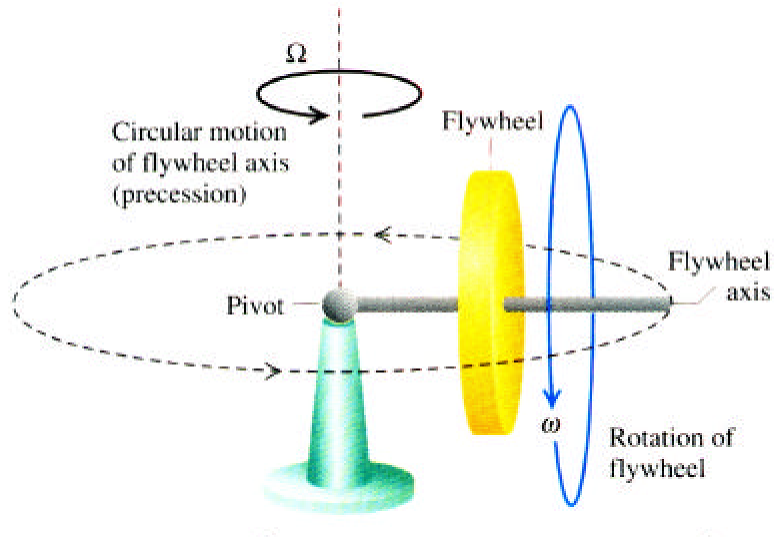


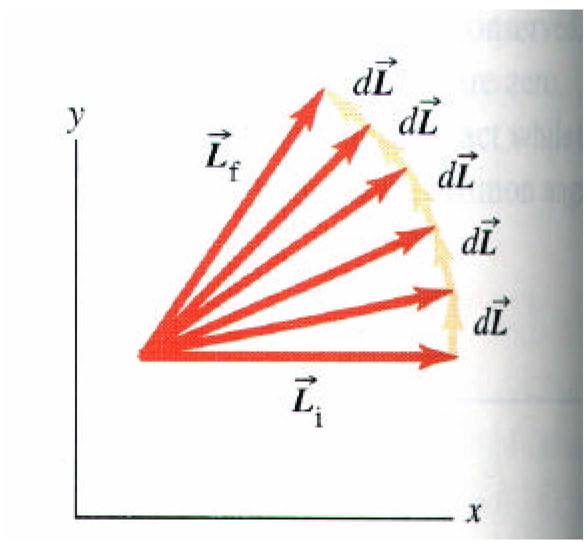


- **Gyroscope:**
  - without torques, keeps pointed in same direction
  - with torque: must change angular momentum
- **The torque (from gravity) must make the angular momentum change**
  - but the torque (normal to angular momentum) cannot change magnitude, only direction

Precession of Gyroscope (cultural)





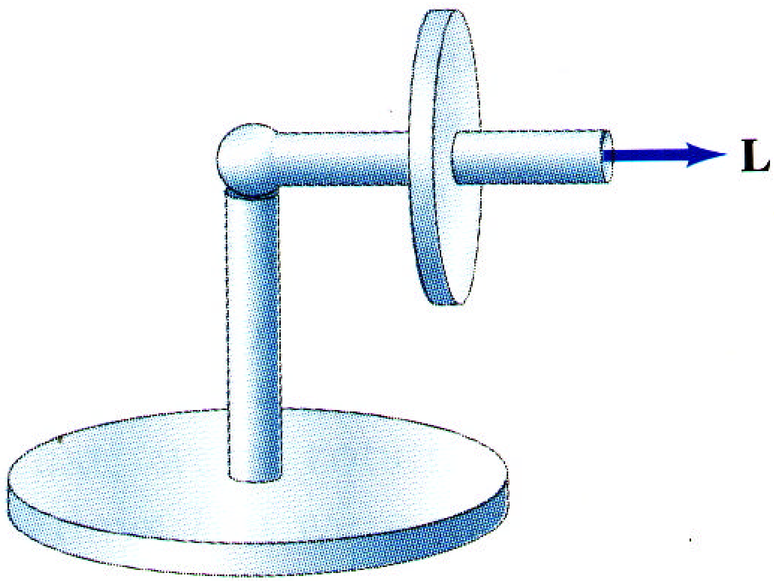


$$\vec{t} = \frac{d\vec{L}}{dt} = \vec{\Omega} \times \vec{L}$$

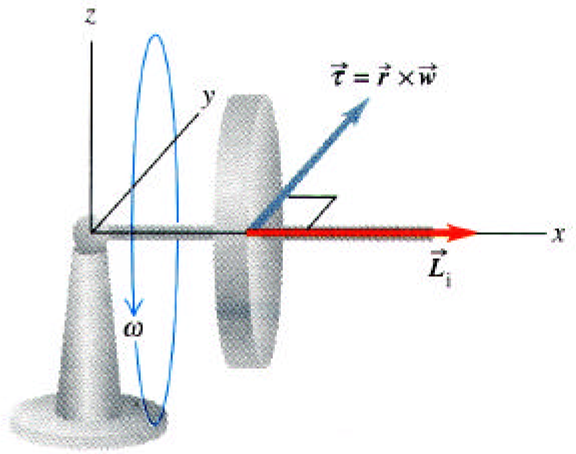
$$wr = \Omega I w$$

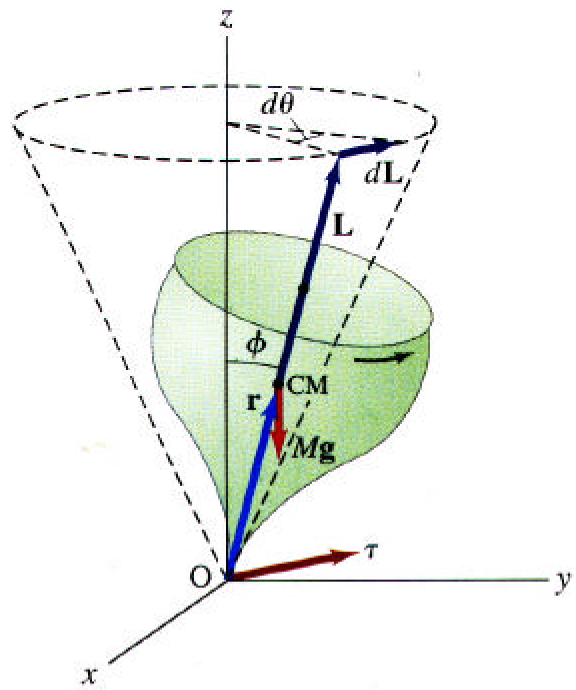
$$\Omega = \frac{wr}{I w}$$

Gyroscopes and Tops (culture)



- Examples of more complicated rotations
- We work out simplest case: precession





Equilibrium (Ch 13)

- System remains at rest if
  - All forces, in all directions, sum to zero
  - All torques, about any axis, sum to zero
- Important mechanism for finding what the real forces are that act on system
  - or whether a system will remain (not fall apart)

$$\sum \vec{F}_i = 0$$

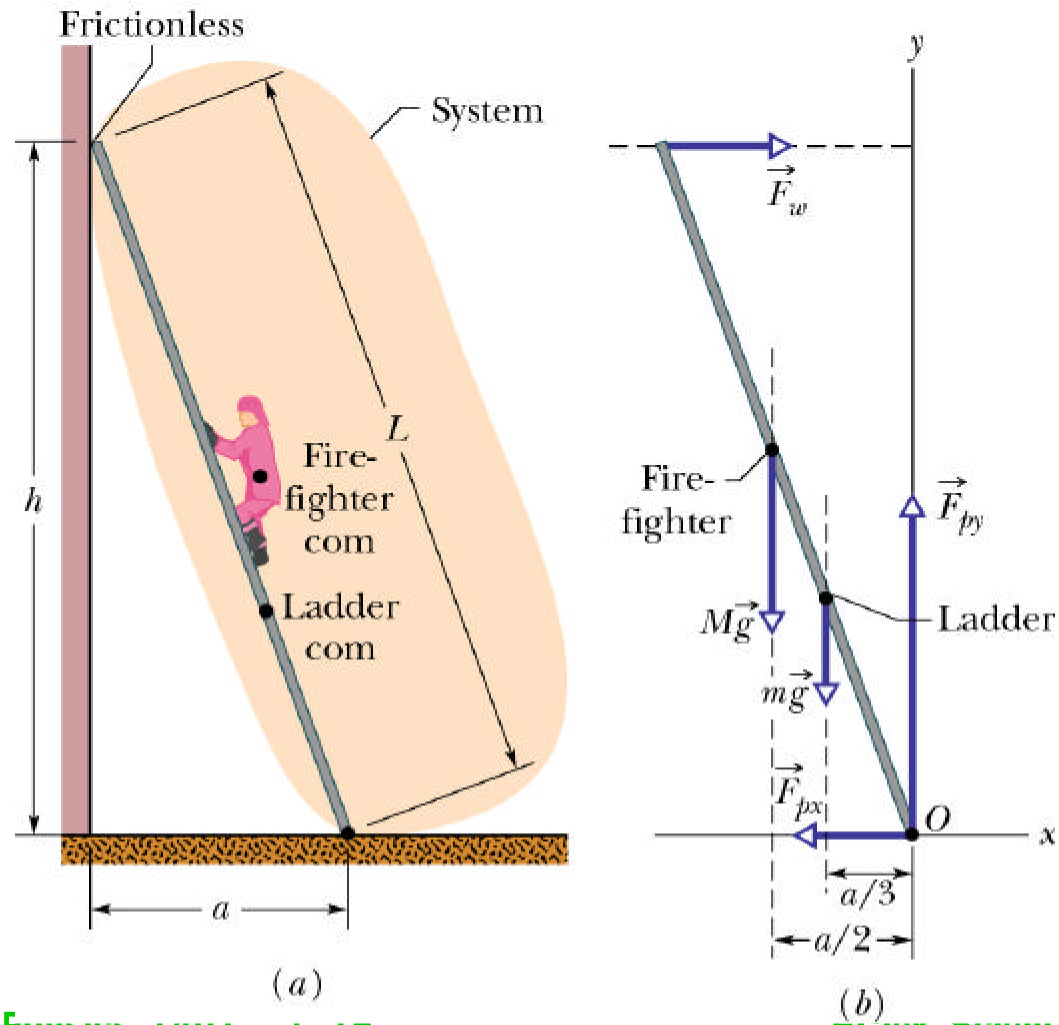
- for all external forces on system

$$\sum \vec{t}_i = 0$$

- torques about <u>any</u> axis

Sample Problem 13-2

Forces from wall and ground?



$$F_{px} = F_w$$

$$F_{py} = (M + m)g$$

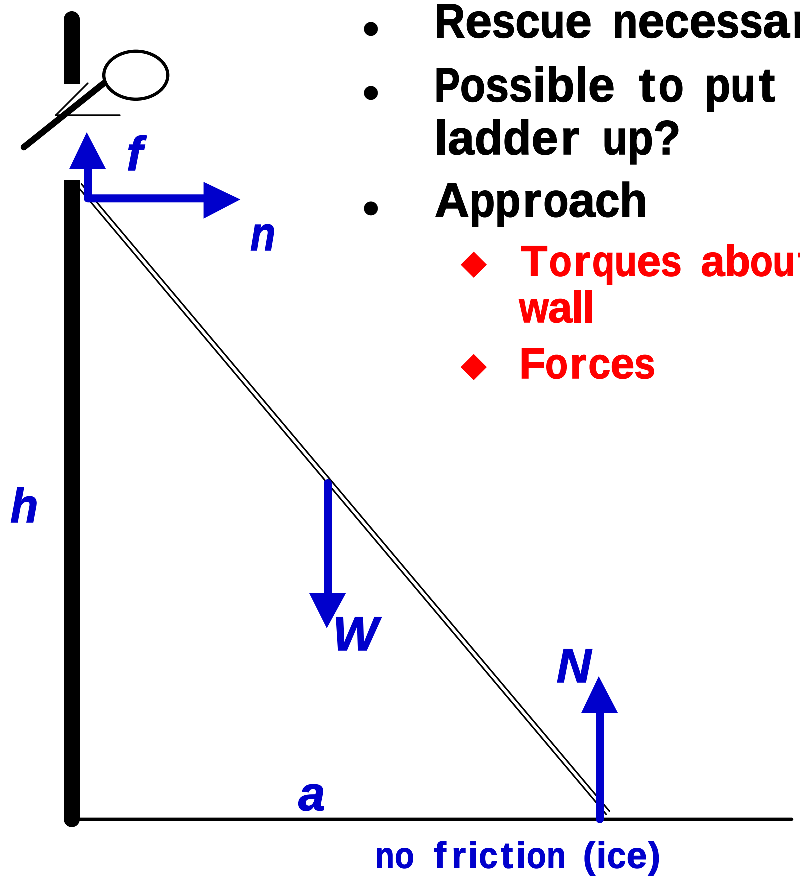
$$F_w h = Mg\,\frac{a}{2} + mg\,\frac{a}{3}$$

$$F_{px} = \frac{ga}{h}\left[\frac{M}{2} + \frac{m}{3}\right]$$

1. Make good figure
2. Balance forces
3. Balance torques

What if no friction on ground also?

Realistic firefighter rescue problem



- Icy pavement
- Rescue necessary
- Possible to put ladder up?
- Approach
  - Torques about wall
  - Forces

$$Na = W\frac{a}{2} \quad \text{or} \quad N = \frac{W}{2}$$

$$f + N = W \quad \text{or} \quad f = \frac{W}{2}$$

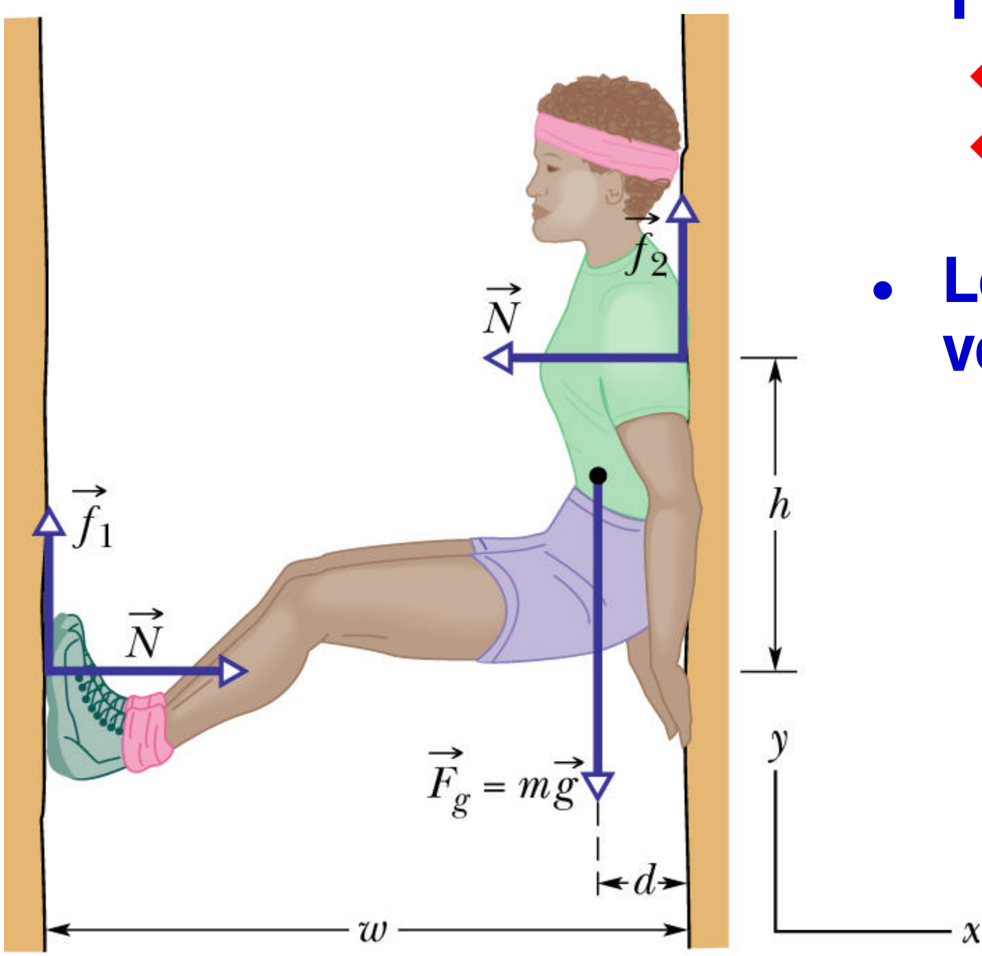
$$n = 0$$

$$\therefore m \equiv \frac{f}{n} \to \infty$$

Answer: either

- use a bracket at window
- or remove ice …

Sample Prob 13-4



- **Given coefficents of static friction, what is required**
  - **Normal force?**
  - **Vertical separation between feet and back?**
- **Look at situation just on the verge of slipping**

$$f_1 = m_1 N \qquad f_2 = m_2 N$$

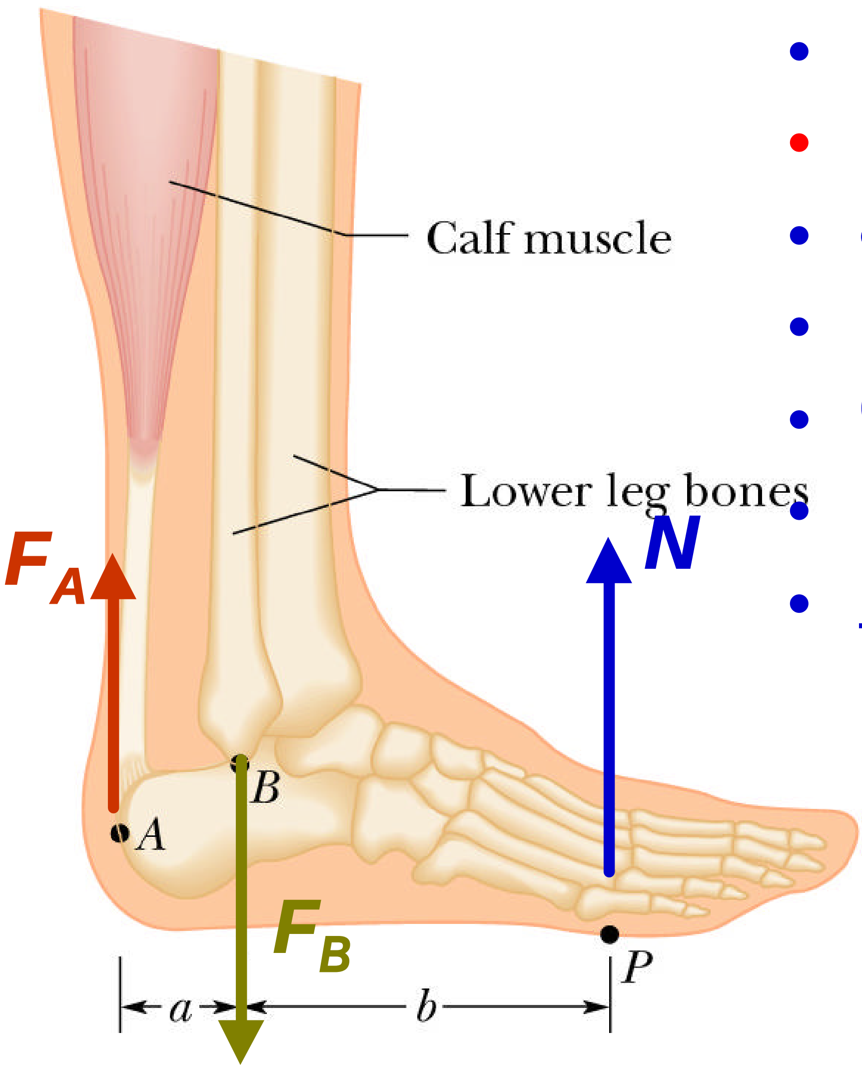
$$f_1 + f_2 = mg$$

$$N = \frac{mg}{m_1 + m_2}$$

$$hN + f_2 w = mg(w - d)$$

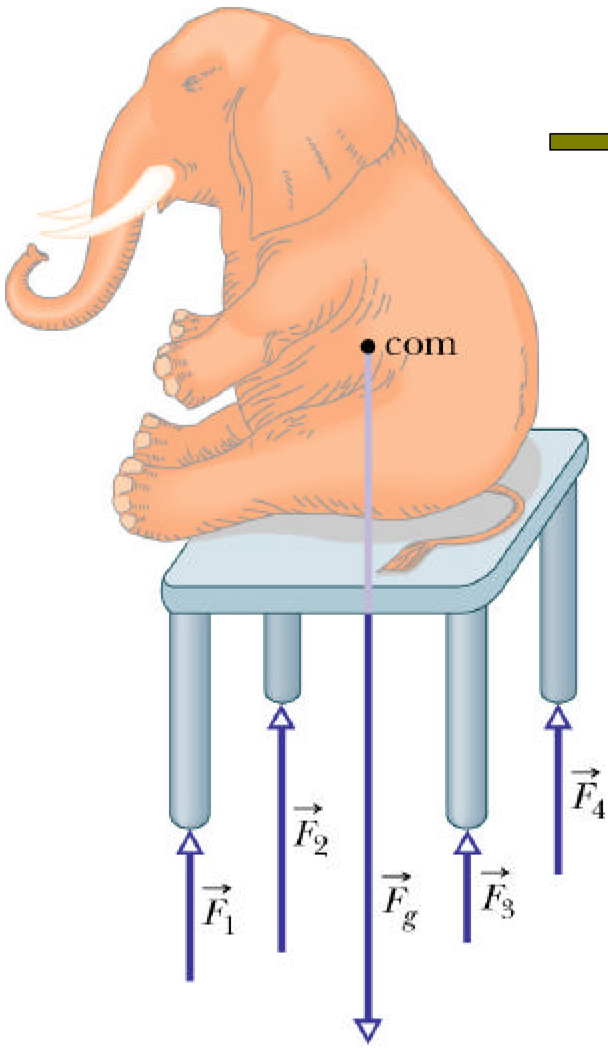
$$h = m_1(w - d) - m_2 d$$

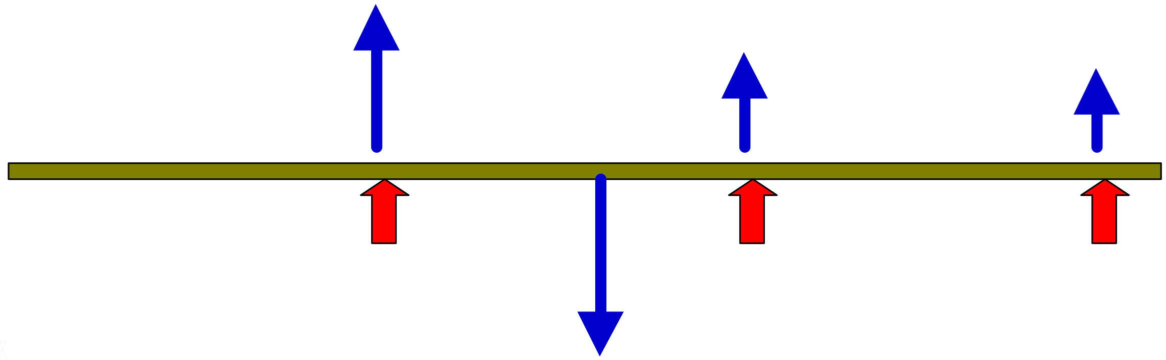
Problem 13-14 (not assigned)



- Person weight W
- Rest on floor at *P* (tiptoe)
- a = 5cm
- b = 15 cm
- Consider foot as system
- Forces on foot at *P*,*A*, and *B*
- <u>Only point of contact at *P*</u> must support body weight, W
  - So *N*=*W* since the gravitational force on body must equal force at only point of contact

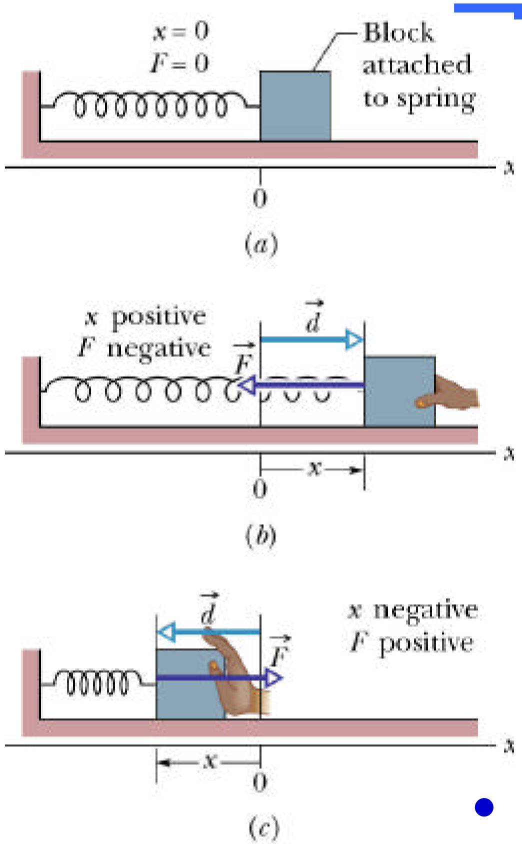
When more forces than necessary - elasticity



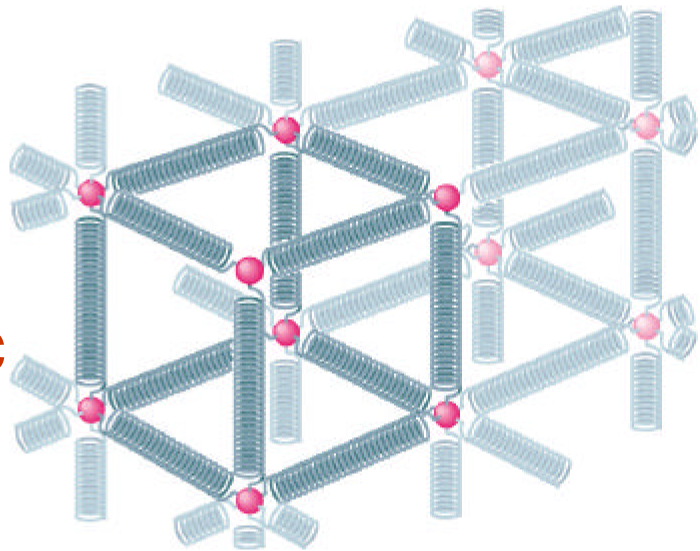


- Consider bar subject to two points of contact
- All forces determined by constraints
  - Add another point of contact … system is <u>underconstrained</u>
  - Need additional info
  - Also text elephant
  - "Give" of material

Elasticity

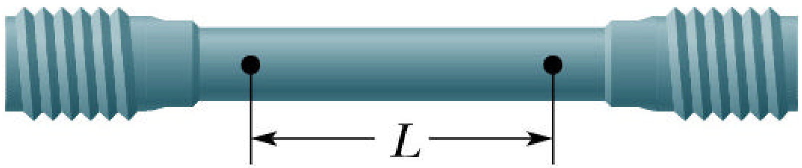


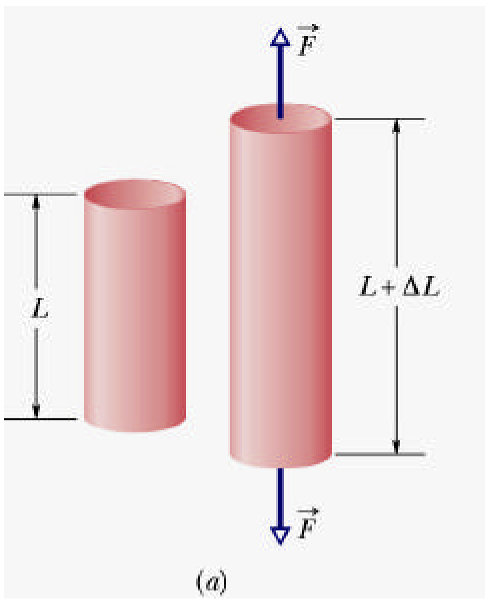
- Actually, of course, not springs
- Electro-magnetic forces on charge constituents of atoms



- Already familiar with "spring forces" obeying Hooke's Law: $F = -kx$
- Solid materials act like the molecules are held together by springs

Tensile or Compressive Stress

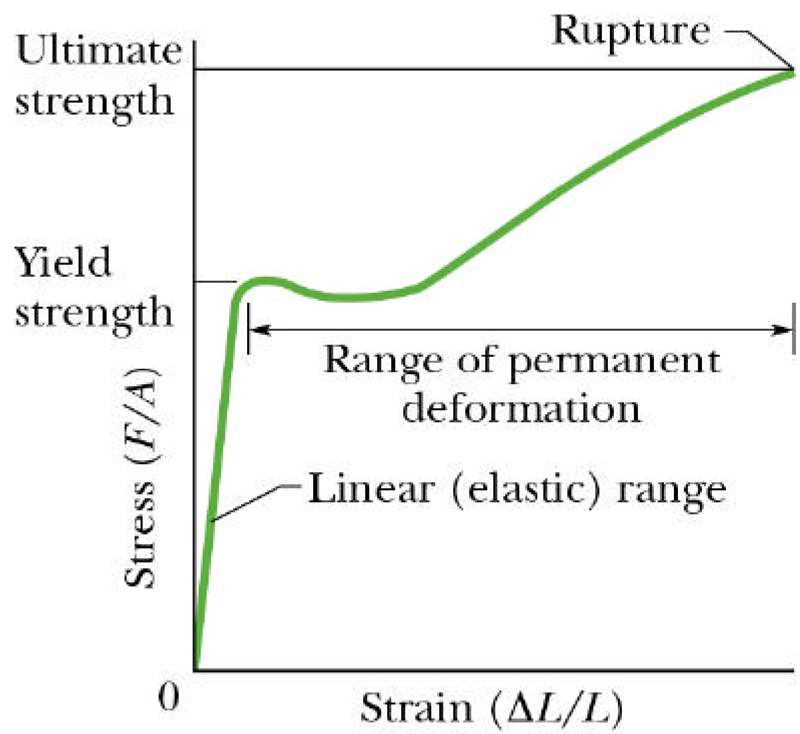




$$\textit{Stress} \equiv \frac{F}{A}$$

$$\textit{Strain} \equiv \frac{\Delta L}{L}$$

$$\frac{F}{A} = E\,\frac{\Delta L}{L}$$

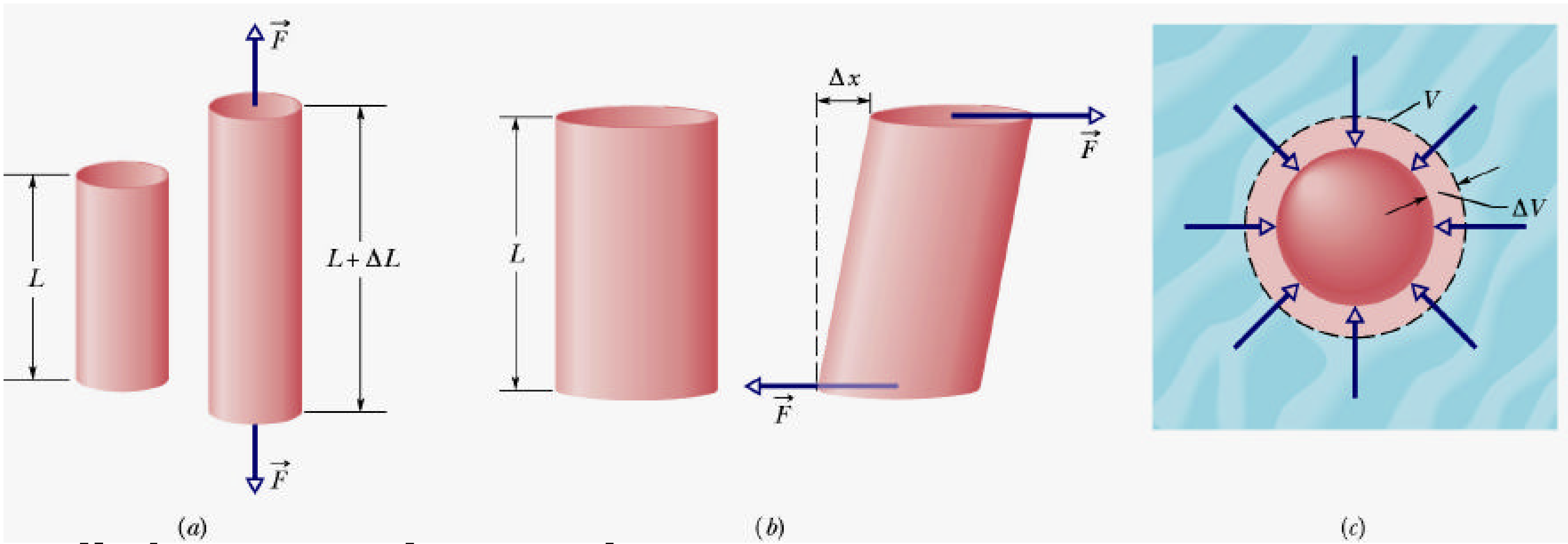


- Most solids, *Stress* $\propto$ *Strain* over substantial range of *F*
- Reach "elastic limit"
- Finally, solid ruptures

Young's Modulus

Check Table 13-1 for some values of YM and yield strengths for several materials

Other deformations



Tensile/Compressive

$$\frac{F}{A} = E\,\frac{\mathrm{D}L}{L}$$

E o Young's Mod.

Shear Stress

$$\frac{F}{A} = G\,\frac{\mathrm{D}x}{L}$$

G o Shear Mod.

Hydraulic Stress

$$\frac{F}{A} = p = B\,\frac{\mathrm{D}V}{V}$$

B o Bulk Mod.

Each works over a finite (low) range of forces

Bulk stress relevant for fluids also

Conclusion

- Do problems in ch 12-13
- Remember HW 7 due Wednesday
- No lecture next Monday, 11/3 (academic holiday)
  - Note both 11/3-4 are academic holidays
- Midterm 2 in class a week from Wednesday (Nov 5) will cover mainly chapters 7 – 13
  - Sample exam and solution at website
  - Sample exam will be reviewed at this week's recitation session