This course: learn how things work

- make trains run at high speed (later)
- make sure they stop (soon)
Physics + Math

• Today ... learn about describing motion
• Starting, stopping, and everything in between
• Beyond basic issues
  • uncertainties --- errors
  • evaluating them left for labs
Today's Lecture: See Ch 2, 3

- Position, velocity, and acceleration in one dimensional motion
- Motion in 2 and 3 dimensions
  - vectors
Position versus time

- Pix from text shows location versus time
Definitions of velocity and acceleration

Average velocity

\[ \bar{v} \equiv \frac{\Delta x}{\Delta t} \]

Average acceleration

\[ \bar{a} \equiv \frac{\Delta v}{\Delta t} \]
Velocity

- **Average velocity**
  - Interval dependent

- **Instantaneous velocity**
  - Limit of interval = 0

\[ v_{\text{avg}} = \bar{v} = \frac{\Delta x}{\Delta t} \]

**Case shown**
\[ \bar{v} = \frac{6 \text{ m}}{3 \text{ s}} = 2 \text{ m/s} \]

\[ v_{\text{inst}} = v = \lim_{\Delta t \to 0} \left[ \frac{\Delta x}{\Delta t} \right] = \frac{dx}{dt} \]
Instantaneous velocity

\[ v_{\text{inst}} = v = \lim_{\Delta t \to 0} \left( \frac{\Delta x}{\Delta t} \right) = \frac{dx}{dt} \]

\( x(t) \)

\( t (s) \)

(a) Position at \( t = 0 \)

(b) \( \Delta x = 2 \text{ m} - (-4 \text{ m}) = 6 \text{ m} \)

\( \Delta t = 4 \text{ s} - 1 \text{ s} = 3 \text{ s} \)
Velocity as slope of $x$ vs $t$

- Instantaneous velocity can also change with time.
- For this case, it does!
- Red points show $v = dx/dt$ (in m/s) as a function of time.
- Note that velocity changes!!
Movie illustration

Position, velocity, and acceleration in one-dimensional motion

x-v-a-1min.MOV
Acceleration = rate of velocity change

- Note that velocity changes = acceleration (a) !!

\[ a = \lim_{\Delta t \to 0} \left[ \frac{\Delta v}{\Delta t} \right] \equiv \frac{dv}{dt} \]

- Units of acceleration

\[ \frac{m}{s} = \frac{m}{s^2} \]
**Constant acceleration**

\[
x = x_0 + v_0 t + \frac{1}{2} a t^2
\]

\[
v = v_0 + a t
\]

\[a = \text{constant}\]
Table 2-1 in text: constant acceleration

\[ v = v_0 + at \]

\[ x = x_0 + v_0 t + \frac{1}{2} at^2 \]

\[ v^2 = v_0^2 + 2a(x - x_0) \]

\[ x - x_0 = \frac{1}{2} (v_0 + v) t \]

\[ x - x_0 = vt - \frac{1}{2} at^2 \]

Derived already

Derived already

eliminate \( t \) from eqns

eliminate \( a \) from eqns

eliminate \( v_0 \) from eqns
Real Up and Down 1D Motion (on Earth)

If all other forces (friction, air drag, ...) are small and can be neglected, then for a free body:
- horizontal motion has zero acceleration
- vertical motion has a universal value of acceleration:
  \[ a = g \approx 9.8 \text{ m/s}^2 \approx 32 \text{ ft/s}^2 \]

GOT IT RIGHT!

Aristotle (384-322 BCE):
- celestial motion circular
- earthly motion linear

GOT IT WRONG
or at least incomplete!
Marginal predictive power!
Physics Examples of 1D Motion

- Ignore complications of friction, air drag, etc–free bodies move in simple ways!

- Horizontal Motion
  - Horizontal: zero acceleration

- Vertical Motion
  - “Free fall”: substantial and constant acceleration
  - Universal gravitational acceleration at Earth surface: \( g=9.8 \text{ m/s}^2 \)

grav acc - ball.MOV
More than 1 dimension: VECTORS
Chapter 3

- Mathematically represent quantities with size & direction
- Vector points from one point to another in graph
- Vector completely specified by its components along axes
- Magnitude represented by length of vector
- Angle(s) required to provide direction
- Properties independent of vector location

\[ a_x = a \cos \theta \quad a_y = a \sin \theta \]

\[ a = \sqrt{a_x^2 + a_y^2} \quad \tan \theta = \frac{a_y}{a_x} \]
Position Vectors - 3D

- Real object in real space requires 3D description
  - 3 components or projections
- Get away with fewer when it happens to move
  - along a line (1D)
  - in a plane (2D)
Position, velocity, acceleration vectors

- Position vector from origin to present location
- Velocity vector is along trajectory
- Acceleration vector can be any direction

\[
\vec{v} = \frac{d\vec{r}}{dt}
\]
\[
\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}
\]
Vector equation shorthand

Simple short way of writing three equations

\[ \mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \]

\[ \mathbf{v} = \frac{d\mathbf{r}}{dt} \]

\[ \mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2} \]

\[ \mathbf{v}_x = \frac{dx}{dt} \]

\[ a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2} \]

example:
Chapters 2 and 3

- Make sure you understand them
- We will use all of the mathematical concepts
- Get comfortable with using vectors
  - Work through examples on your own
  - Work out problems to satisfy yourself
- Vector products ... understand ...
  - We will use them later (for work, energy, ...)
  - Assume you know the math