General Points

- Finish required material today
- End of this and the next lecture will be expanded discussion of section 21-7 (on entropy) … not on final
- By next Monday (Dec 8), will post sample final exam and solutions.
- Will have regular office hours on Monday afternoon next week. Special office hours next Friday … to be scheduled - check website.
- Review session next week, to go over sample exam; schedule today with Mr. Bansal.
- Real final on Monday, December 15, 1 PM here
  - Must be taken
  - You may bring your own double-sided handwritten formula sheet (8½ X 11 inch).
Thermodynamics and Gases

Last Time

- Specific Heats more generally
- Adiabatic Expansion
- Reversible and Irreversible Processes
- Entropy
- 2nd Law of Thermodynamics

Today

- More entropy
- Uses in engines, heat pumps, refrigerators
  - End of material for final exam
- Lead to more insightful view of “entropy”
  - not on the final exam
  - Probability distributions
2nd Law of Thermodynamics: gases

- Net entropy change of any process of a closed system is either zero (reversible) or greater than zero (irreversible)
  - discussion section 21-2
- Entropy is a state function; depends only on parameters of the system: calculate general change for gas

\[ \Delta S \geq 0 \]

\[ dQ = dE_{\text{int}} + dW \]

\[ dQ = nC_v dT + pdV \]

\[ \Delta S_{i \rightarrow f} = S_f - S_i \]

\[ = \int_i^f \frac{dQ}{T} \]

\[ \Delta S_{\text{rev}} = \Delta S_{\text{irrev}} \]

entropy change for ideal gas

\[ \Delta S = \int_i^f \frac{dQ}{T} = nC_v \int_{T_i}^{T_f} \frac{dT}{T} + nR \int_{V_i}^{V_f} \frac{dV}{V} \]

\[ \Delta S = nC_v \ln \frac{T_f}{T_i} + nR \ln \frac{V_f}{V_i} \]
Steam Engines (also nuclear power, etc)

- All use the same principles
  - furnace putting heat into water (boiler) and converting to hot steam
  - heat energy does work by pushing on a piston
  - some heat energy carried away, either to condenser (to be reused via pump) or vented to air --- water is condensed to recycle
  - essential to return the gas in front of piston in the same state as at the beginning of the cycle $\Delta S = 0$
Carnot Cycle has highest efficiency attainable for engine

- a-b: isothermal expansion of gas from hot reservoir (at $T=T_h$) as piston is pushed out
- b-c: further adiabatic expansion as piston moves to furthest extent
- c-d: isothermal compression of piston at temperature of cool reservoir ($T=T_l$)
- d-a: further adiabatic compression back to original state

Back at a completes one cycle ... gas is in its initial state and the next cycle can begin
Carnot Cycle

Completely reversible gas cycle: $\Delta S = 0$

Take: $Q_H, Q_L$ to be +ve

$W = Q_H - Q_L$

Carnot cycle has the highest ideal efficiency of any thermodynamic cycle

\[
\Delta S = \frac{Q_H}{T_H} - \frac{Q_L}{T_L} = 0
\]

\[
\frac{Q_L}{Q_H} = \frac{T_L}{T_H}
\]
Efficiency of “Ideal” Steam Engine

\[ \Delta S = \frac{Q_H}{T_H} - \frac{Q_L}{T_L} = 0 \]

\[ \frac{Q_L}{Q_H} = \frac{T_L}{T_H} \]

- How much work get out for energy put in?
- Cost is to heat the boiler: \( Q_H \)
- Useful output \( W \)
- Lost energy: \( Q_L \)

\[ \varepsilon = \frac{W}{Q_H} = \frac{Q_H - Q_L}{Q_H} \]

\[ \varepsilon = 1 - \frac{T_L}{T_H} \]

\[ \varepsilon_{\text{real world}} \leq 1 - \frac{T_L}{T_H} \]
Other engines

Stirling Engine (left)
- Isochoric rather than adiabatic legs of cycle
- Text incorrect: same ideal efficiency as Carnot cycle

Internal Combustion engine (below)
- Otto cycle: similar to Stirling but adiabatic compression ... efficiency different (lower)
Refrigerators and Air Conditioners

- Common system in refrigerators, air conditioners, (heat pumps)
- Do work and remove heat from cold environment
- deliver heat to warm environment
Refrigerators

- How much work to put in for heat energy taken out
- Cost is work: $W$
- Useful output: $Q_L$
- Vented energy: $Q_H$
- Define “coefficient of performance” (cop)

\[ Q_H = W + Q_L \]
\[ \text{cop} = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L} \]
\[ \text{cop} = \frac{T_L}{T_H - T_L} \]

\[ \Delta S = \frac{Q_H}{T_H} - \frac{Q_L}{T_L} = 0 \]
\[ \frac{Q_L}{Q_H} = \frac{T_L}{T_H} \]
Heat Pump

- Similar to refrigerator, only in reverse
- Note \( \text{cop} > 1 \)
- So \( Q_H > W \)

\[ \Delta S = \frac{Q_H}{T_H} - \frac{Q_L}{T_L} = 0 \]

\[ \frac{Q_L}{Q_H} = \frac{T_L}{T_H} \]

\[ Q_H = W + Q_L \]

\[ \text{cop} \equiv \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_L} \]

\[ \text{cop} = \frac{T_H}{T_H - T_L} \]
Origin of Entropy

- Short description in text (21-7)
  - Remainder today and Monday, we will try to provide more elaborate answer than given section 21-7
  - This and following will not be on the final exam

- Short answers:
  - Entropy is a measure of the disorder of the system
  - Entropy is a measure of probability of finding state in a given configuration
Entropy increase of free expansion of gas

\[ \Delta S = \frac{Q}{T} = \frac{W}{T} = \frac{nRT}{T} \ln \frac{V_f}{V_i} \]

\[ \Delta S = nR \ln \frac{V_f}{V_i} \]

- This is the entropy change of the gas for both processes

- More general entropy change for ideal gas

\[ \Delta S = \int_{i}^{f} \frac{dQ}{T} = nC_v \int_{T_i}^{T_f} \frac{dT}{T} + nR \int_{V_i}^{V_f} \frac{dV}{V} \]

\[ \Delta S = nC_v \ln \frac{T_f}{T_i} + nR \ln \frac{V_f}{V_i} \]

irreversible in a closed system
reversible isothermal equivalent

Physics 1401 - L 24
Frank Sciulli
Macroscopic vs Microscopic for Gases

- Gas is said to be in a specific “macrostate”
- **Entropy** is a measure of the gas “state probability”
- A specific macrostate may have many microstates (different ways of arranging atoms to get same macrostate)

- Macroscopic Quantities
  - temperature
  - pressure
  - volume
  - internal energy
  - entropy = $\Delta Q/T$
  - Specific macroscopic “state” of gas can have many configurations or microstates

- Microscopic States
  - velocity vector each molecule
  - position vector each molecule
  - Specific “microstates”, with definite values for $p_x$, $p_y$, $p_z$, $x$, $y$, $z$ for each particle will contribute to the macrostate
  - $w$ = number of different “microstates”, or configurations, in the macrostate
Bouncing Atoms

- For gas, as one specific time in a particular macrostate, the status of each molecule is described by parameter values of
  - $x$, $y$, $z$
  - $v_x$, $v_y$, $v_z$
- The gas molecules are characterized by a probability distribution for each parameter
- Wider range of probability for these six parameters necessarily implies more microstates available to the molecules
- Examples:
  - twice the volume = 2 $\times$ # pos states
  - increase temp = large # velocity states
Boltzmann’s Ansatz

Many “microstates” or combinations of particles may give same macrostate

Statistical mechanics: all microstates are equally probable

Formula for entropy:

\[ S_{\text{molecule}} = k \ln w \]

\[ w = \# \text{ microscopic states in macrostate} \]

\[ S_{\text{gas}} = \sum_{\text{all molecules}} S_{\text{molecule}} \]

- Statistical mechanics: not on exam (later courses will discuss more fully)
- Do simple illustrations here
- We choose for illustration an analogy with “discrete” states available to N coins
- If these were atoms, the number of different microstates would give the entropy of the macrostate (which will be discussed)
Boltzmann applied

- More microstates = more entropy
- For atoms and microstates, the number of states are delineated by:
  - Position possibilities
  - Velocity possibilities
- for each atom, enormous number of possibilities ... return to this later
- Total entropy = Sum over all atoms (also enormous number)
- Look first at simpler physical system: limited number of coins

\[
S_{\text{molecule}} = k \ln w
\]

\[
w = \# \text{ microscopic states in macrostate}
\]

\[
S_{\text{gas}} = \sum_{\text{all molecules}} S_{\text{molecule}}
\]
Order vs Disorder

- **Order** implies that the system is in a state with very few microstates (or a state of very low probability).
- **Disorder** = system is in a highly probable state (many microstates).
Boltzmann: Macrostates (and Microstates)

- Two scenarios at right
  - each has several possible microstates
  - depend on which molecules are on which side
- Analyze with coins

\[ S = k \ln W \]

\[ W = \text{number of microstates} \]
4 pennies—macrostates and microstates

- All heads: one microstate
- One tail: 4 microstates
- 2 heads, 2 tails: six microstates
- In total: 16 microstates
- Pix shows relative probabilities
Binomial Coefficients

- Binomial Coefficients give a formula for the number of ways in which to combine \( N \) coins to have \( n \) tails and \( (N-n) \) heads.

\[
\binom{N}{n} = \frac{N!}{n!(N-n)!}
\]

- \( \binom{4}{0} = \frac{4!}{0!(4)!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{1} = 1 \)
- \( \binom{4}{2} = \frac{4!}{2!(2)!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 6 \)

Four-Penny Macrostates and Microstates

<table>
<thead>
<tr>
<th>Macrostate (1): 4 heads (H) and 0 tails (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Penny 1</td>
</tr>
<tr>
<td>H</td>
</tr>
<tr>
<td>(1 microstate)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Macrostate (2): 3 heads and 1 tail</th>
</tr>
</thead>
<tbody>
<tr>
<td>Penny 1</td>
</tr>
<tr>
<td>H</td>
</tr>
<tr>
<td>H</td>
</tr>
<tr>
<td>H</td>
</tr>
<tr>
<td>T</td>
</tr>
<tr>
<td>(4 microstates)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Macrostate (3): 2 heads and 2 tails</th>
</tr>
</thead>
<tbody>
<tr>
<td>Penny 1</td>
</tr>
<tr>
<td>H</td>
</tr>
<tr>
<td>H</td>
</tr>
<tr>
<td>T</td>
</tr>
<tr>
<td>T</td>
</tr>
<tr>
<td>H</td>
</tr>
<tr>
<td>(6 microstates)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Macrostate (4): 1 head and 3 tails</th>
</tr>
</thead>
<tbody>
<tr>
<td>Penny 1</td>
</tr>
<tr>
<td>H</td>
</tr>
<tr>
<td>T</td>
</tr>
<tr>
<td>T</td>
</tr>
<tr>
<td>T</td>
</tr>
<tr>
<td>(4 microstates)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Macrostate (5): 0 heads and 4 tails</th>
</tr>
</thead>
<tbody>
<tr>
<td>Penny 1</td>
</tr>
<tr>
<td>T</td>
</tr>
<tr>
<td>(1 microstate)</td>
</tr>
</tbody>
</table>
10 coins - contrast

- 5 each is 252 times more probable than all heads
- as more coins are added, equal partitioning = equal numbers heads and tails gets more probable relative to all other possibilities
  - especially those near the extremes (list top and bottom)
Thermodynamics and Gases

Today

- 2nd Law of Thermodynamics
- Entropy
- Uses in engines, heat pumps, refrigerators

Next Time (not on final)

- Continue discussing origin of Entropy
  - measure of system disorder
  - statistical mechanics
General Points

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