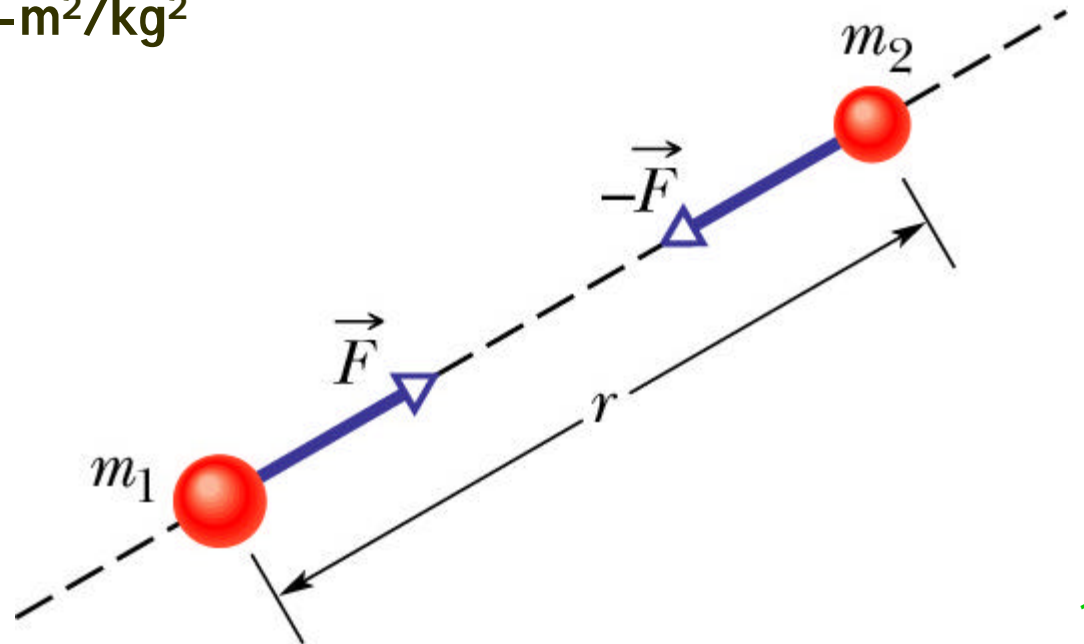


review

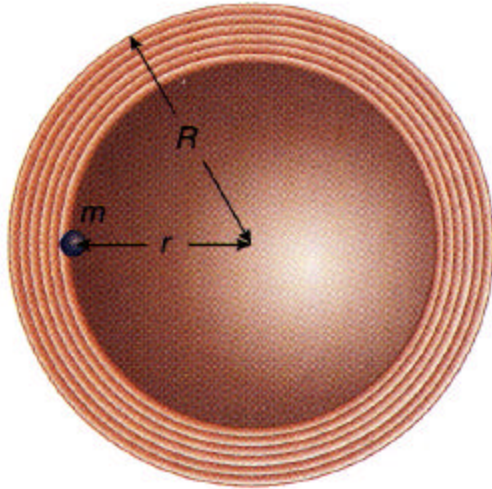
What is Gravity? (Newton)

- Attractive force between any two mass elements of the universe; force is:
 - ◆ always attractive
 - ◆ proportional to masses
 - ◆ inversely proportional to square of separation
 - ◆ dependent on a new universal constant, G
 - $G = 6.67 \cdot 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$

$$F = G \frac{m_1 m_2}{r^2}$$

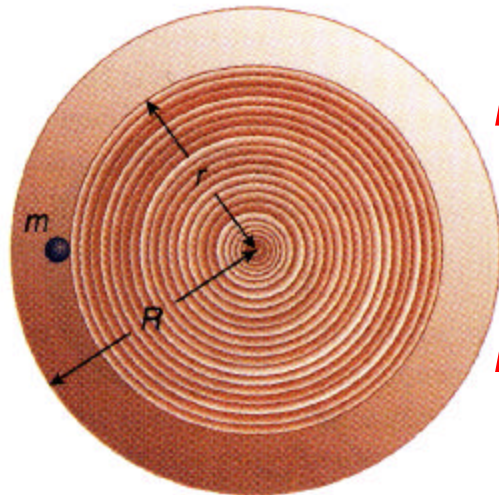


review



Consider Earth as continuous, uniform set of shells

- Shells outside ($r > R$) contribute NO net force on m
- Shells inside ($r < R$) act as if all their mass concentrated at the center
- Only that part of the Earth mass inside r is relevant to the force on m
- Check out problems 14-22 and 23



$$F = G \frac{mM_{\text{inside } m}}{r^2}$$

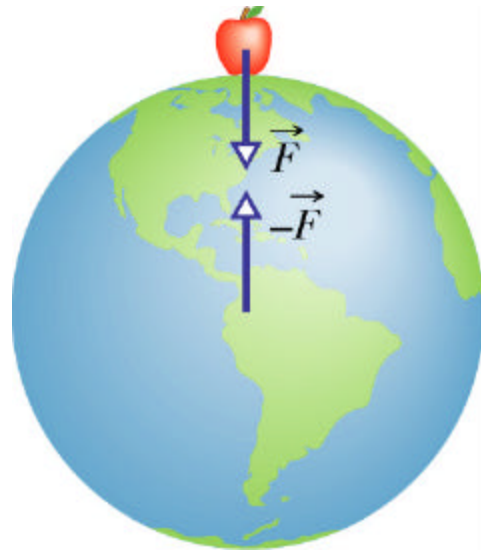
Actually Earth not uniform (previous lecture) but general point valid

$$F = G \frac{m}{r^2} M_{\text{tot}} \frac{\frac{4}{3}pr^3}{\frac{4}{3}pR^3} = \left[G \frac{mM_{\text{tot}}}{R^3} \right] r$$

attractive to center ... like spring force

review

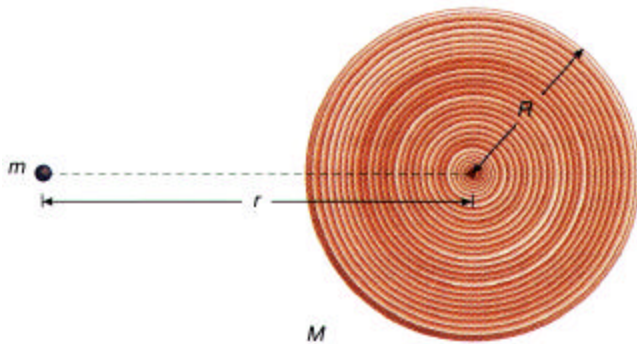
Earth Gravity (g) described by G and Earth properties



$$G \frac{Mm}{R^2} = mg$$

$$g = G \frac{M}{R^2}$$

$M = \text{Earth mass}$
 $R = \text{Earth radius}$
 $G = \text{universal gravitational constant}$



• SHOWN in last lecture!!

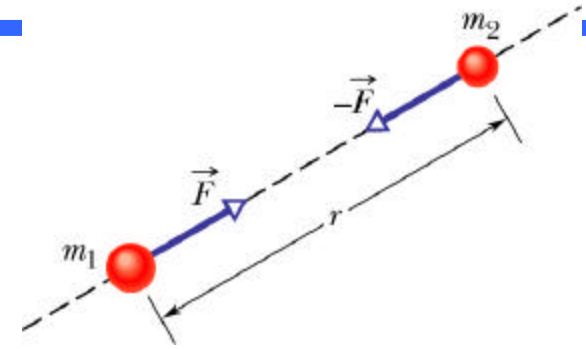
✎ Only assumption is that the Earth consists of a continuous set of shells, and each shell is uniform

✎ Note that, disentangling G from M not simple

✎ First done: Henry Cavendish (in 1798)

Newton's Gravity Solved Two Problems

$$F = G \frac{m_1 m_2}{r^2}$$



- Provides clear explanation and testable predictions about motions on Earth
 - ◆ Consolidation of previous knowledge
 - ◆ The "falling apple"
 - ◆ Issues already discussed
- Simultaneously provides quantitative explanations about astronomy that previously had no underlying explanation
 - ◆ Earth, Sun, planets, ...
 - ◆ Lots went before here too

UNIFICATION

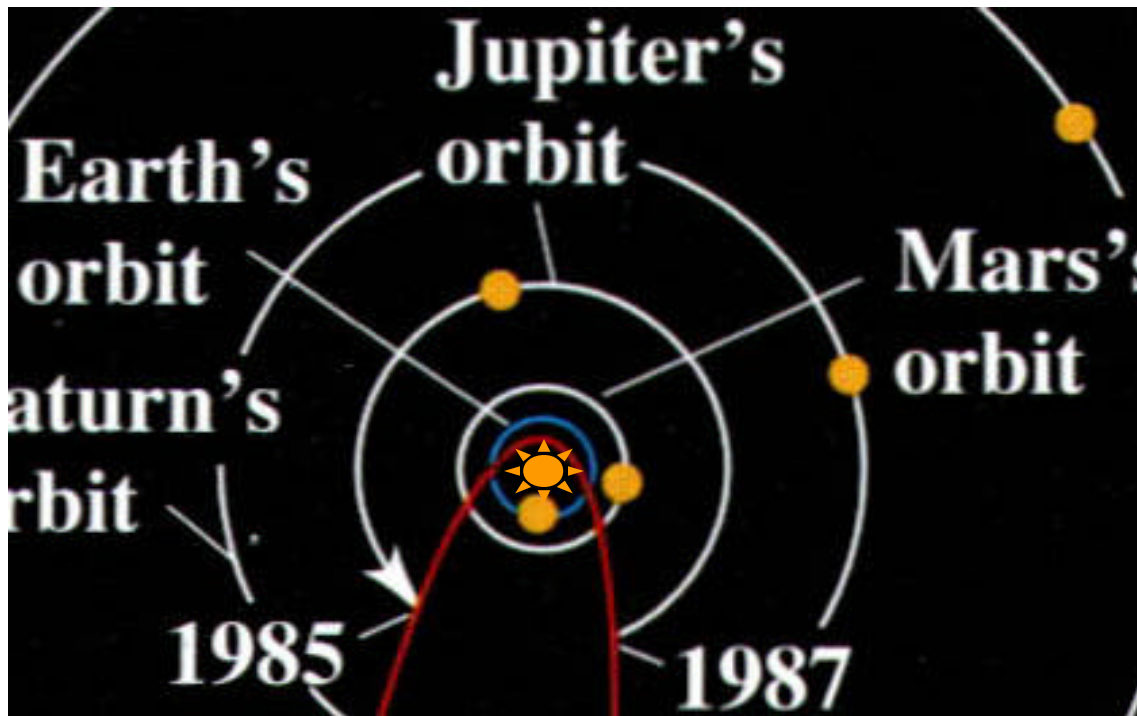
Ptolemaic Astronomy – Observational Problems

- Ptolemaic system (adopted by religions) said Earth center of Universe
- “Obvious” to those who look at the night sky
 - ◆ As if stars, etc sit on a “celestial sphere” that rotates about axis (through the North Star) once a day
- Exception to those who look carefully:
 - ◆ Moon: on its own sphere
 - ◆ “Wanderers”, or planets, do something different



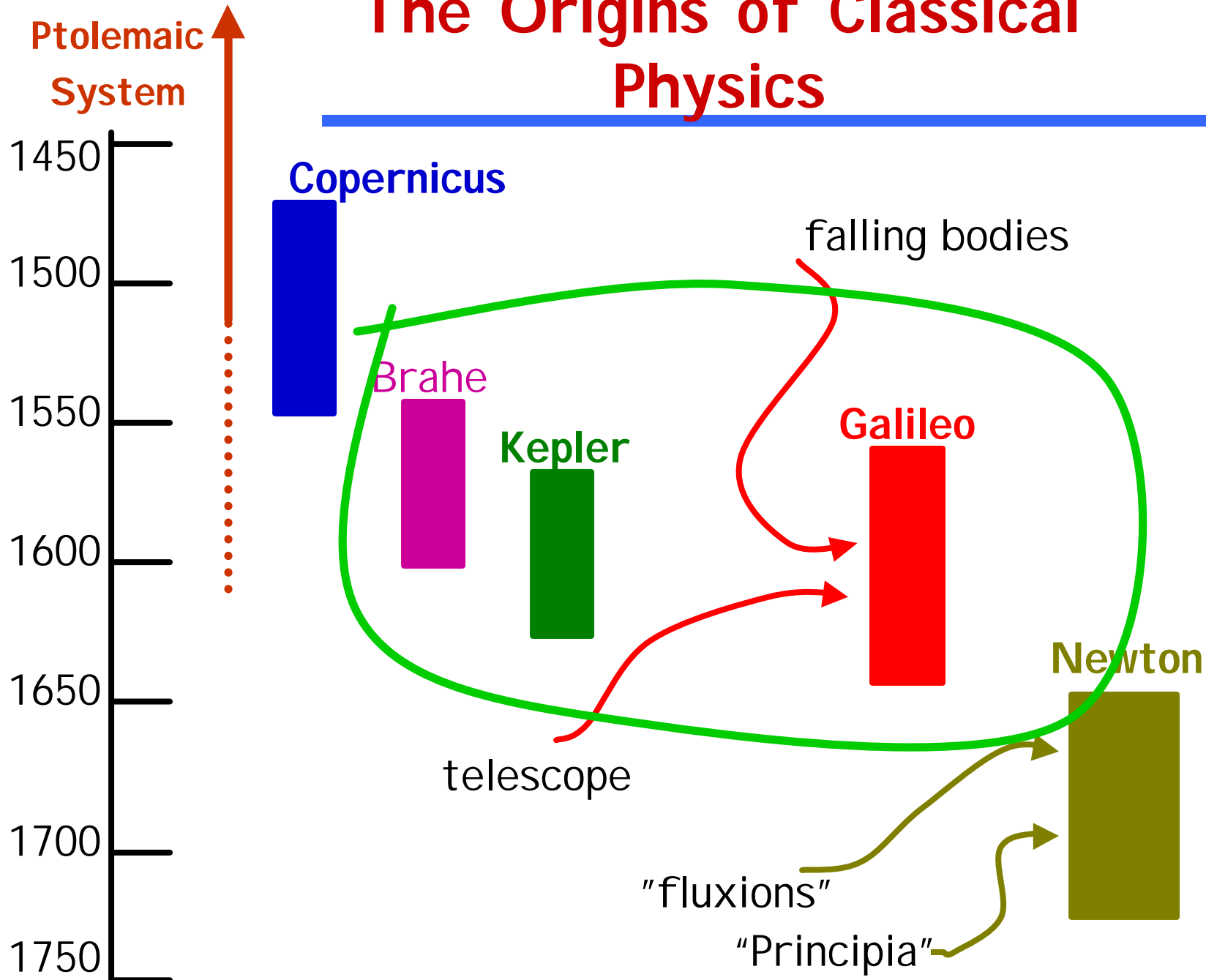
Motion of Mars (1971) against the background of the celestial sphere ... retrograde motion

Copernicus in 16th century – Sun more natural for center



- His idea ... Sun at center of "celestial sphere"
- Planets on their own spheres ... circular
- Explains retrograde motions
- Explains why only see from Earth small angular separations of inner planets from Sun
- BUT, quantitative Copernican descriptions not much better than Ptolemaic-Arab systems

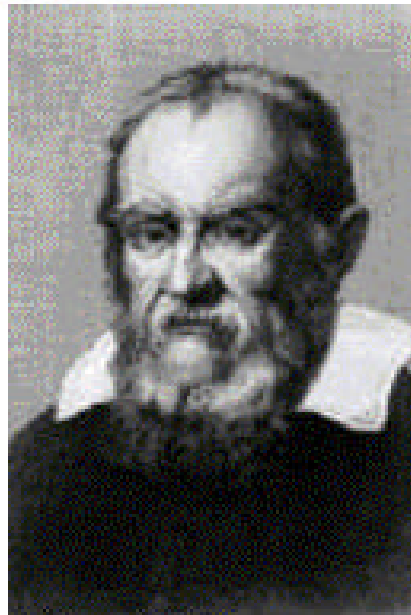
The Origins of Classical Physics



Brahe & Galileo



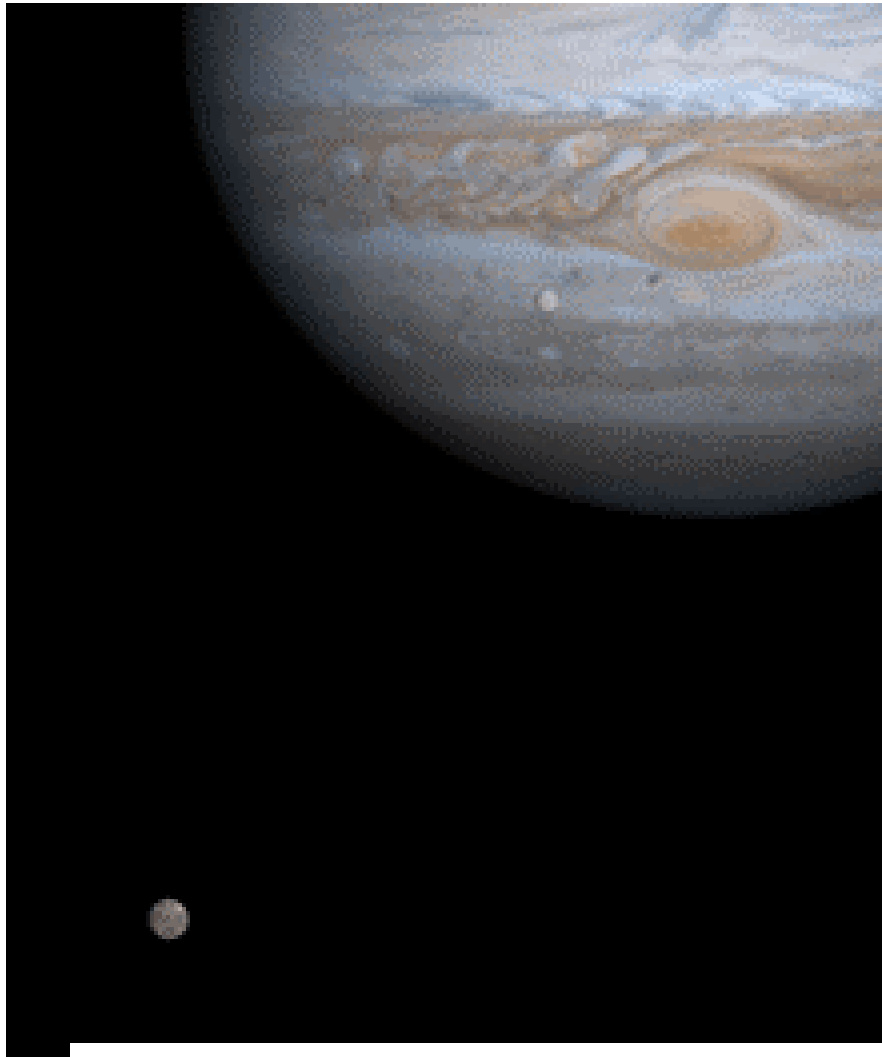
Tycho Brahe
Library of Congress



Galileo Galilei
Library of Congress

- **Brahe (1546-1601)**
 - ◆ Danish astronomer well-funded by kings (Denmark-Norway, Bohemia)
 - ◆ careful and accurate observations of planets
 - ◆ incorrect model of universe
 - ◆ measurements used by Kepler
- **Galileo (1564-1642)**
 - ◆ copied early telescope from the Dutch
 - ◆ first observed Jupiter and discovered 4 moons

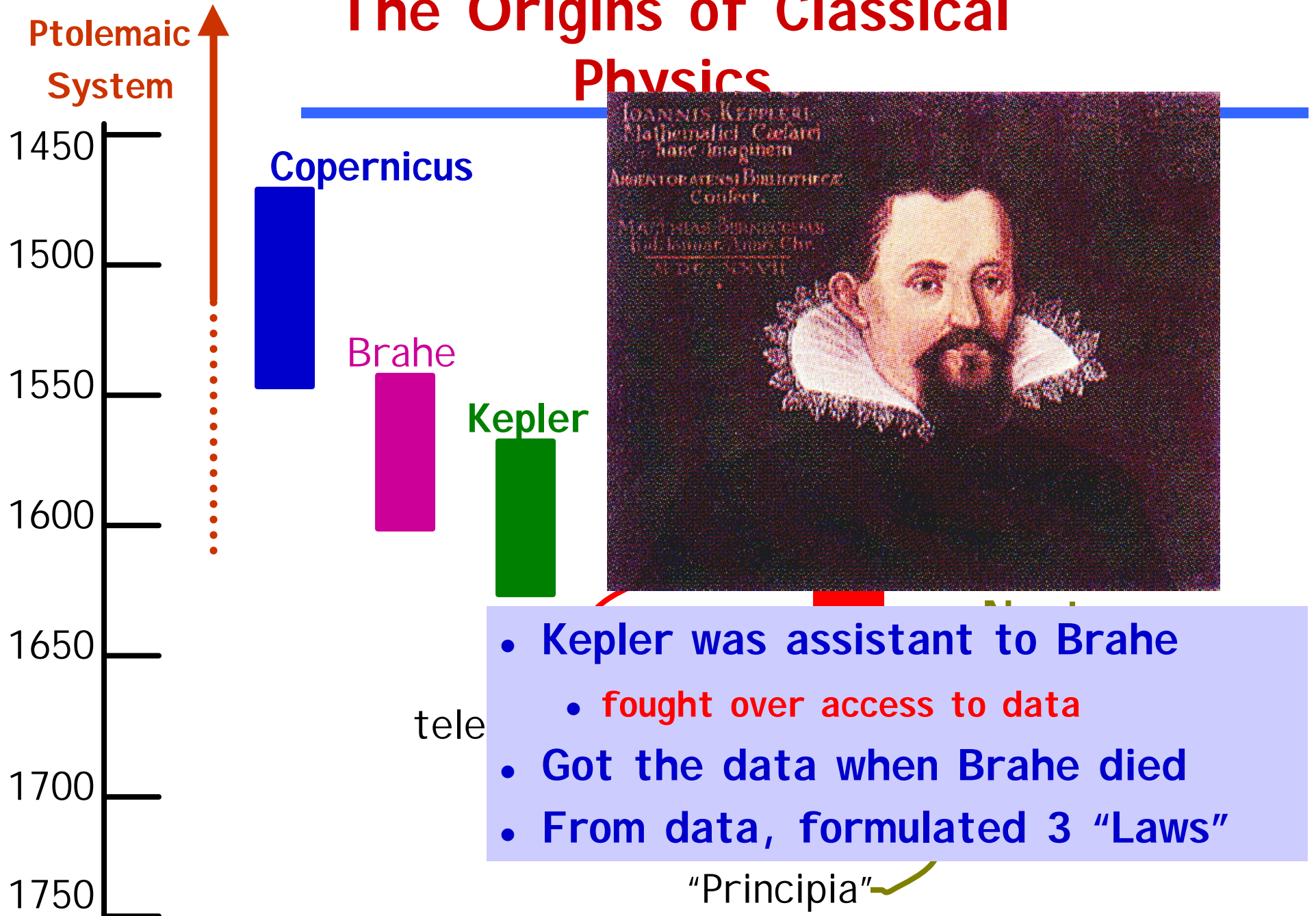
Galileo and Jupiter's Moons



- March 1610, Galileo discovered four of the moons of Jupiter
- Showed clearly that all celestial bodies do not revolve about the Earth!!

Io, Callisto and Jupiter

The Origins of Classical Physics



- Kepler was assistant to Brahe
 - fought over access to data
- Got the data when Brahe died
- From data, formulated 3 "Laws"

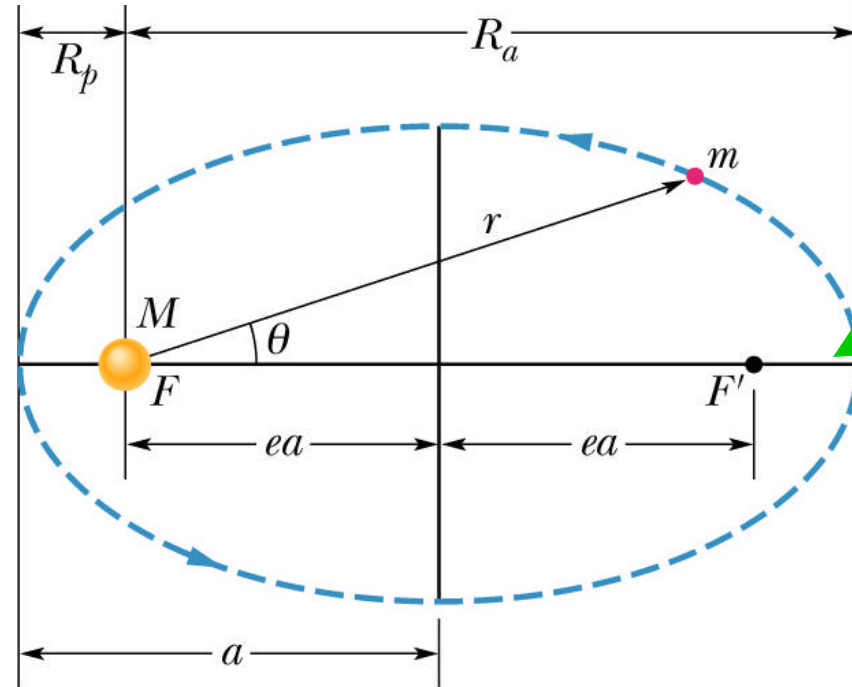
"Principia"

Gravity-Kepler's Laws

Copernican (Sun center) system with exaggerated ellipse

• Kepler's Three "Laws"

1. All planets move in elliptical orbits, with Sun at one focus
2. Vector from Sun to planet sweeps out equal areas in equal times
3. Square of Planet's period is proportional to the cube of the semimajor axis of the orbit



- Newton recognized that these empirical "Laws" were consequences of his Laws of Motion + Gravitation
- We treat deductively - Newton's genius was inductive

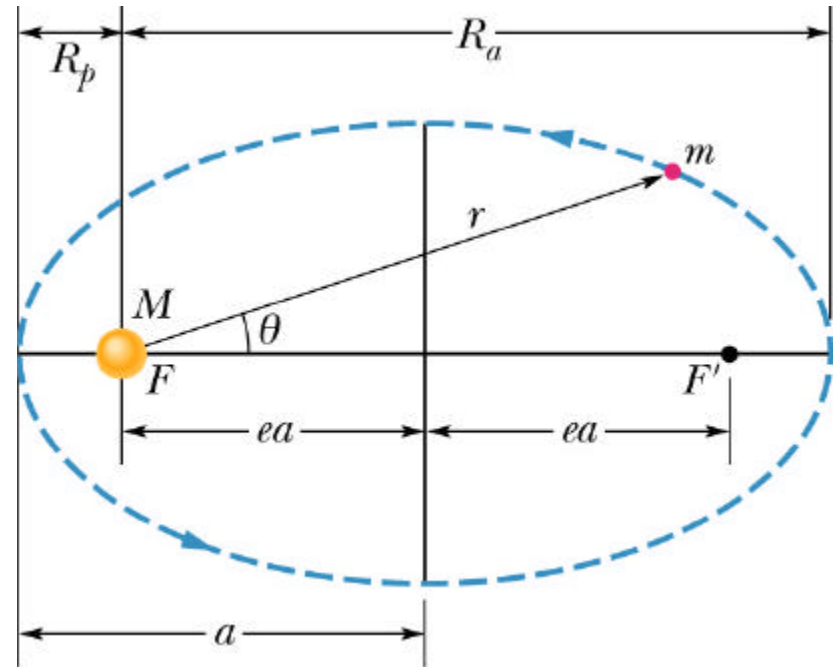
Kepler's First Law

1. All planets move in elliptical orbits, with Sun at one focus

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b = a\sqrt{1 - e^2}$$

- Ellipse characterized by two independent parameters:
 - semi-major axis, a
 - eccentricity, $0 < e < 1$
- Most planets have very small e
 - near circular orbits
 - $e \sim 0$; $a \sim$ radius
- (Note that bodies with $e > 1$ will not be bound in orbit)

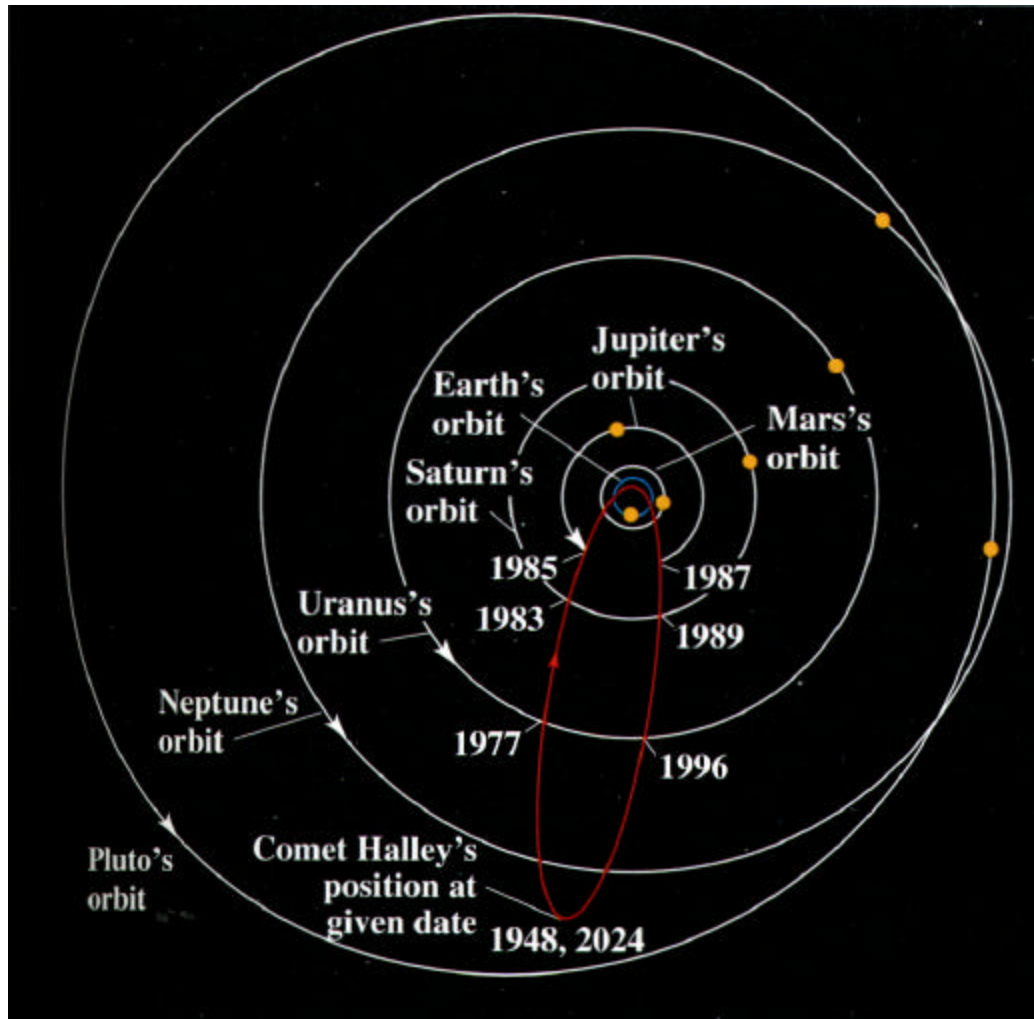


Unique prediction of Newton

$$-G \frac{Mm}{r^3} \vec{r} = m \frac{d^2 \vec{r}}{dt^2}$$

$$E = -\frac{GMm}{2a} = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

Most planets near circular (and in same plane- ecliptic)



- Present knowledge shows most (even inner planets not shown) almost circular
- Even so, deviations were important
 - ◆ Copernicus' circle predictions not very accurate
- Note different Pluto with large e
 - ◆ discovered last century
 - ◆ Today arguments prevail that Pluto is NOT planet but "Kuiper Belt Asteroid"
- Note even more eccentric Halley's comet

Kepler's Second Law

2. **Vector from Sun to planet sweeps out equal areas in equal times**

• Now know this is a consequence of Conservation of Angular Momentum

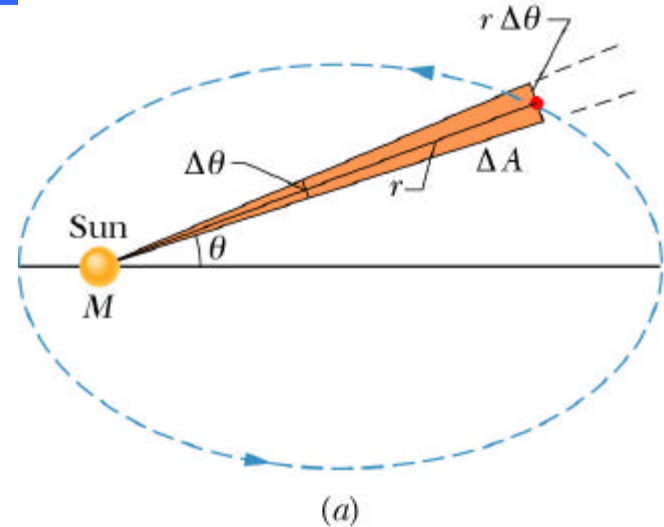
◆ would work for any 'central' force law (no torque)

$$\Delta A = \frac{1}{2} r (r \Delta q)$$

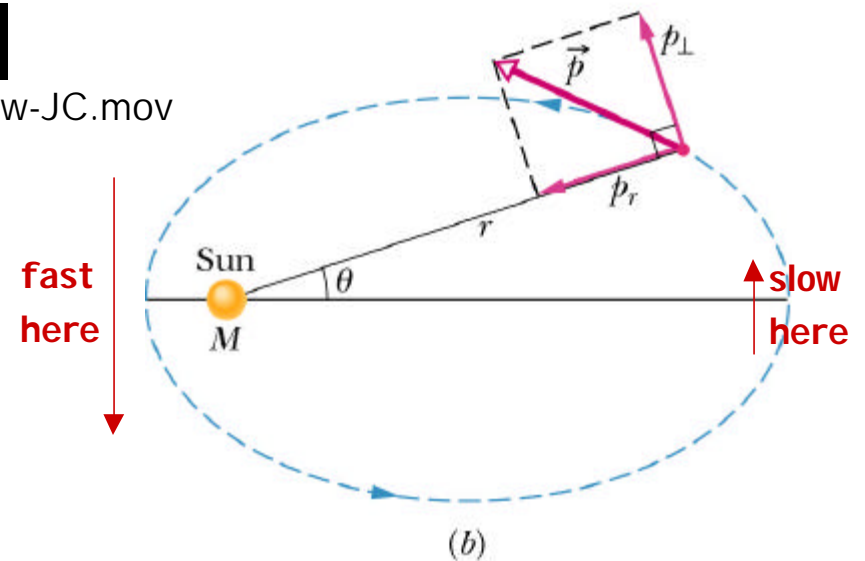
$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{dq}{dt} = \frac{1}{2} r^2 \omega$$

$$L = r(p_{\perp}) = r(mv_{\perp}) = r(mr\omega)$$

$$\frac{dA}{dt} = \frac{L}{2m} = \text{constant}$$



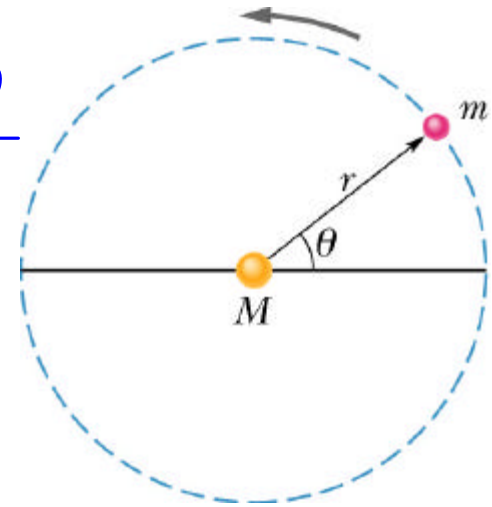
kepler area law-JC.mov



Kepler's Third Law

- ◆ Square of Planet's period is proportional to the cube of the semimajor axis of the orbit
- ◆ Is a consequence of Newton's Force Law
- ◆ Illustrate with circular orbits

$$w = \frac{2p}{T}$$



$$F = ma = mrw^2$$

$$\frac{GMm}{r^2} = mr \left(\frac{2p}{T} \right)^2$$

$$\frac{T^2}{r^3} = \left(\frac{4p^2}{GM} \right)$$

Body	Mass (kg)	Mean Radius (m)	Period (s)	Distance from Sun (m)	$\frac{T^2}{r^3} \left(\frac{s^2}{m^3} \right)$
Mercury	3.18×10^{23}	2.43×10^6	7.60×10^6	5.79×10^{10}	2.97×10^{-19}
Venus	4.88×10^{24}	6.06×10^6	1.94×10^7	1.08×10^{11}	2.99×10^{-19}
Earth	5.98×10^{24}	6.37×10^6	3.156×10^7	1.496×10^{11}	2.97×10^{-19}
Mars	6.42×10^{23}	3.37×10^6	5.94×10^7	2.28×10^{11}	2.98×10^{-19}
Jupiter	1.90×10^{27}	6.99×10^7	3.74×10^8	7.78×10^{11}	2.97×10^{-19}
Saturn	5.68×10^{26}	5.85×10^7	9.35×10^8	1.43×10^{12}	2.99×10^{-19}
Uranus	8.68×10^{25}	2.33×10^7	2.64×10^9	2.87×10^{12}	2.95×10^{-19}
Neptune	1.03×10^{26}	2.21×10^7	5.22×10^9	4.50×10^{12}	2.99×10^{-19}
Pluto	$\approx 1.4 \times 10^{22}$	$\approx 1.5 \times 10^6$	7.82×10^9	5.91×10^{12}	2.96×10^{-19}
Moon	7.36×10^{22}	1.74×10^6	—	—	—
Sun	1.991×10^{30}	6.96×10^8	—	—	—

Energies

Circular Orbit:

$$U = -G \frac{Mm}{r}$$

$$m \frac{v^2}{r} = G \frac{Mm}{r^2}$$

$$v^2 = G \frac{M}{r} = - \frac{U}{m}$$

$$E = \frac{1}{2} mv^2 + U$$

$$E = - \frac{1}{2} G \frac{Mm}{r}$$

- Note some general features of objects in orbit
 - ◆ Potential energy is twice the magnitude of kinetic energy (and opposite sign)
 - ◆ Velocity, energies, ... completely specified by radius
- For elliptical orbit, same expression for energy, but substitute $r \textcircled{R} a$
- Note as $a \textcircled{R} \infty$, energy is zero
- $E > 0$ corresponds to unbound

Homework Problems

- **Should be straightforward use of**
 - ◆ **Newton's Laws of Motion and Gravitation**
 - ◆ **Kepler's Laws (deducible from Newton's)**
- **Includes**
 - ◆ **Problem (14-51) on four of the 12 satellites of Jupiter**
 - ◆ **Geosynchronous orbit (prob 14-48): means that the satellite stays over the same spot on the Earth**
 - Satisfy yourself what this requires ...
 - Only can happen at the equator
 - Satellite must have the same rotation period as the day
 - Can only occur at a specific distance from the center of the Earth

Beyond the Solar System

- We are in a spiral galaxy much like the one shown here.



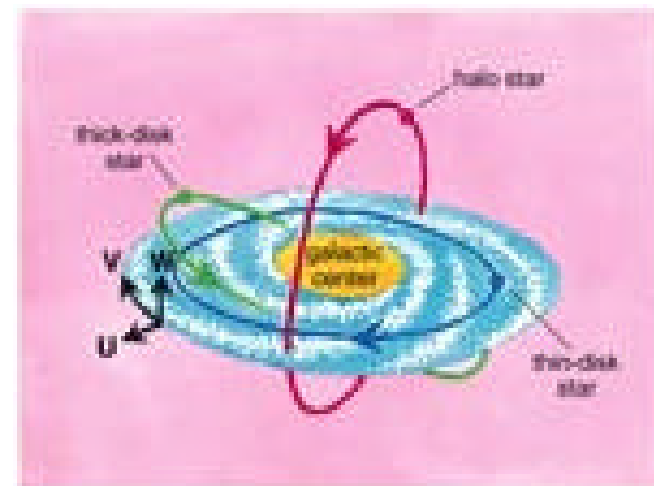
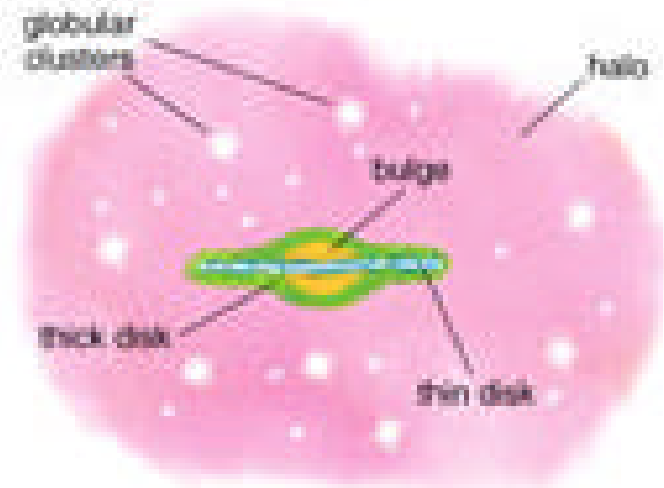
- Our sun (star) rotates around the galactic center with path described by Newton's Laws



sun motion around galaxy.MOV

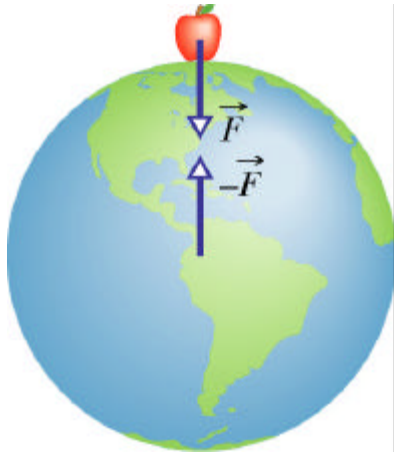
Structures of Galaxies

- Typically globular clusters formed earliest
- Then accumulation of mass (mainly hydrogen) formed stars
- Inside stars heavier elements made - generations
- Strong evidence that at centers of galaxies are Black Holes



culture

Black Holes



$$E > 0$$

$$\frac{1}{2} m v_{\text{esc}}^2 - G \frac{Mm}{R} = 0$$

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}} \text{ or } R = \frac{2GM}{v_{\text{esc}}^2}$$

- Consider now massive body emitting light (or any massless particle) while collapsing
- We all know that light travels at finite velocity $c=3 \cdot 10^8$ m/s

For

$$R < R_s$$

$$R_s = \frac{2GM}{c^2}$$

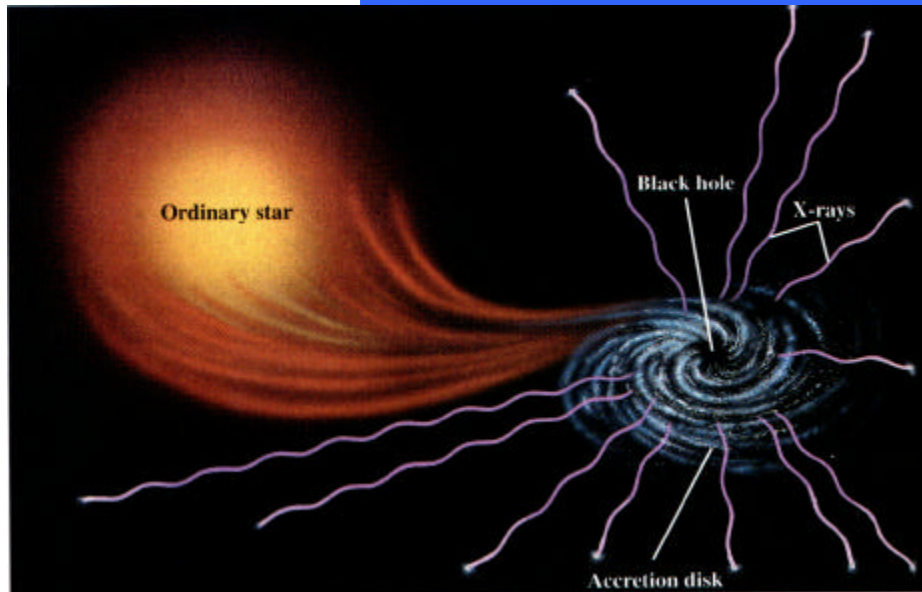
light inside trapped and everything else

- e.g. 2 suns in radius of ~ 6 km instead of $\sim 10^6$ km

• Actually, relativity makes formula right by lucky accident

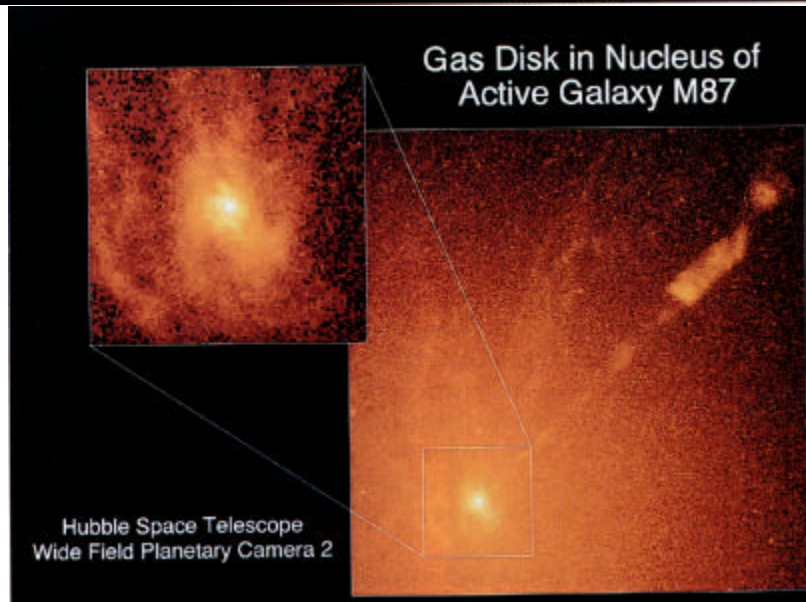


Two Kinds of Black Holes Observed



culture

- Black holes in orbit around ordinary stars ... see radiation from accreting material
- Black holes at the centers of galaxies



culture

Milky Way-like Galaxies

On edge



ESO PR Photo 36/97 (21 March 1997)

Spiral Galaxy NGC 4945
(MPG/ESO 2.2-m + WFI)

© European Southern Observatory



From above
Spiral Galaxy NGC 1232 - VLT UT 1 + FORS1

ESO PR Photo 37d/98 (23 September 1998)

© European Southern Observatory



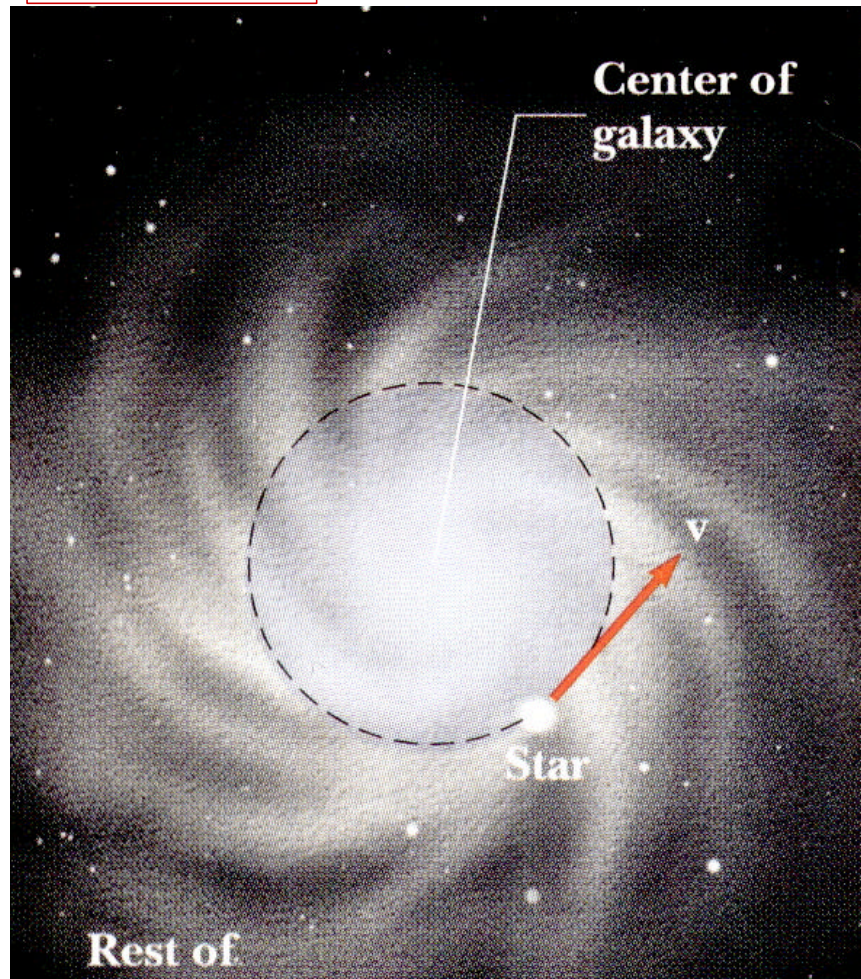
Beyond Galaxies



- Any small piece of the sky that is viewable ... see lots of galaxies
- Groups of galaxies
- Billions of galaxies
- Billions of groups of galaxies
- Each galaxy with \gg billions of stars
- Do Newton's Laws still work?
 - ◆ Well ... yes, but

The Missing Mass (Dark Matter) Problem

culture



- Stars on outskirts of a galaxy
- Or galaxies in orbit around a dense group of galaxies
- Expectations

$$G \frac{M_{\text{inside } r} m}{r^2} = m \frac{v^2}{r}$$

$$v = \sqrt{\frac{GM_{\text{inside } r}}{r}}$$

- Doesn't behave this way

Still unresolved exactly what makes up the dark matter!

- v bigger at large r
- M bigger ... unseen mass

Next Time

- Back to earth - read chapter 15

- **Fluids**

