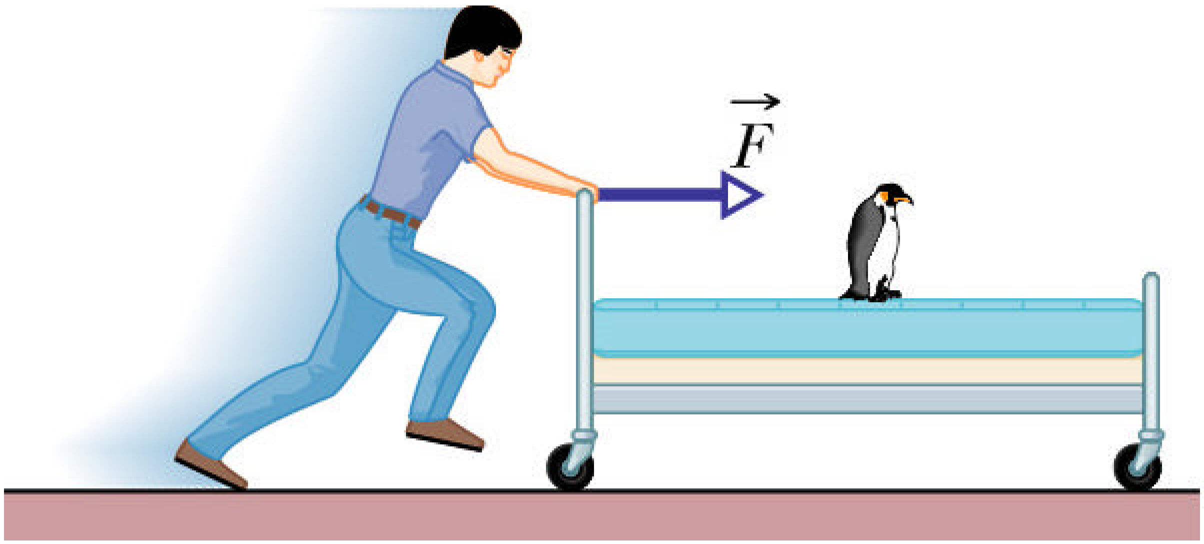
Announcements and Reminders

- Midterm on Monday covers chapters 1-6
- Sample exam posted with solutions
- Third homework not due until next Wednesday
- Thursday recitation will go over homework 2 and the sample midterm posted at the site
- Today's lecture:
  - Continue discussing work and energy
  - See where they come from and some uses

review

Quantitative Definition of Work



Work done =

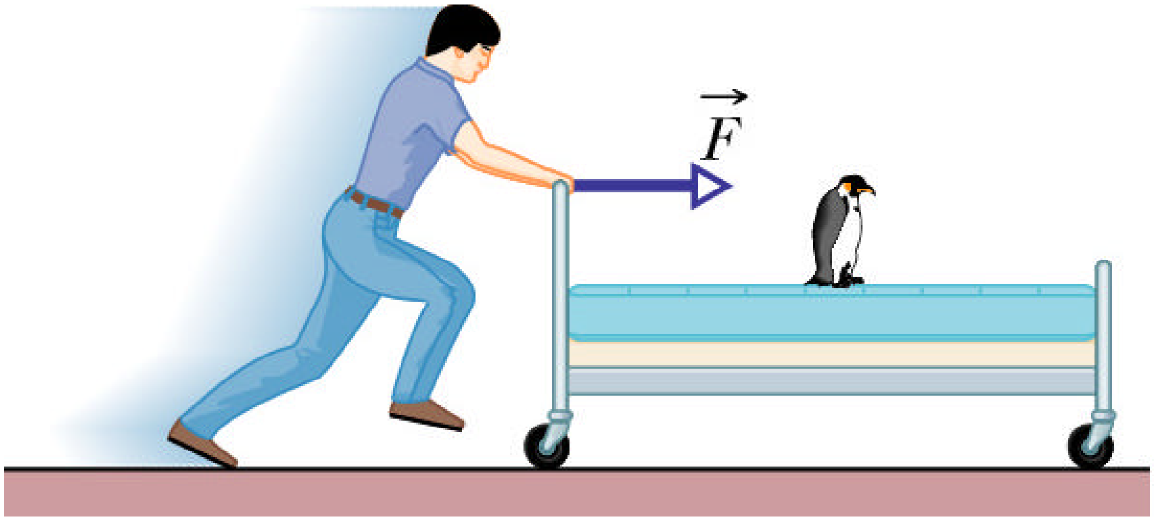
$$W = F \times \begin{Bmatrix} \text{displacement} \\ \text{in same direction} \end{Bmatrix}$$

$$\Delta W = \vec{F} \bullet \Delta\vec{r}$$

- **Man does work on <u>system</u> of bed and penquin**
- **Note that normal and gravitational forces do NO work in this case.**
- **Only force moving system in <u>same</u> direction does work**

review

Work-Energy Theorem



Take $\boldsymbol{F}$ in $\boldsymbol{x}$-direction and $\boldsymbol{F}$ dependent on $\boldsymbol{x}$:

Each increment in $\Delta \boldsymbol{x}$ produces $\Delta \boldsymbol{W} = \boldsymbol{F} \Delta \boldsymbol{x}$

Add up all the work to get total work: $\boldsymbol{W}$

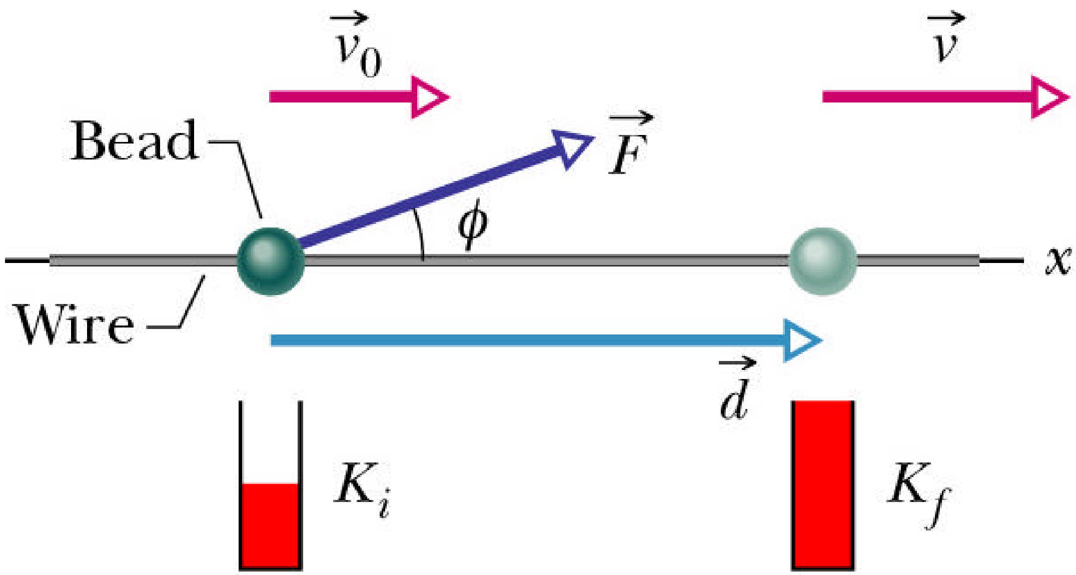
$$W = \tfrac{1}{2} m v^2 - \tfrac{1}{2} m v_0^2$$

$$W = K - K_0$$

Work=change in Kin. Energy
General Theorem
so long as no energy dissipated

review

Bead on a Wire with constant Force (and acceleration)



$$a_x = \frac{F_x}{m} \qquad W = F_x d$$

$$v^2 = v_0^2 + 2\frac{F_x}{m}d$$

$$W = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2$$

$$W = K_f - K_i$$

$$K \equiv \tfrac{1}{2}mv^2$$

No friction

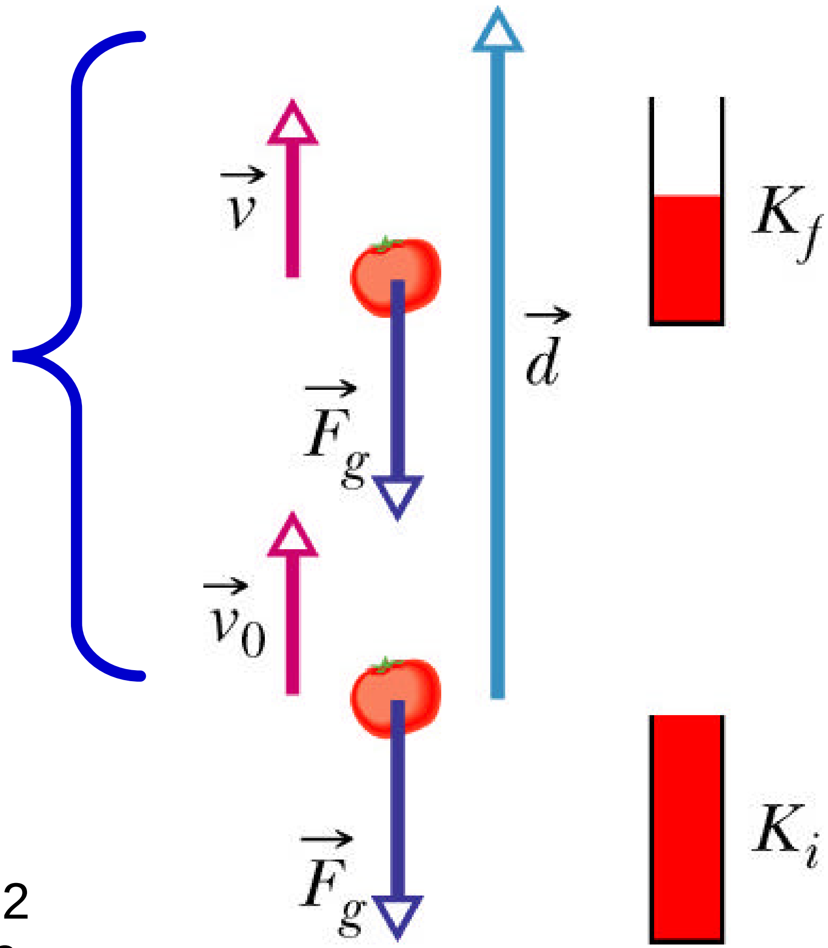
- Work is + or -, depending on whether force has component in the same direction as displacement or opposite to the displacement
- Positive work can increase Kinetic Energy and *vice versa*

1D example: free body moving up

$$K = \tfrac{1}{2} m v^2$$

$$W = -mgd$$

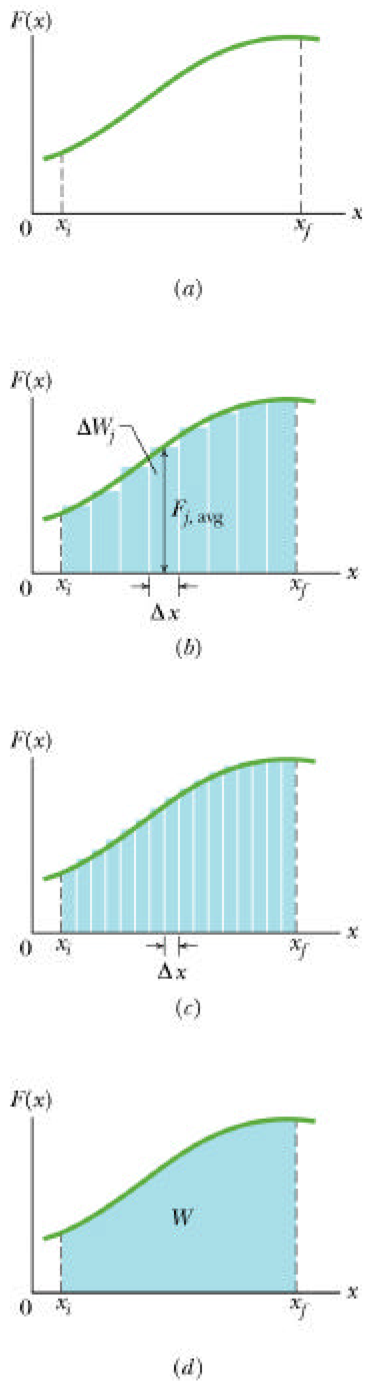
$$K_0 = \tfrac{1}{2} m v_0^2$$



$$W = K - K_0$$

$$-mgd = \tfrac{1}{2} m v^2 - \tfrac{1}{2} m v_0^2$$

$$v^2 = v_0^2 - 2gd$$



Work-Energy Theorem – 1D (General force and accel.)

Take ***F*** in ***x***-direction and ***F*** dependent on ***x***:

Each increment in Δx produces $\Delta W = F \Delta x$

so $W = \int_{x_1}^{x_2} F(x)\,dx$

Work=change in Kin. Energy
General Theorem
so long as no energy dissipated

Work-Energy Theorem correct in >1D also

For F in any direction, $\Delta W = \vec{F} \bullet \Delta\vec{r}$

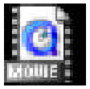
$$\text{so } W = \int_{x_1}^{x_2} F_x dx + \int_{y_1}^{y_2} F_y dy + \int_{z_1}^{z_2} F_z dz$$

$$\text{or } W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \bullet d\vec{r}$$

Work=change in Kin. Energy

General Theorem

so long as no energy dissipated



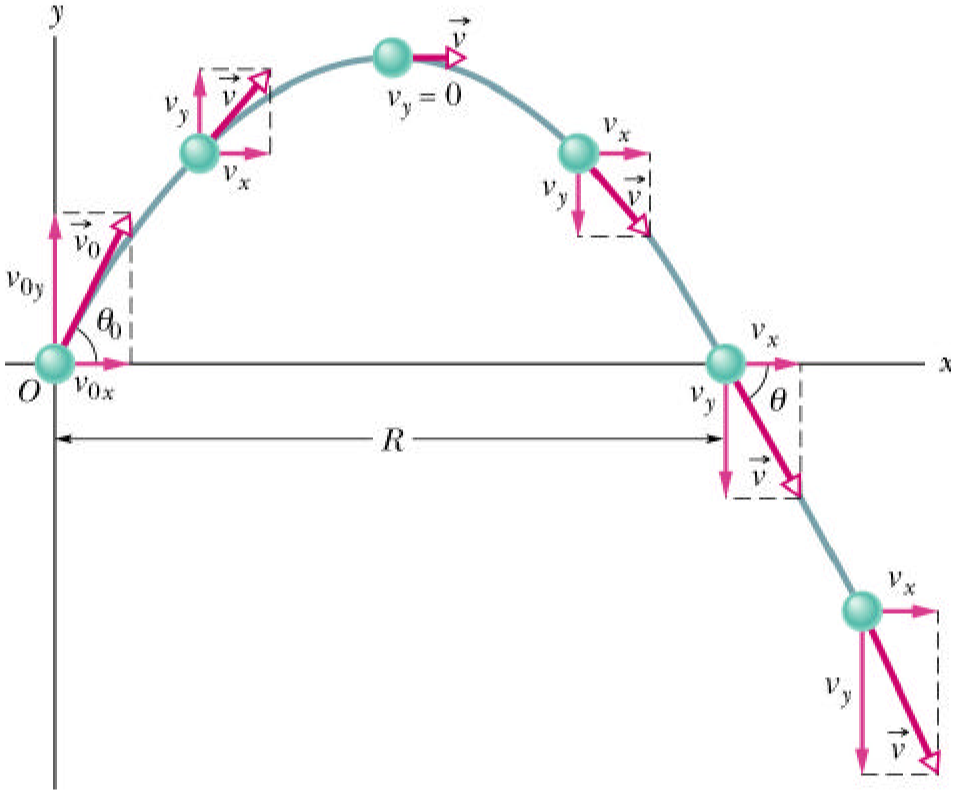
work by grav on irregular motion.MOV

Example is gravity force when object moves in arbitrary direction

First part of

"work by grav on irregular motion.MOV"

Body in gravitational trajectory



- **Even though object moves in 2 (or 3) dimensions, only the force direction contributes work**
- **Gravity force is in *–y* dir**

Work done by gravity in allowing object to fall *H* is

$$W=+mgH$$

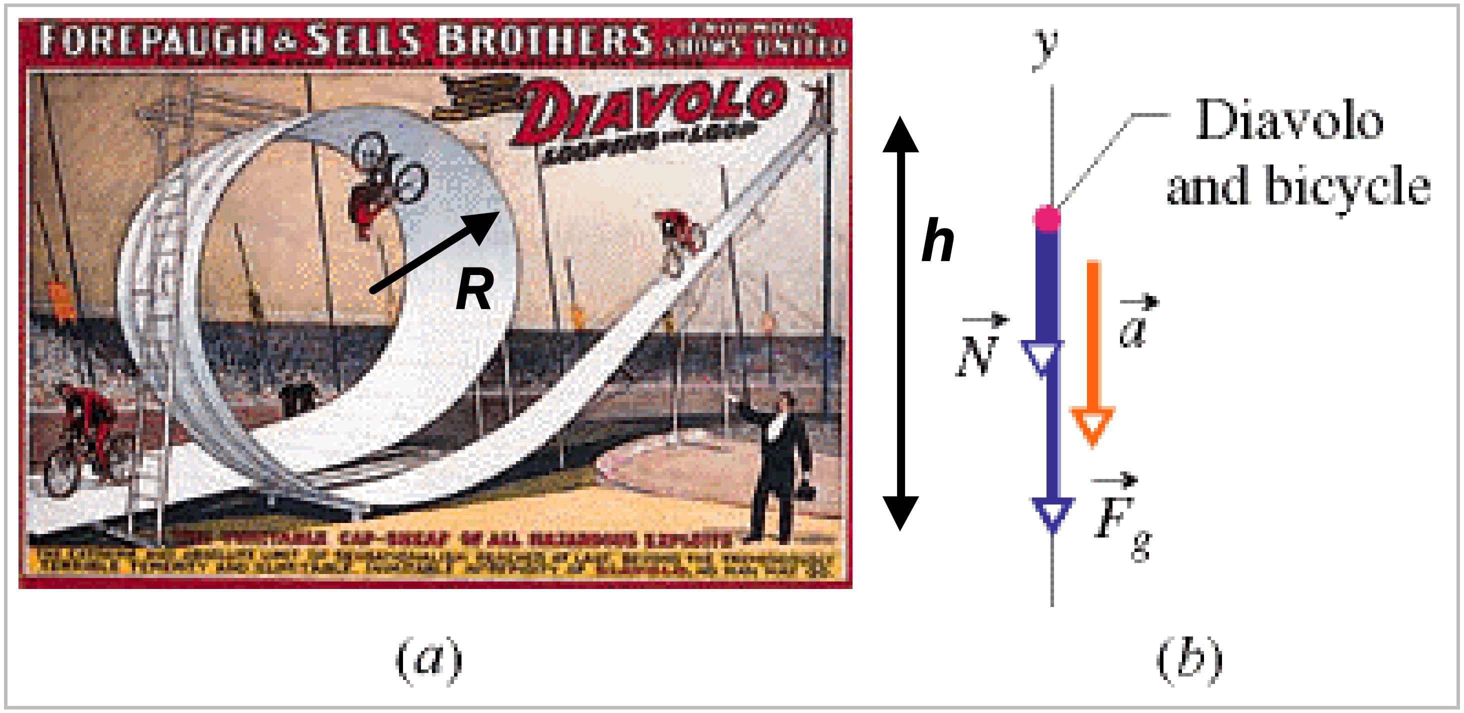
Work done by gravity in raising an object by *H* is

$$W=-mgH$$

Sample Prob 6-7: combining concepts

Our "hot wheels" demo!

What v req'd at top?



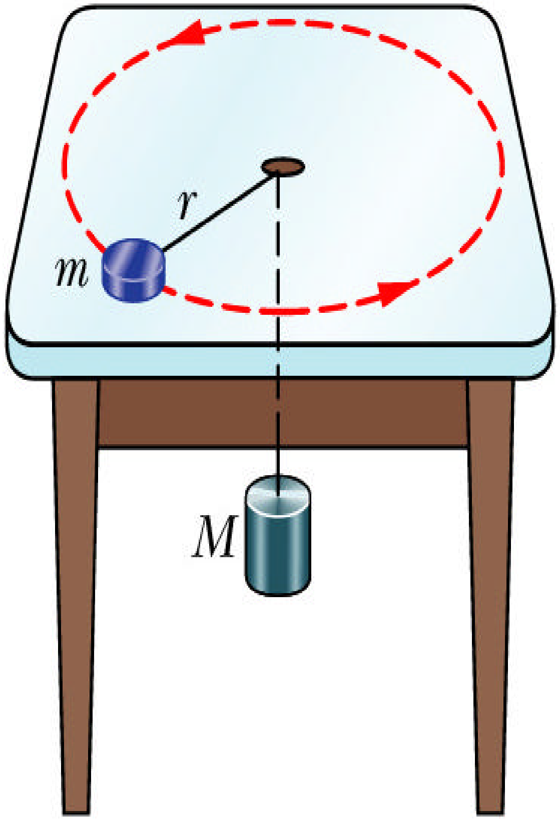
(a) (b)

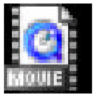
How high *h* above ground to release?

No pedaling!

Work for puck moving in circle

- Pix "work on object in uniform circle-SNDRS.MOV"
- Note that, in general, it takes <u>no work</u> to keep an object moving in a circle
- other examples:
  - ◆Conical pendulum
  - ◆Moon around Earth
  - ◆Earth around Sun

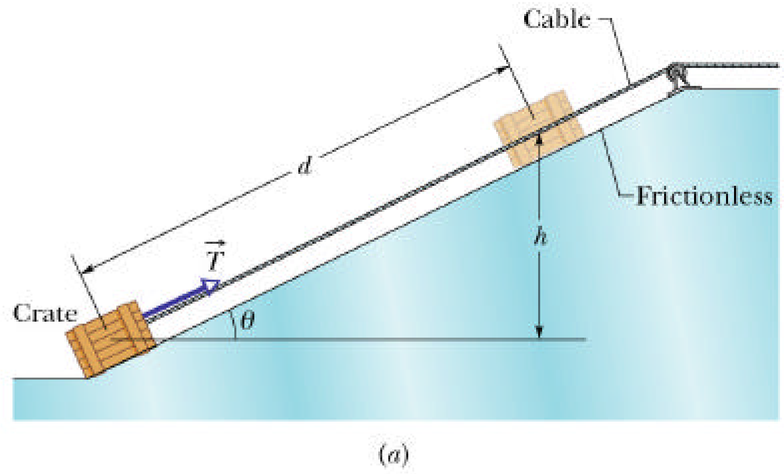


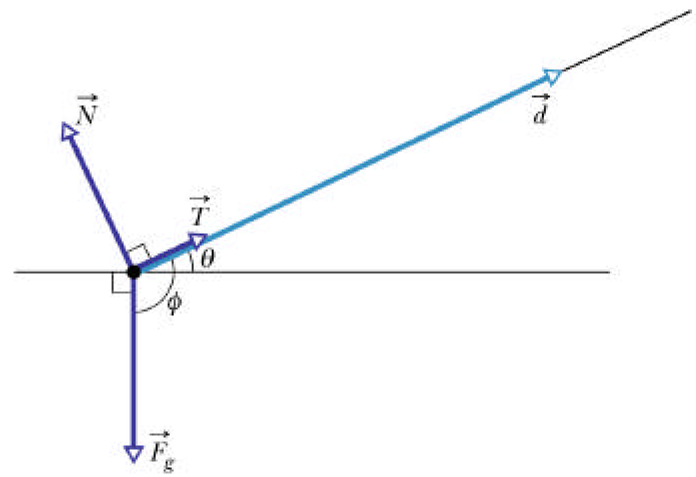


work on object in uniform circle.MOV

Sample Problem 7-5

- **Find the work required to gently move the crate up the ramp (no friction)**
- **W=-mgh (from our theorem) is done by the gravitational force**
- **Check how much work done by the person pulling the rope**
  - ***T = mgsinq***
  - ***W=Td=mgdsinq=+mgh***
  - **In order to move it up, we do work equal and opposite to work done by gravity**





Work by gravity is NOT converted to Kinetic Energy

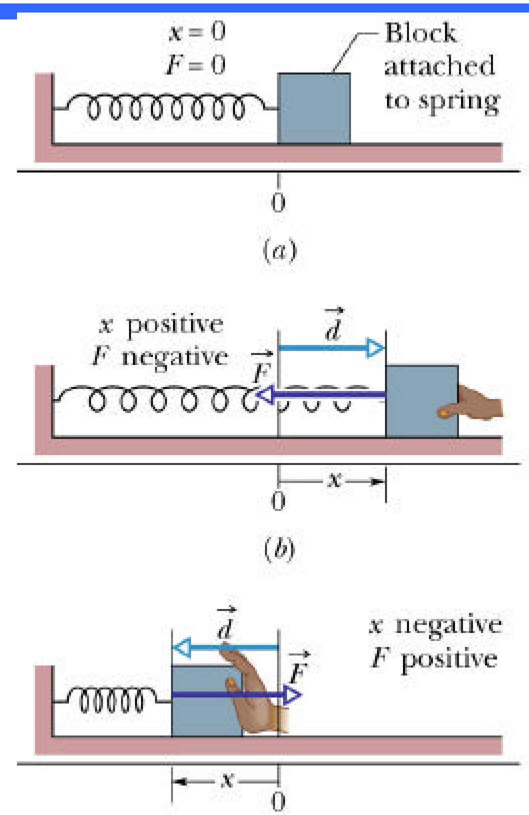
This energy is <u>stored!!</u>

Spring

- **Another case where do work, but not to create kinetic energy**
- **Work that must be done to push or pull on a spring a distance, *x=d* , is different:**
  - ***F* is <u>changing</u> while *x* is changing**
  - ***F(x)=-kx***
- **From <u>equilibrium</u>, work to move to x is**

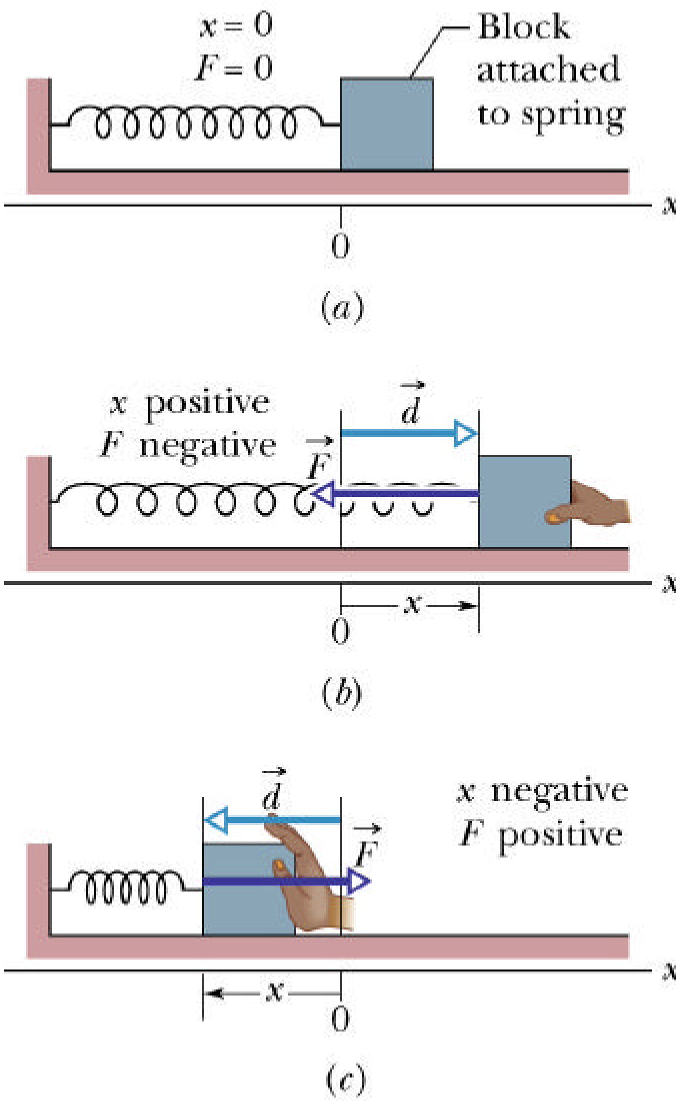
$W = -kx^2/2$

$$W = \int_{x_1}^{x_2} F\,dx$$
$$= -k\int_{x_1}^{x_2} x\,dx$$
$$= -\tfrac{1}{2}kx^2 \Big|_{x_1}^{x_2}$$
$$= \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2$$



At the end, this energy also stored … but in the spring

Let the spring and mass go!



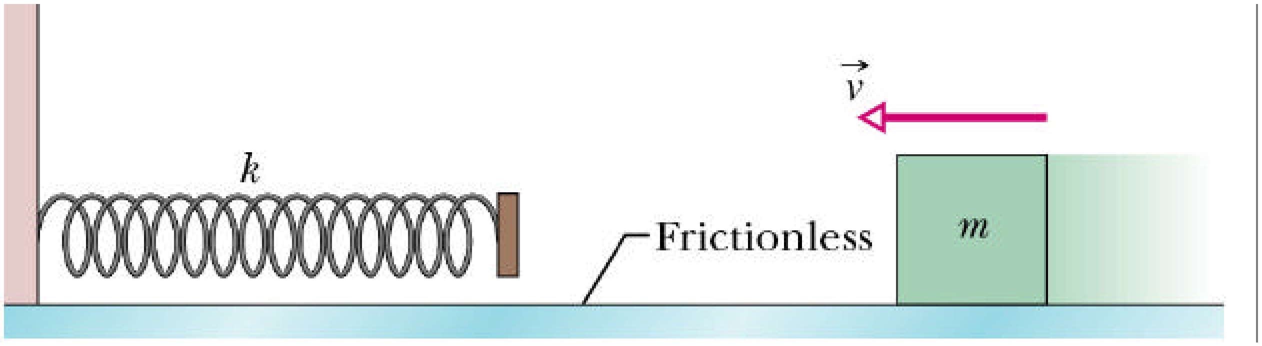
- If compress or stretch the spring by $\Delta x = \pm d$ and then let it go, the spring does work and the mass gains <u>kinetic energy</u>
- Find speed when it passes equilibrium ($x=0$)

$$\Delta W = K_2 - K_1$$

$$\tfrac{1}{2}k(\pm d)^2 = \tfrac{1}{2}mv^2$$

$$v = d\sqrt{\frac{k}{m}}$$

Sample prob 7-8 (reverse of last case)



- **Convert kinetic energy into stored energy of spring**
- **Mass encounters spring (*k*)**
  - mass does work on the spring, and loses *K*
  - *m* slows down and stops
  - how far does it go?

$$K_i = \tfrac{1}{2} m v^2$$

$$K_f = 0$$

$$K_f - K_i = W = -\tfrac{1}{2} k d^2$$

$$\tfrac{1}{2} m v^2 = \tfrac{1}{2} k d^2$$

$$d = v\sqrt{\frac{m}{k}}$$

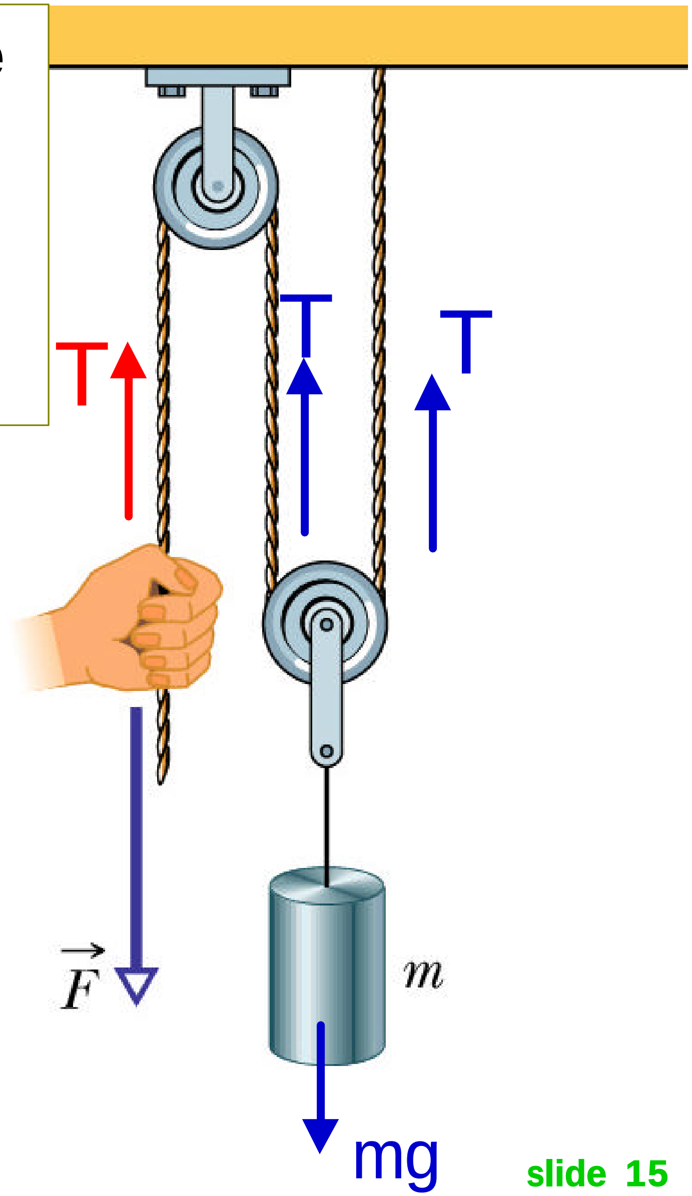
Problem 7-15 – counterintuitive? … think again!

$$F = T$$

$$2T = mg$$

$$F = \frac{mg}{2}$$

- Raise weight with force equal to only half its weight! Cheat?
- No! Called <u>mechanical advantage</u>! Happens!

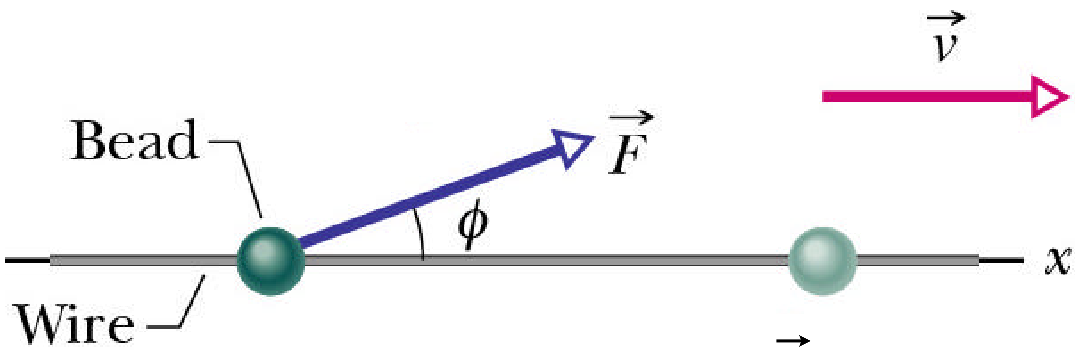


Hand moves distance *d*, then the mass *m* moves up by *d/2*

$$Fd = mgd/2$$

So Work done by hand = Work done by gravity

Power



$$P \equiv \frac{dW}{dt} = \frac{d(\vec{F} \bullet \vec{r})}{dt}$$

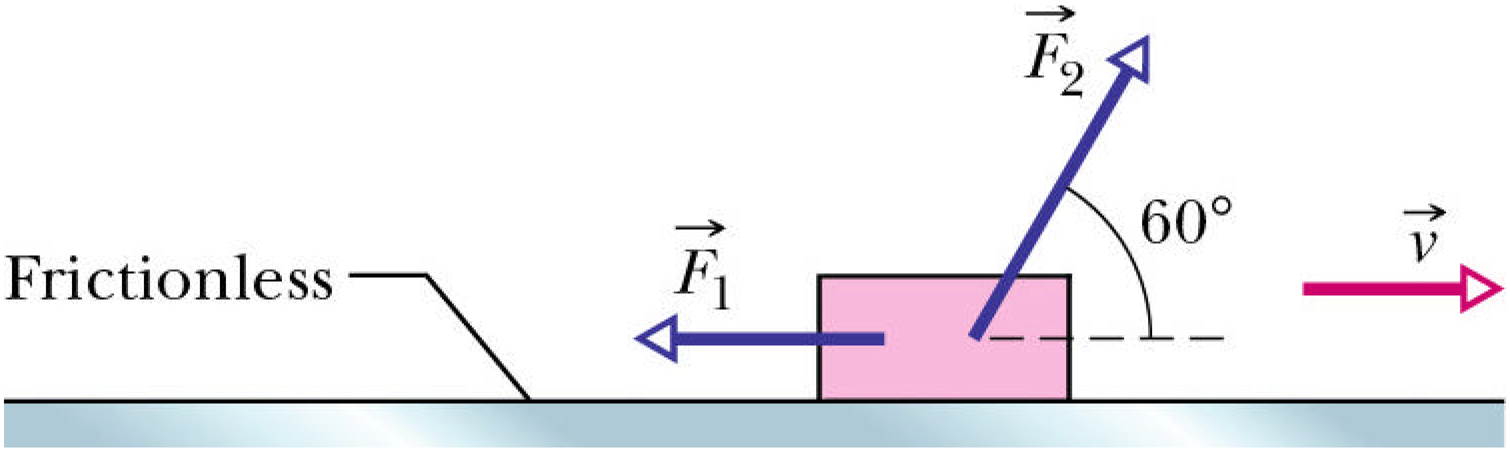
$$= \vec{F} \bullet \frac{d\vec{r}}{dt} = \vec{F} \bullet \vec{v}$$

- Bead moving with velocity *v* being pushed with constant force, *F*
- Bead's motion measured by r (vector)
- Power º Rate at which work is done
- Power depends on force and speed along direction of force

Units of Power are (all the same)

N-m/s or **Joules/sec** or **Watts**

Sample Prob 7-10



- Note gravity does no work? Why?
- F_1=2.0 N
- F_2=4.0 N
- v =3.0 m/s
- How much power?

$$P_2 = F_2 v \cos 60^\circ = +6W$$

$$P_1 = -F_1 v = -6W$$

Keep in Mind

- Review and do lots of problems to prepare for midterm on Monday on chapters 1 – 6
- Remember
  - Thursday recitation session …
  - Third homework due next Wednesday
- Next time (1 week)

  Continue with the saga of work and energy

  Read chapter 8