Review – 1D motion

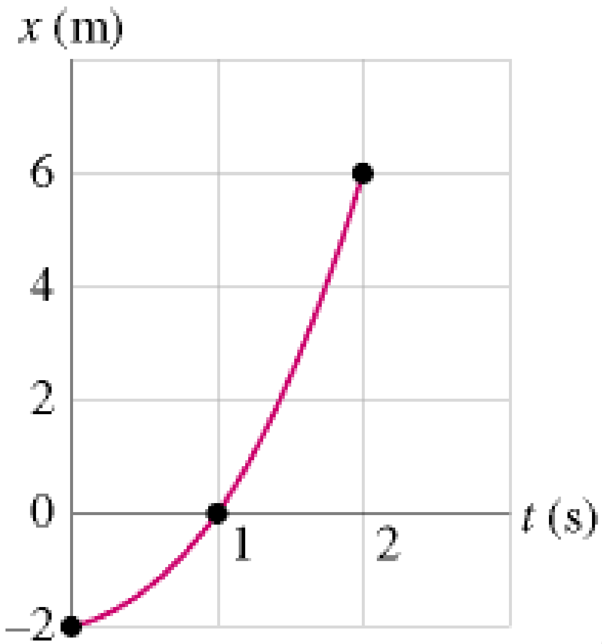
- **Constant acceleration**
- **Position (*x*) and velocity (*v*) as a function of time**
- **Note when clock starts (*t=0*), the position and velocity are x_0 and v_0**

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

Other 1D equations (in table 2-1) are restatements of these

One of HW Problems

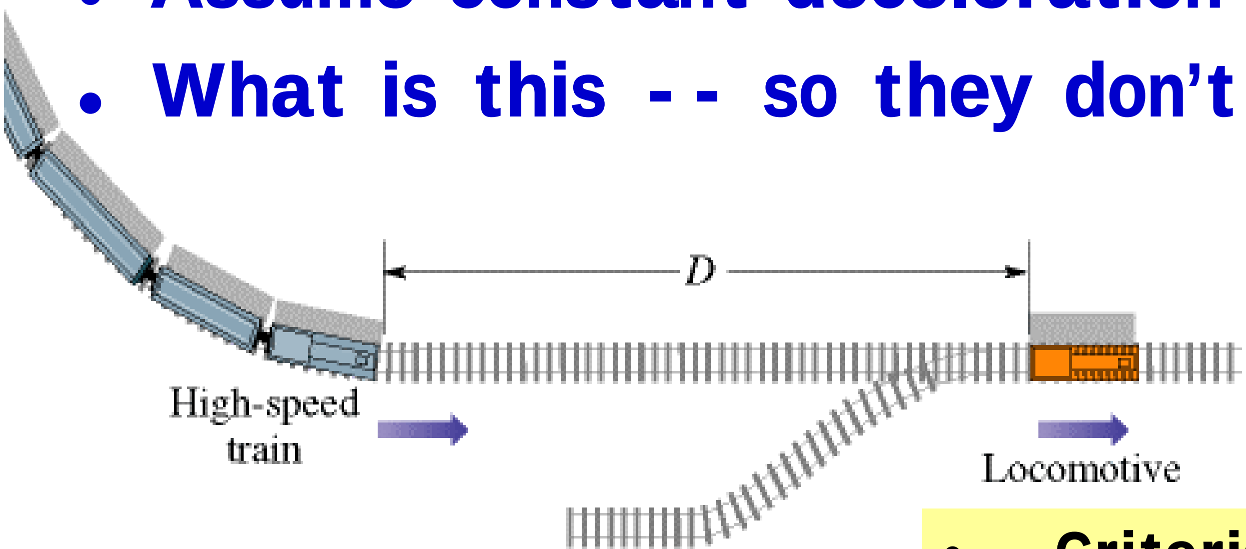


$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

- **Graph given is an expression of one of the derived equations.**
- **In this case, we are given every position (x) for every t for 0<t<2s**
  - **In particular, three black points satisfy equation**
- **Here, what we <u>don't know</u> are x_0, v_0, *and a*.**
- **Three relationships among x_0, v_0, *and a* are enough to find them!**

Another HW problem

- Initially, train going faster than locomotive
- Assume constant deceleration for train
- What is this -- so they don't collide?



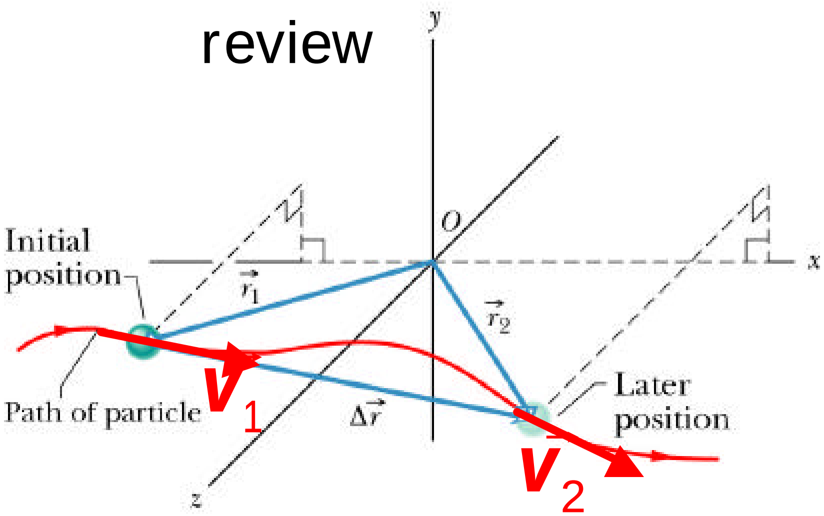
$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

- Criteria (limiting case):
- When they meet
  1. Just touch
  2. Have same speed
- 2 equations – 2 unknowns

Position, velocity, acceleration vectors

review



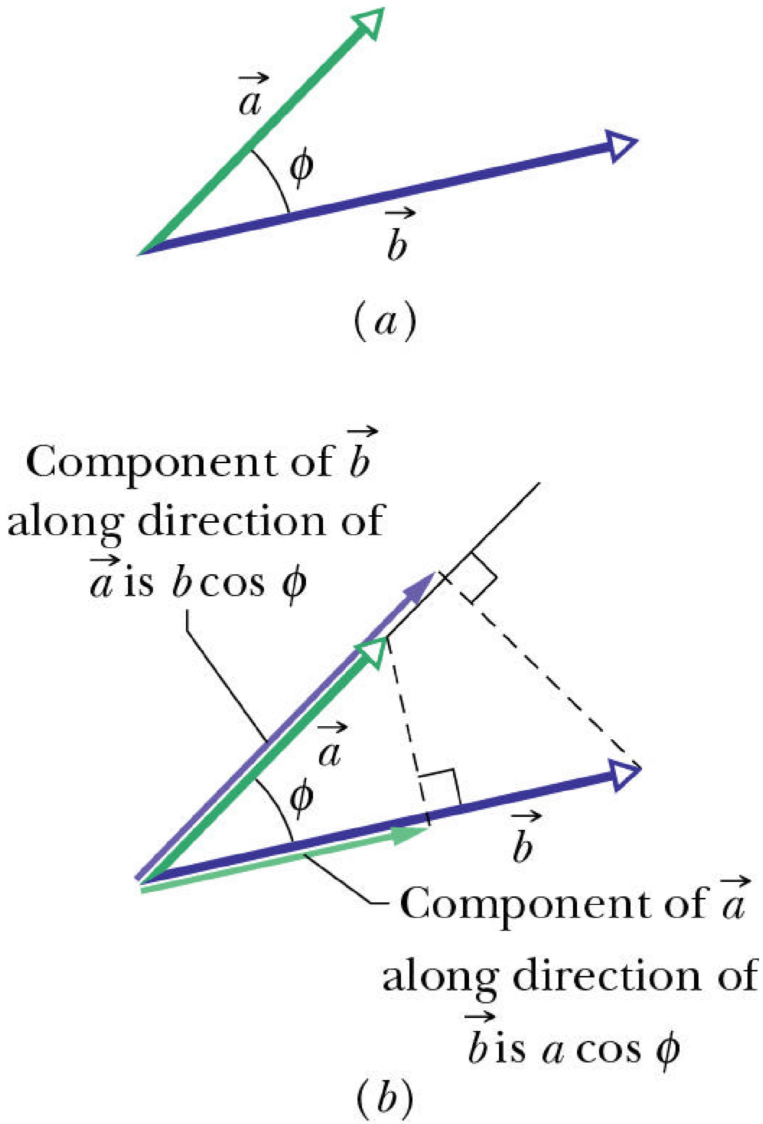
$$\vec{r}$$

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

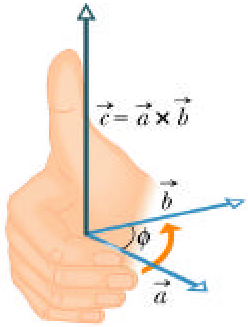
- **Position vector from origin to present location**
- **Velocity vector is <u>along trajectory</u>**
- **Acceleration vector can be any direction**

Scalar Product of Vectors

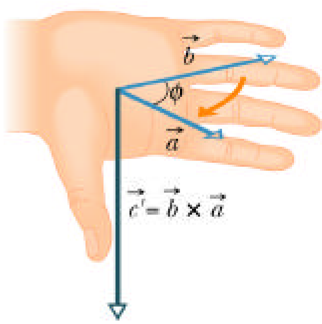


$$S \equiv \vec{a} \bullet \vec{b} = ab\cos f$$
$$= (a\cos f)(b)$$
$$= (a)(b\cos f)$$

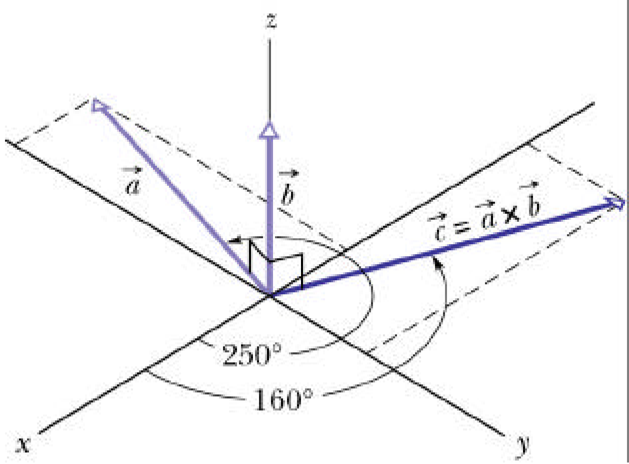
- The product of the <u>common</u> components
- Note limiting cases
  - ◆ f = 0 and f = 90^0
- Example of use: discussion of WORK (later)



(a)



(b)



Vector Product - 1

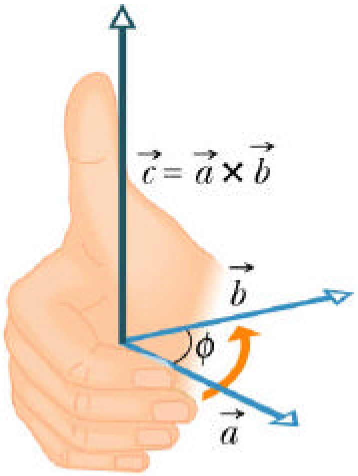
- **Needs 3D for description**
- **<u>Different</u> product of two vectors (from scalar pr.)**
  - **result a <u>vector</u>**
- **Note limiting cases**
  - f = 0 and f = 90^0
- **Examples of use: torque and magnetism**

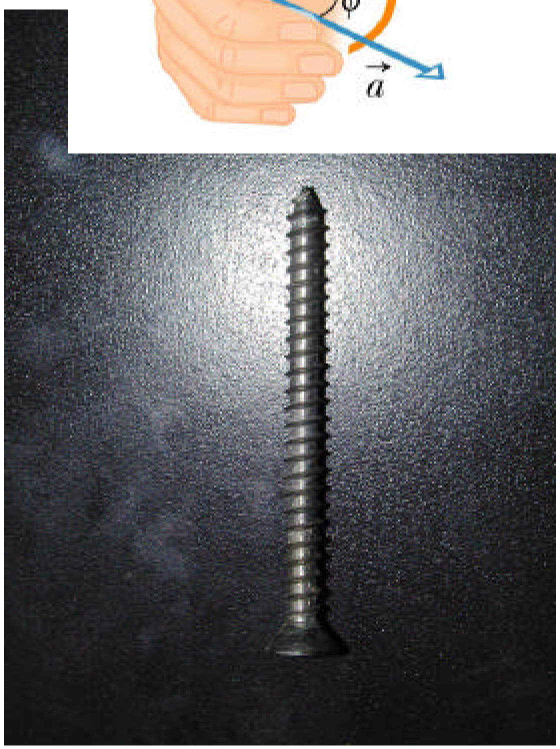
$$\vec{c} \equiv \vec{a} \times \vec{b}$$

$$|\vec{c}| \equiv c = ab \sin f$$

$$\vec{c} \perp \vec{a} \quad \text{and} \quad \vec{c} \perp \vec{b}$$

Vector Product - 2





$$\vec{c} \equiv \vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$$

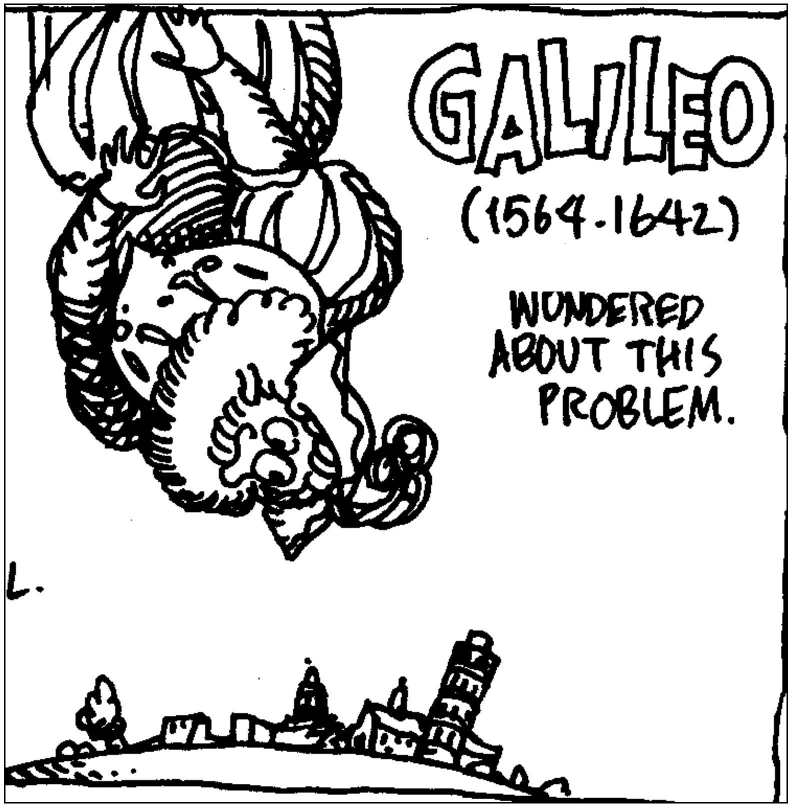
$$|\vec{c}| \equiv c = ab \sin f$$

$$\vec{c} \perp \vec{a} \quad \text{and} \quad \vec{c} \perp \vec{b}$$

- **Two methods to remember direction of vector product**
  - **Right hand rule**
  - **Direction that a right hand screw advances when rotating**

$$\vec{a} \text{ into } \vec{b}$$

Reminder: Real World Motion (on Earth)



GOT IT RIGHT!

If all other forces (friction, air drag, …) are small and can be neglected, then

- **horizontal motion has zero acceleration**
- **vertical motion has a universal value of acceleration:**

$a = g = 9.8\ m/s^2 = 32\ ft/s^2$

DOWN!

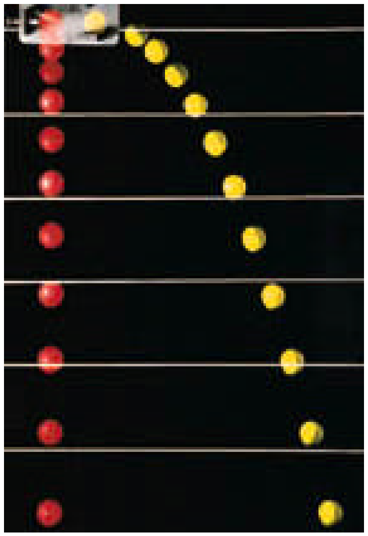
Juggler and Balls

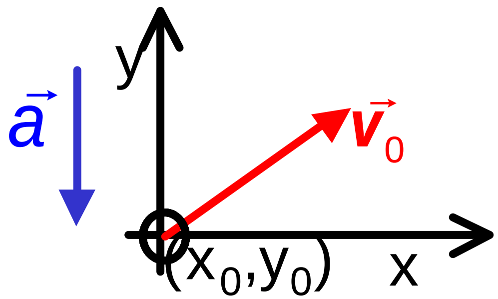
one ball-parabolic path.MOV

3balls-parabolic.MOV

- **horizontal and vertical acceleration when ball is in the air (in FREE FALL)**
- **When ball is <u>touched</u>, acceleration different**
- **Note trajectory is <u>parabola</u>**

2D motion near Earth's surface

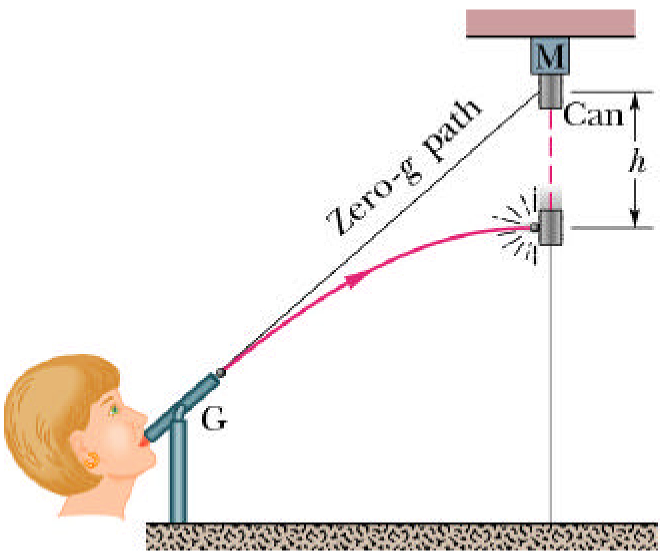




Critical points:

- **Relative to origin, only two vectors:** $\vec{a} = -g\,\hat{j}$ and $\vec{v}_0$
- **So motion in a plane: 2D**
- **<u>All bodies experience</u> acceleration = -g in –y dir independent of the horizontal motion (so long as friction, drag, etc, can be neglected)**

Demo: shoot the monkey!

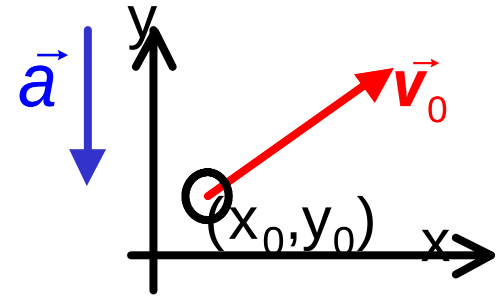


$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

Trajectory: Each dimension has const acceleration

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$



- **Horizontal: $a_x = 0$**
- **So, it terms of initial x-component velocity and position**

$$v_x = v_{0x}$$

$$x = x_0 + v_{0x} t$$

- **Vertical: $a_y = -g$**
- **So, in terms of initial y-component velocity and position**

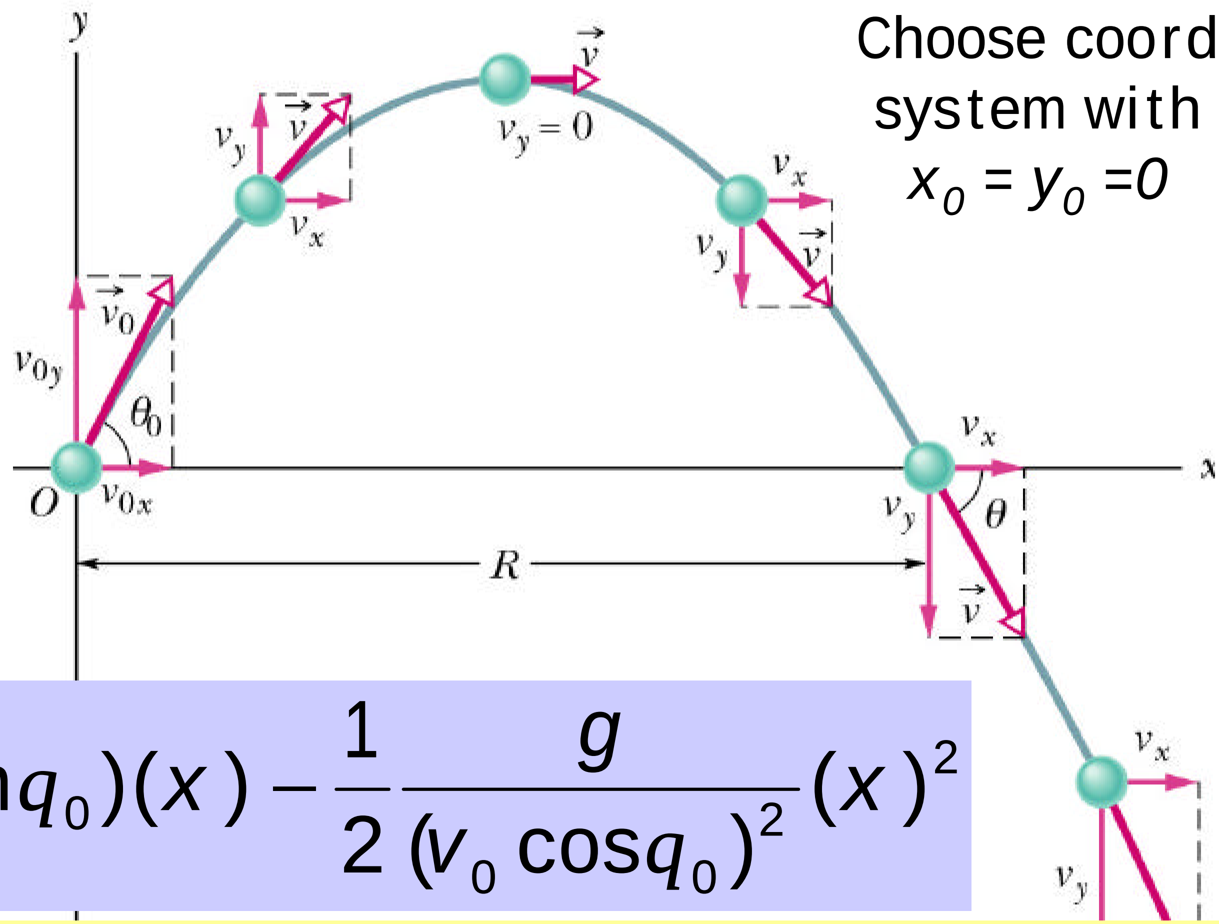
$$v_y = v_{0y} - gt$$

$$y = y_0 + v_{0y} t - \frac{1}{2} g t^2$$

Trajectory equation $\vec{a} = -g\,\hat{j}$

- *x-comp of v is fixed*
- *y-comp of v is getting more negative at a fixed rate (g)*

Choose coord system with $x_0 = y_0 = 0$



$$y = (\tan\theta_0)(x) - \frac{1}{2}\frac{g}{(v_0\cos\theta_0)^2}(x)^2$$

balls-traj eq.MOV

Movie showing parabolic trajectory for "projectile" is ALWAYS the case while ball is in the air (FREE FALL). Specific shape of parabola depends on INITIAL CONDITIONS.

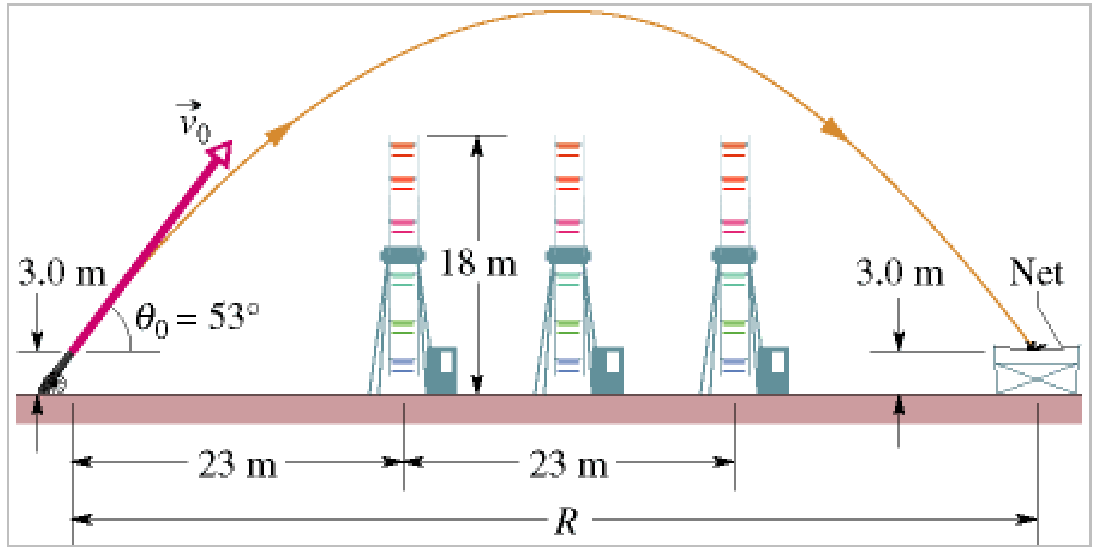
Work on the consequences

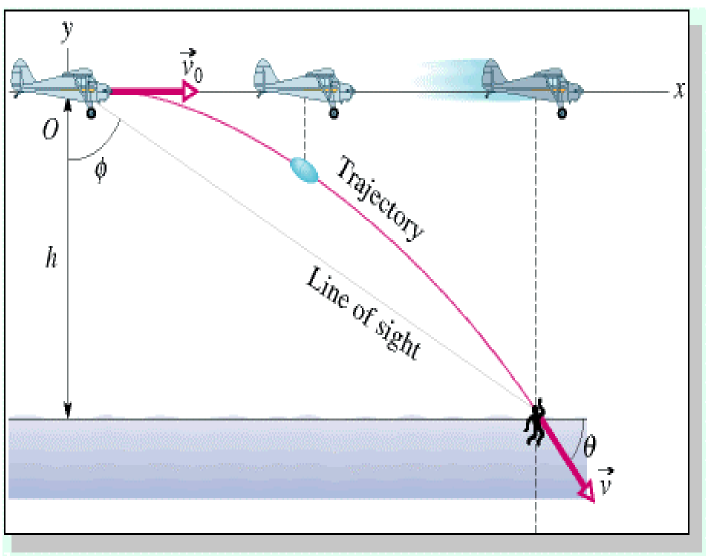
$$y = (\tan q_0)(x) - \frac{1}{2}\frac{g}{(v_0 \cos q_0)^2}(x)^2$$

Range,…

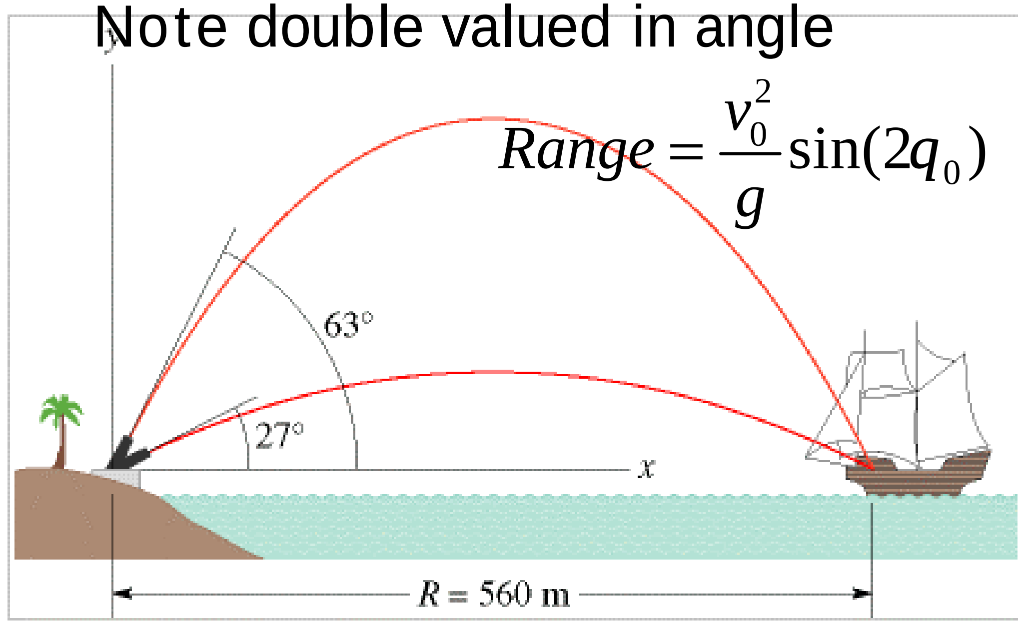
Sample problems: 4-6, 7, 8 + HW probs

Same relation: different initial conditions





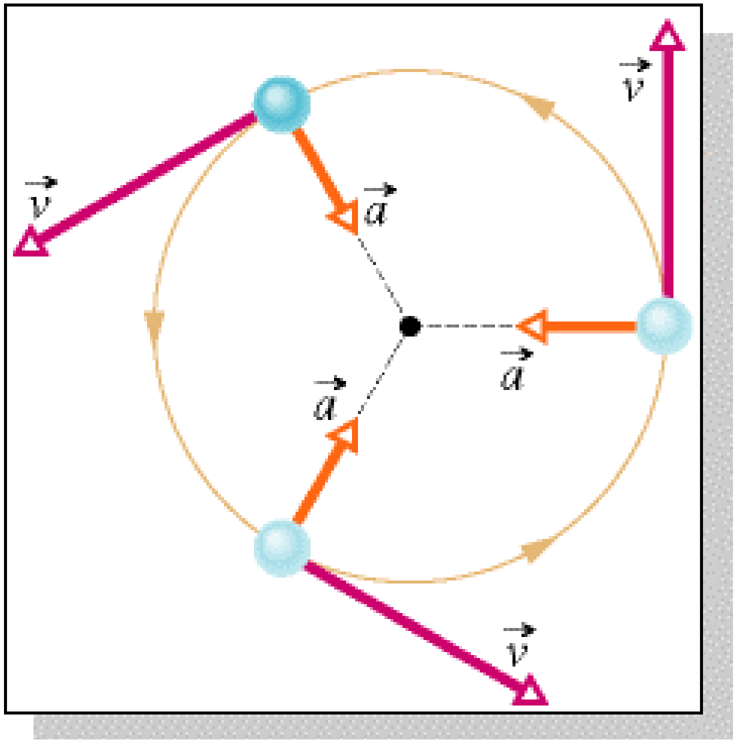
Click on the image to start the simulation



Note double valued in angle

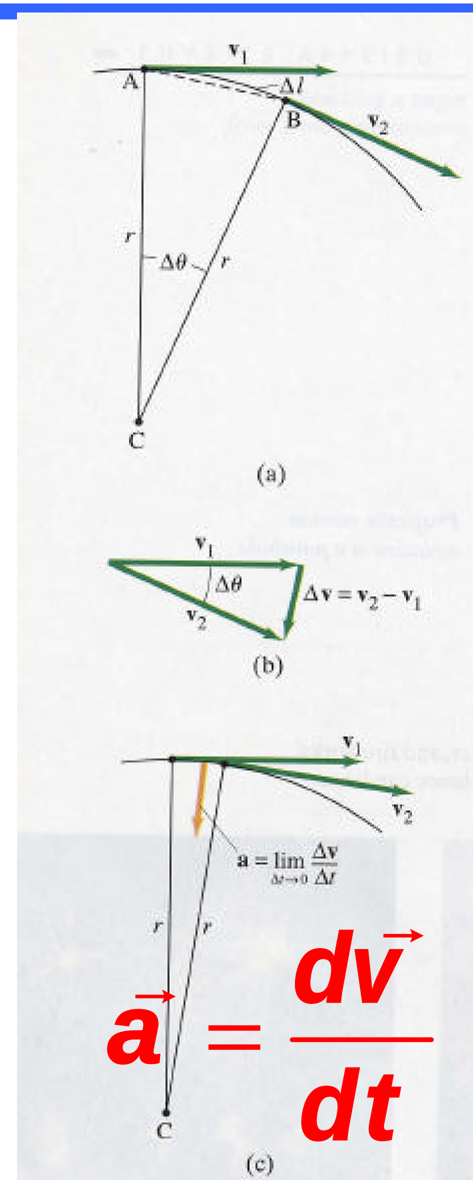
$$Range = \frac{v_0^2}{g}\sin(2q_0)$$

Motion in a Circle at Constant Speed



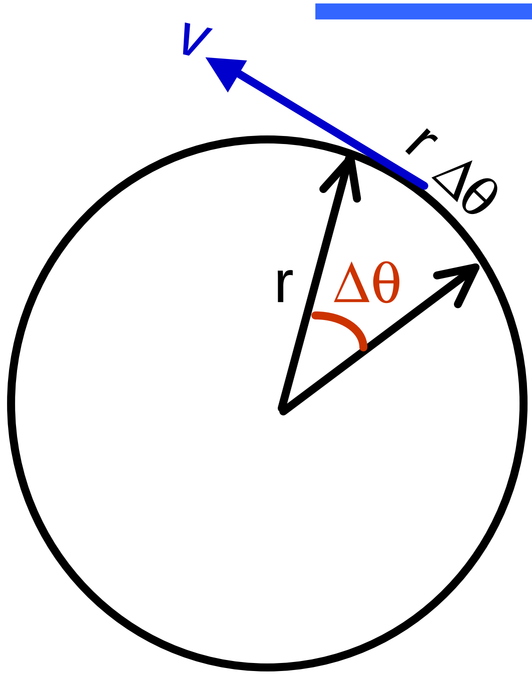
circ motion- v chg.MOV

Graphical argument for the <u>sign</u> of the centripetal acceleration … movie



$$\vec{a} = \frac{d\vec{v}}{dt}$$

Circles and Circular Motion



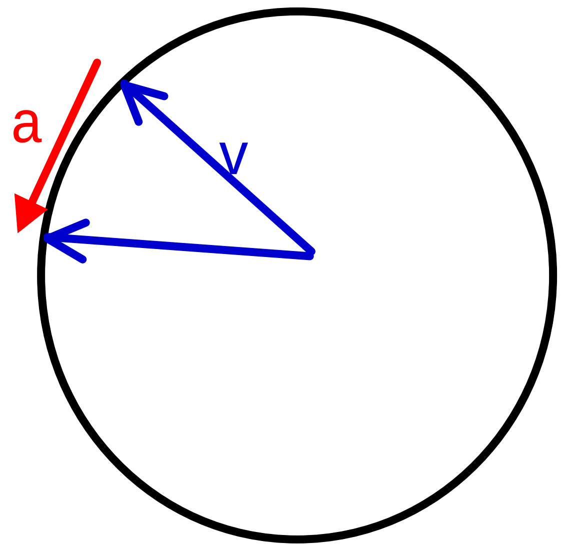
- **Object moving at constant <u>speed</u> *v* in a <u>circle</u>, rad *r***
- **Velocity vector always perpendicular to position vector**
- **Assume time for one circumference is *T***

$\vec{v} \perp \vec{r}$ are changing

$|\vec{v}| = v = \text{constant}$

$|\vec{r}| = r = \text{constant}$

$vT = 2p\,r$



Finish for the Day

- Make sure to understand chapter 4
- We have discussed
  - Trajectory motion
  - Circular motion
- Next time
  - Relative motion (complete chapter 4)
  - Forces (chapter 5)
- Reminder
  - First homework due on Monday (ch 2, 3, 4) … bring to class and deposit in indicated box <u>before</u> class
  - Those unable to make lecture on Monday, make sure it gets to me then or beforehand (under office door) or my mailbox (#1) across from physics dept. office on 7th floor Pupin