Lecture 9

- **Midterm 1 returned today (shelf outside 301 entry. Discussed statistics.**
- **See website for the exam, the solutions, and the statistics associated with the grading.**
  - **Partial credit has been allotted in a consistent manner.**
  - **If you believe there has been an error made in grading your exam, fill in a regrade request, available from Ugrad secretary in the Physics office. (See **exam specifics webpage.) Any request must be made within two weeks of today.**
- **Today … continue potential energy functions and conservative forces; then go on to chapter 9 on how to apply what we know so far to complex systems.**
- **Read carefully chapters 9 and 10**

review

Conservative Force? How do we tell?

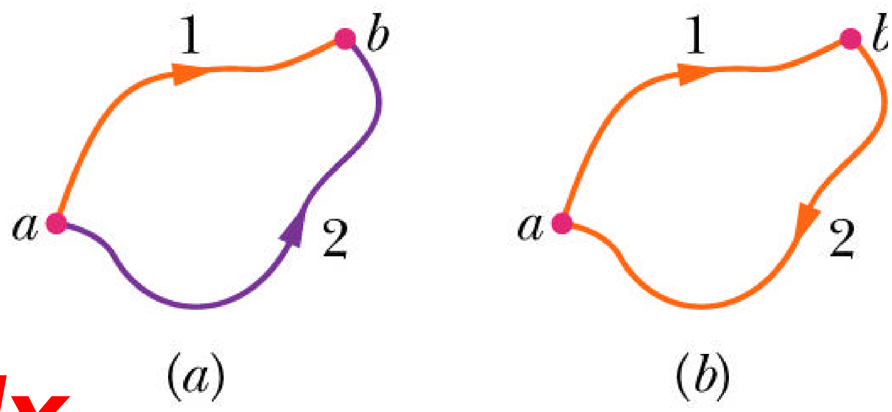
- Recall movie "work by grav"
- Conservative force: doesn't depend on path

$$\Delta U \equiv U_b - U_a$$

$$\Delta U \equiv -W_{ab} = -\int_a^b F dx$$

$$U_{grav}(y) = mgy$$

$$U_{spring}(x) = \tfrac{1}{2} kx^2$$



If depends on path, the potential cannot be defined and force is non-conservative!

review

Work Energy Theorem ®
Conservation of Mechanical Energy

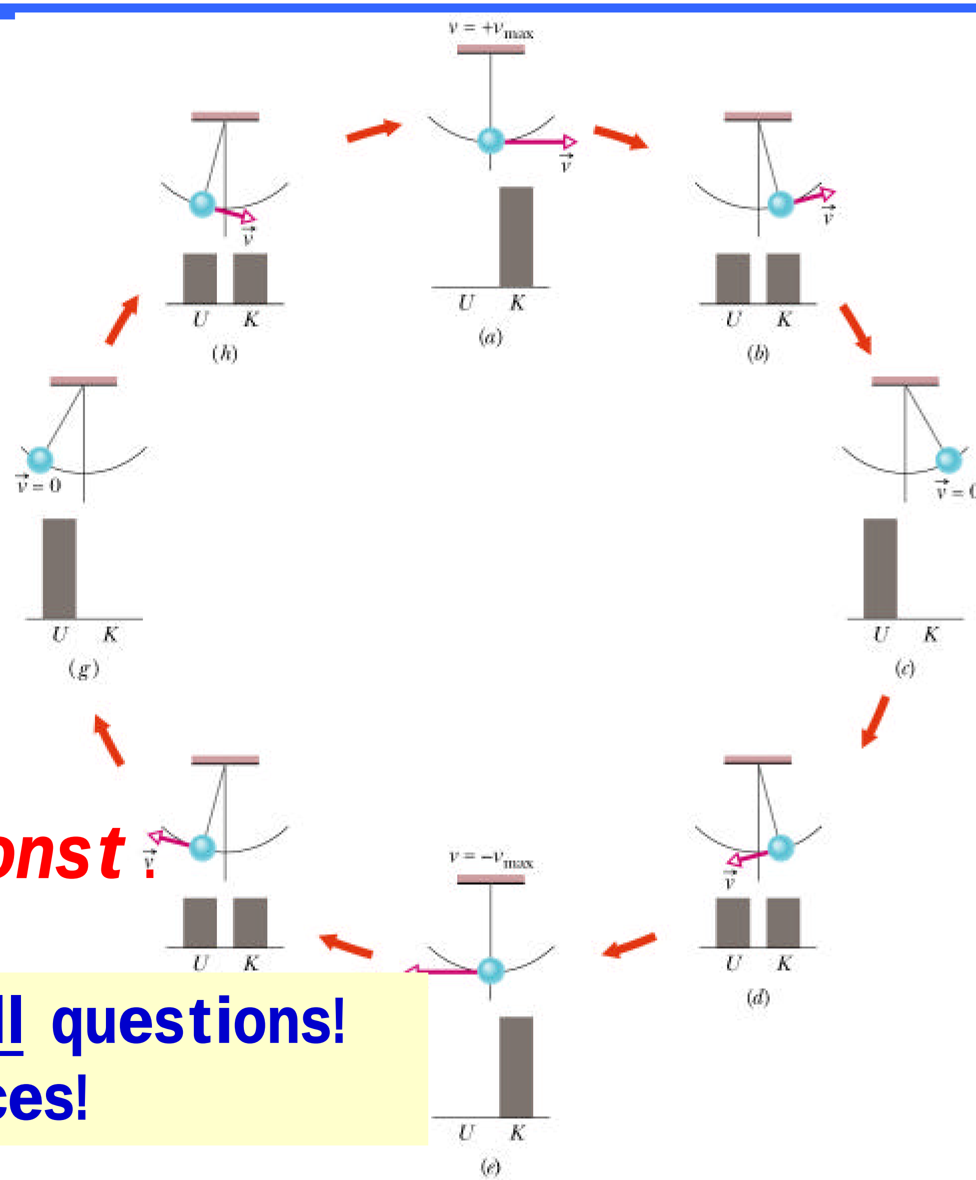
$$\Delta W = K_2 - K_1$$

$$U_2 - U_1 = -\Delta W$$

$$U_1 + K_1 = U_2 + K_2$$

Pendulum illustrates conservation of mechanical energy

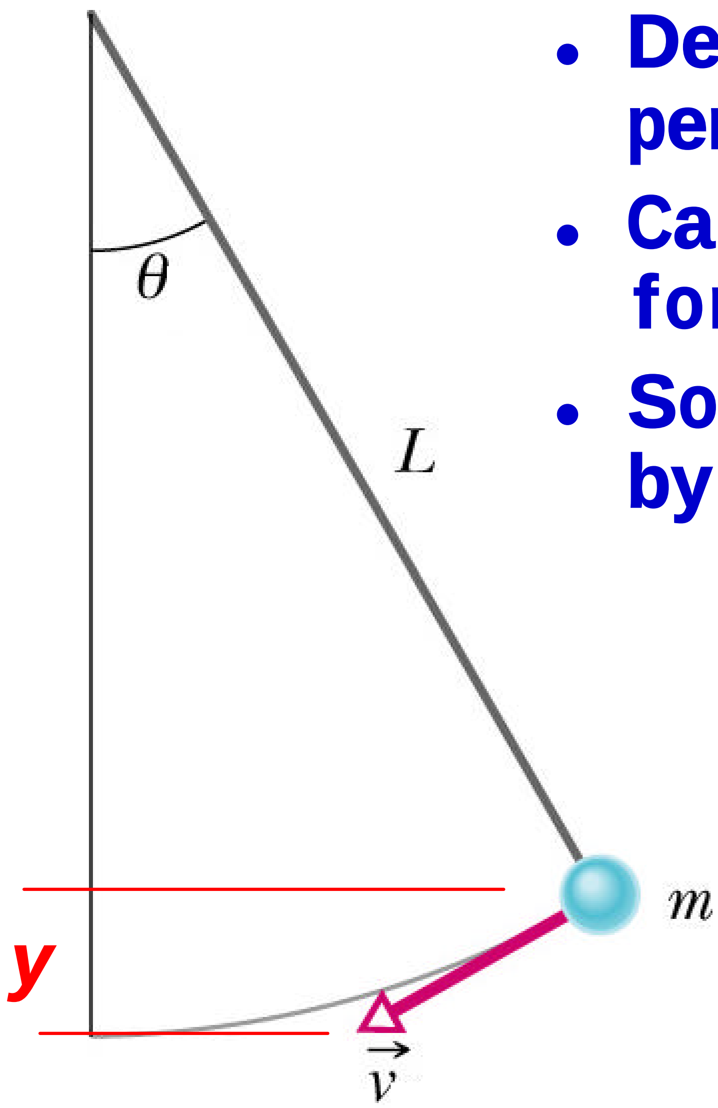
$$mgL(1-\cos q) + \tfrac{1}{2}mv^2 = const.$$



Note that doesn't answer all questions!
For x(t), better to use forces!

review

Pendulum Problem



- Describe the motion of a pendulum
- Can be done by using forces
- Some aspects can be done by using energy

$$y = L - L\cos q$$

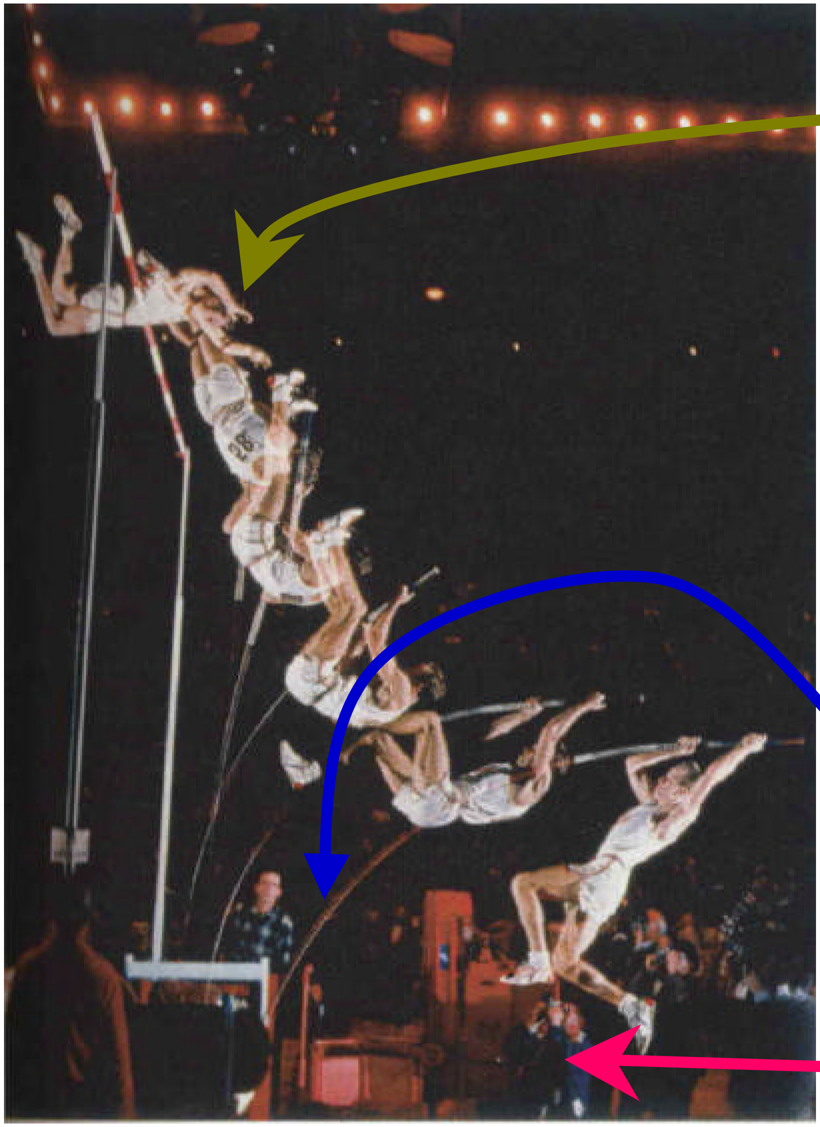
$$U = mgL(1 - \cos q)$$

$$K = \tfrac{1}{2}mv^2$$

$$U + K = E \text{ is constant}$$

review

Changing work into energy in real world



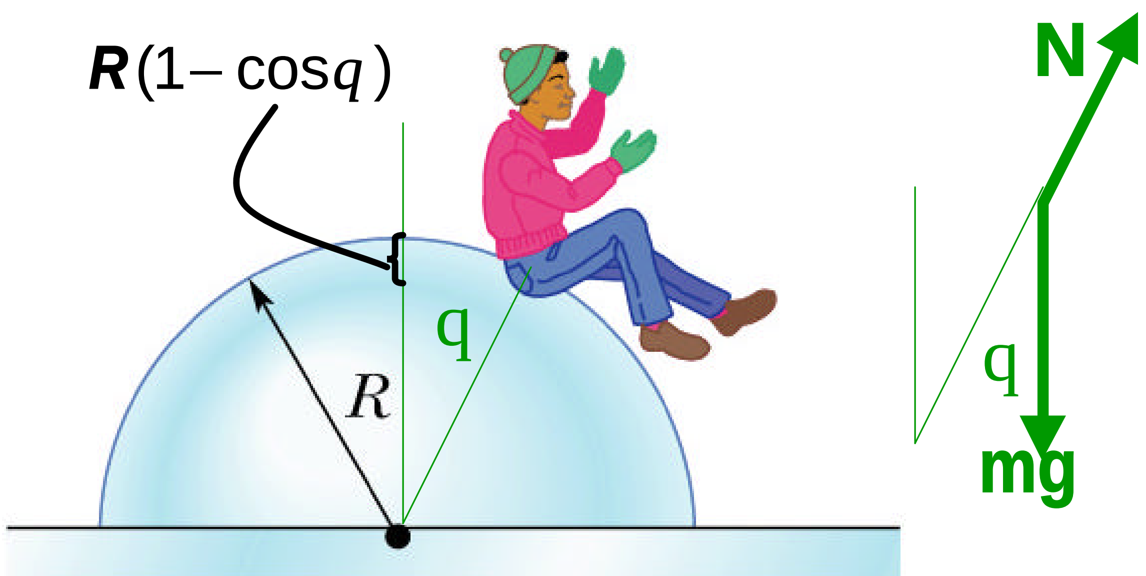
Pole potential "spring" energy becomes man's gravitational potential energy

Kinetic energy transformed into "spring" potential energy by deforming pole

Man gets kinetic energy by doing work pushing against the ground

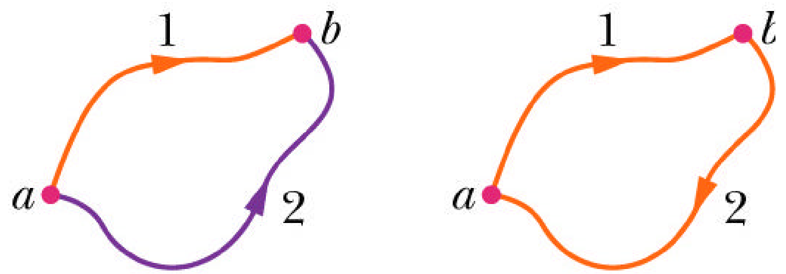
8-35: a problem involving conservative force (gravity)

Boy starts at top of semispherical mound of ice (no friction) and slides down. What angle with vertical when he leaves the ice?



$$q = 48.2^{0}$$

Force from Potential Energy Function (sec 8-5)



(a) (b)

Two Important Uses of Potential Energy

- conservation of mechanical energy
- Potential function to determine force

Simple and direct way to find the relevant force!

$$\Delta U \equiv U_b - U_a$$

For U & F varying with x

$$\Delta U = -W = -\int_a^b F_x dx$$

so for small changes, $\Delta x = x_b - x_a$

$\Delta U = -F_x \Delta x$ which requires

$$F_x = -\lim_{\Delta x \to 0} \frac{\Delta U}{\Delta x} = -\frac{dU}{dx}$$

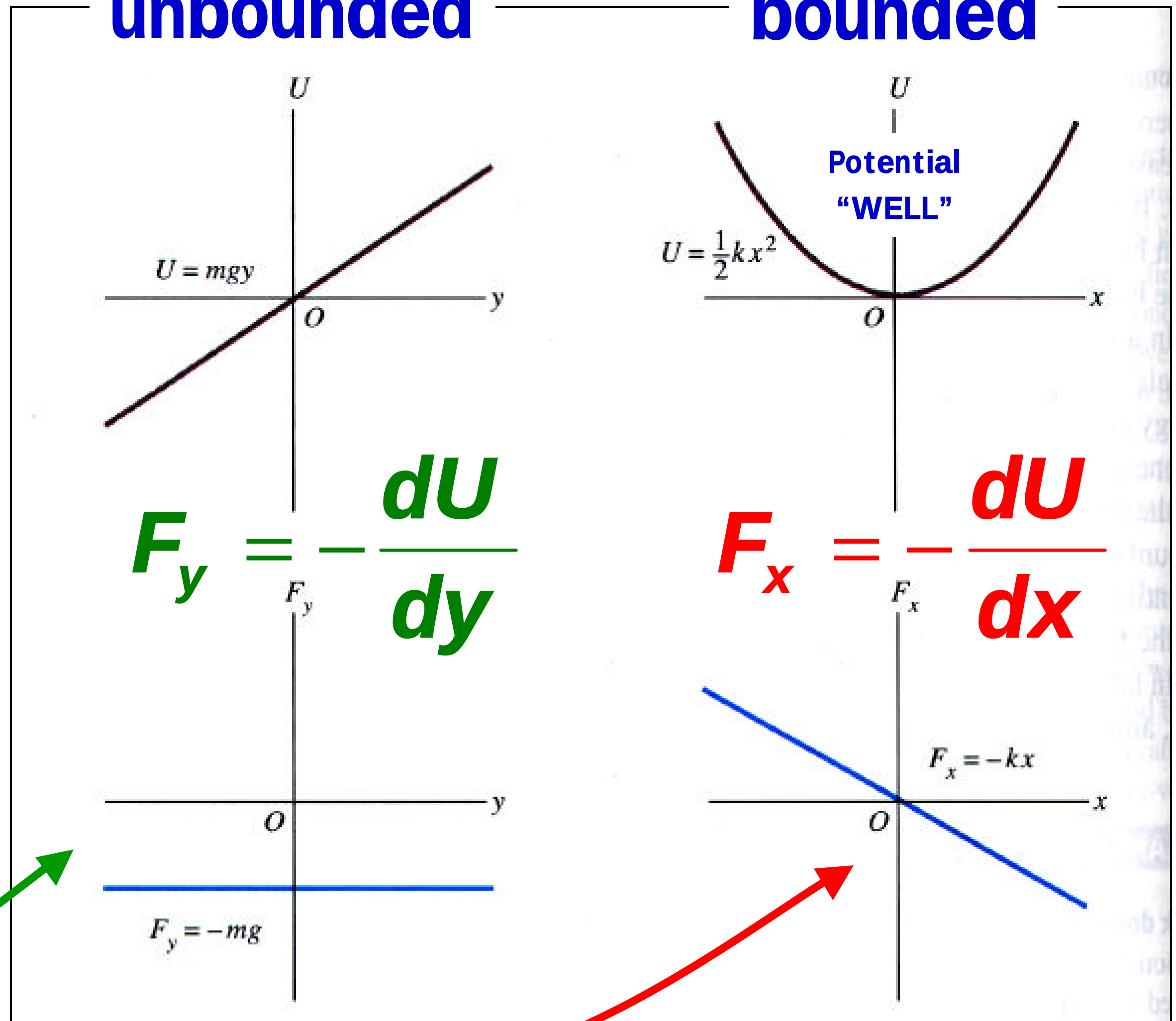
Graphs P. E. for Gravity and Springs

motion

unbounded | bounded

Relationship between force and potential functions for two cases:

- gravity
- spring

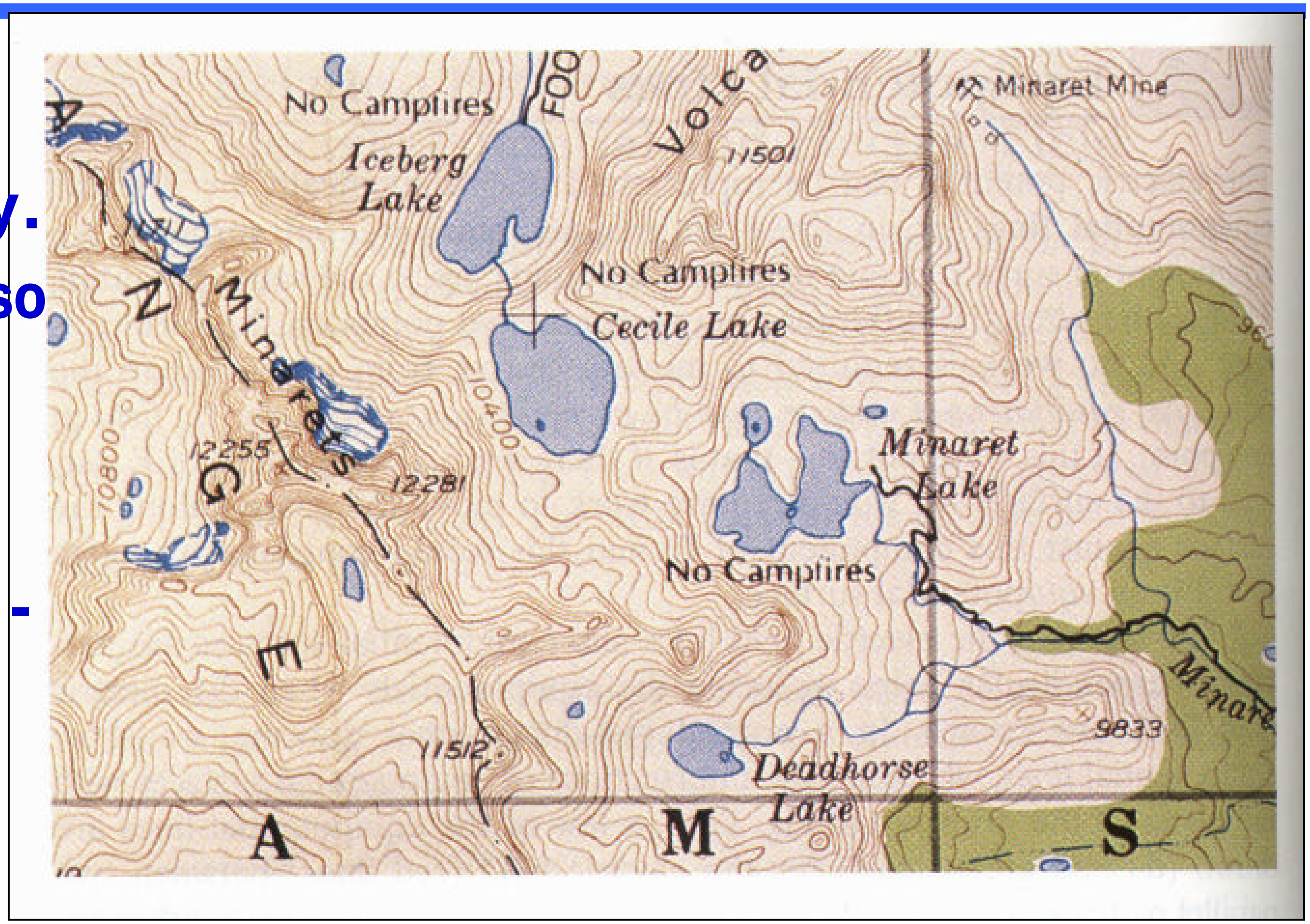


$$F_y = -\frac{dU}{dy}$$

$$F_x = -\frac{dU}{dx}$$

Real World: Topological Map and Equipotentials

- Gives elevation (*h*) versus x and y.
- But $U_{grav} \sim mgh$ so these curves are <u>equipotentials</u>
- Net forces can be determined ---
- Take *s* <u>along</u> terrain … then force normal to terrain has component along terrain equal to ……



$$F_s = -\frac{\Delta U}{\Delta s} = -mg\,\frac{\Delta h}{\Delta s}$$

culture

Generalization of Force from Potential Function

Suppose Potential Function depends on 3D:

$$U(x,y,z)$$

$$\vec{F} = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$

$$\vec{F} = -\vec{\nabla} U(x,y,z)$$

$\vec{\nabla}$ called "gradient"

Aside from math which is another statement of path independence of conservative forces:

$$\oint_{\text{any closed path}} \vec{\nabla} U \bullet d\vec{s} = 0$$

So knowledge of potential function versus position tells us about the force!

Has broad implications! Cultural for now!

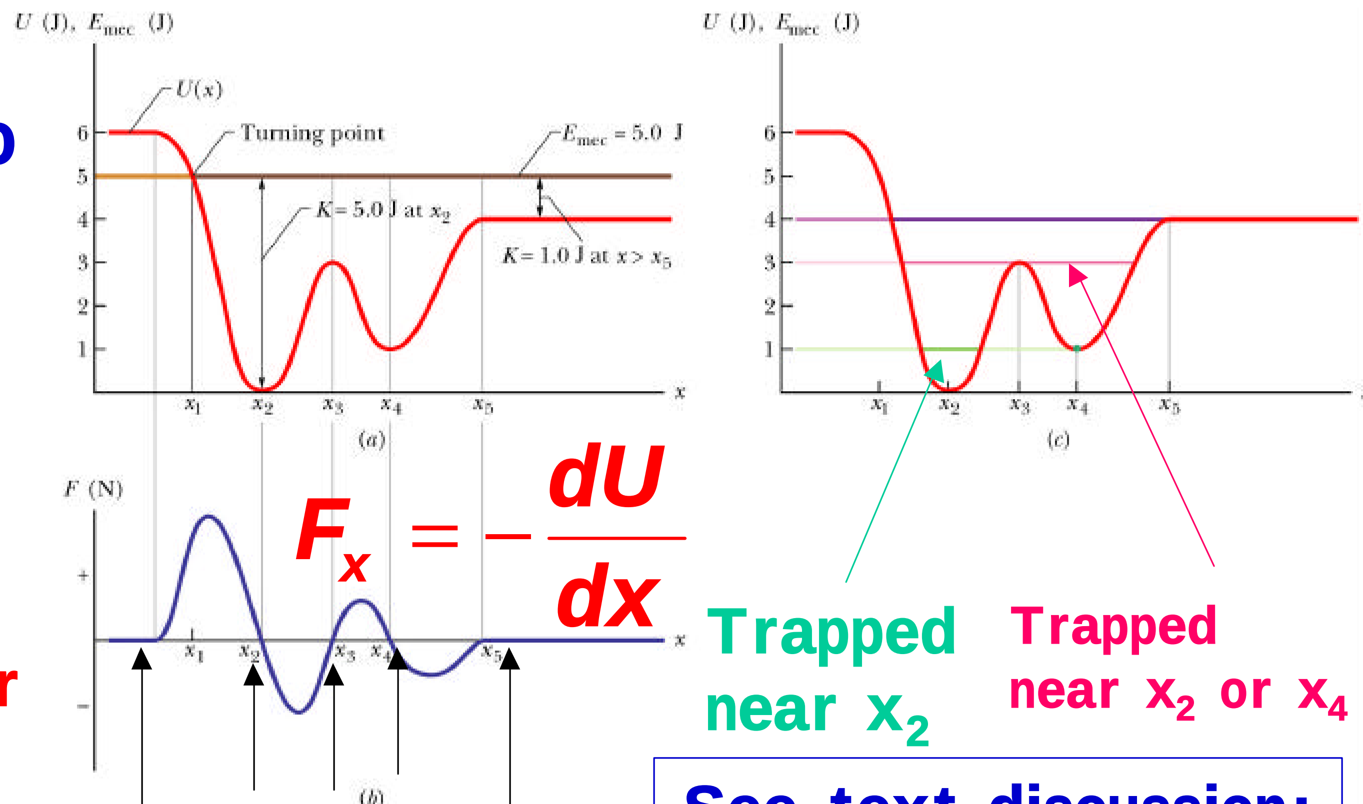
Potential Energy Function- General

- Graph of potential energy function allows one to determine
  - Forces
  - Stable equilibrium locations
  - Subsequent motion after initial start

$$E = U(x) + K(x)$$

$$K(x) = E - U(x)$$

K(x) > 0

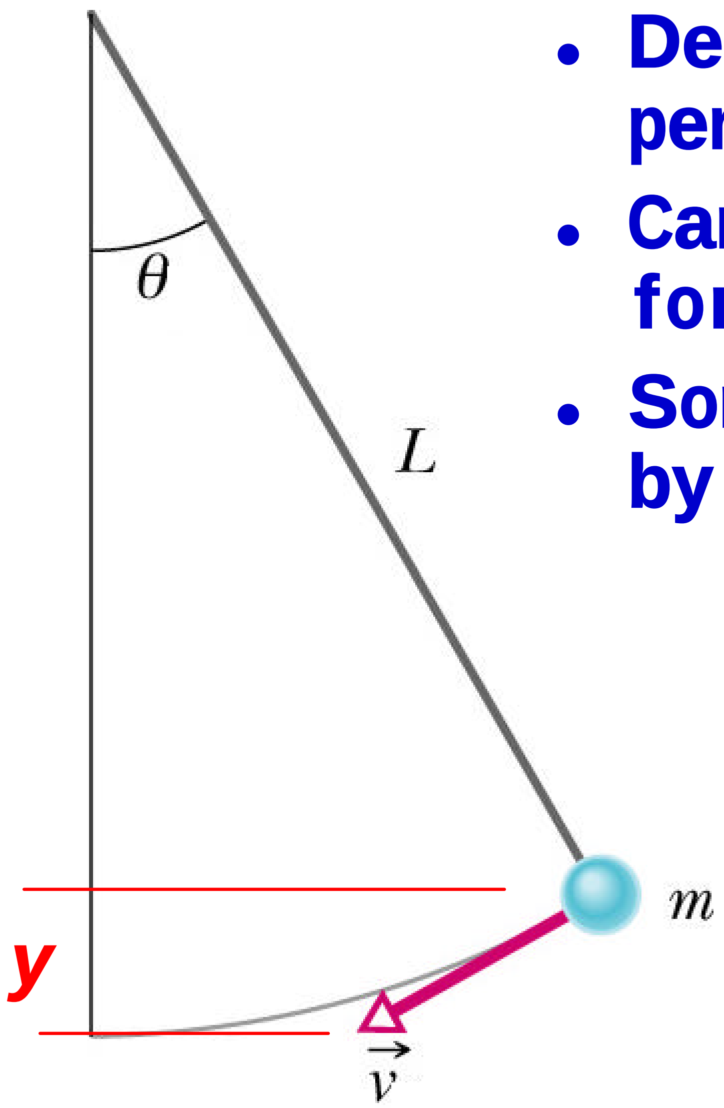


$$F_x = -\frac{dU}{dx}$$

See text discussion: sec 8-5

review

Pendulum Problem



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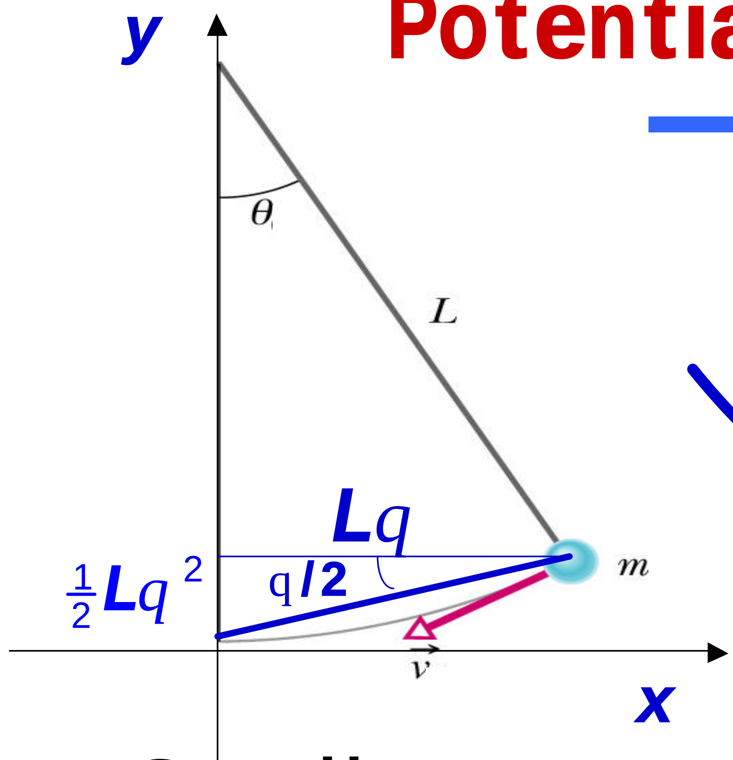
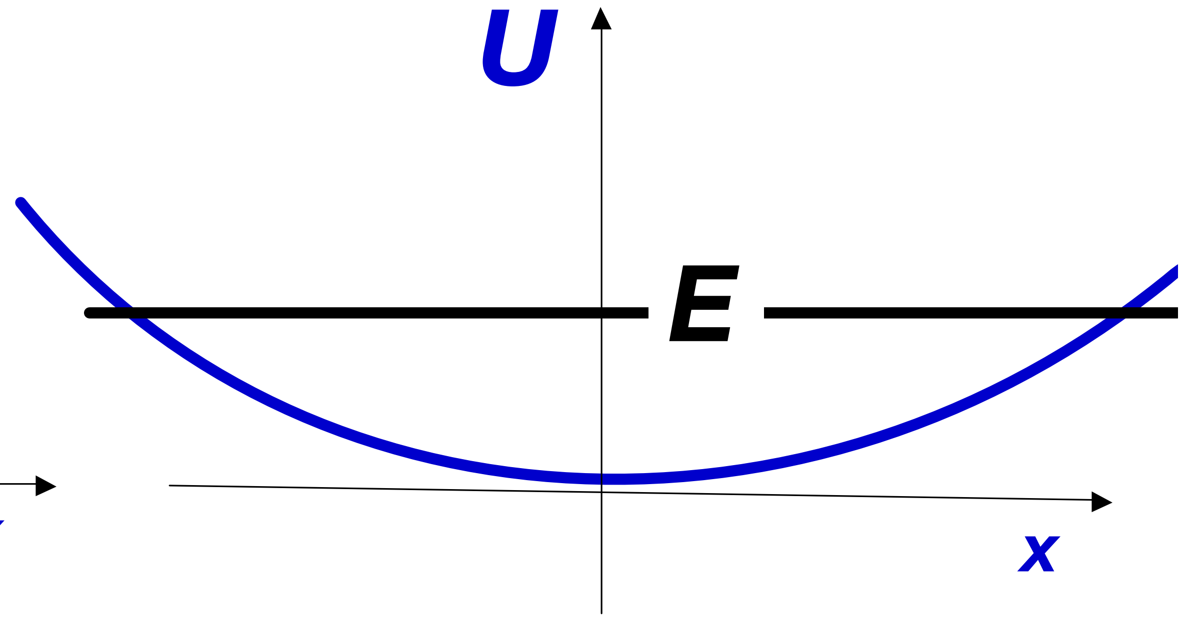
Potential Function of Pendulum





Small q

$$y = L(1 - \cos q)$$

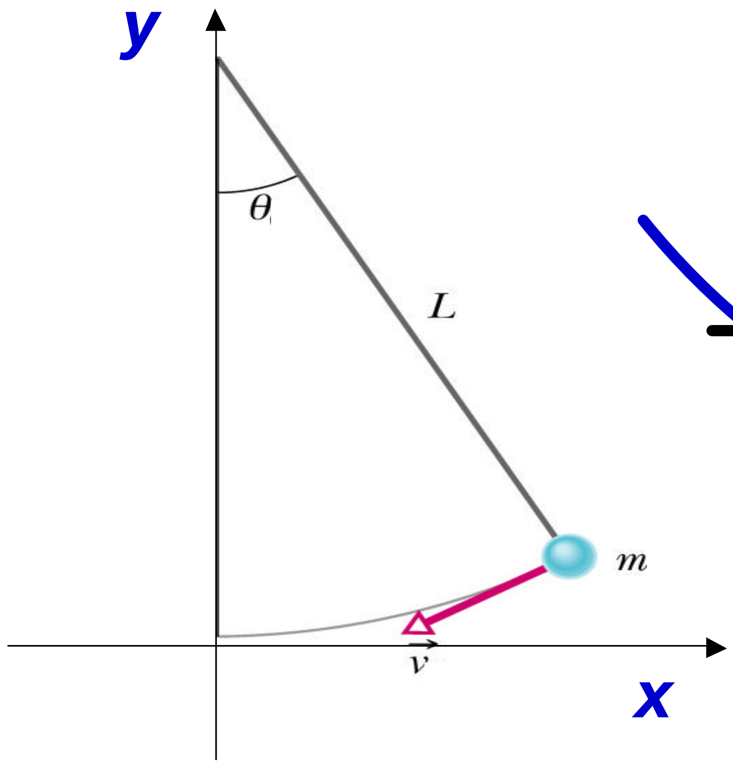
$$\approx \tfrac{1}{2} L q^2$$

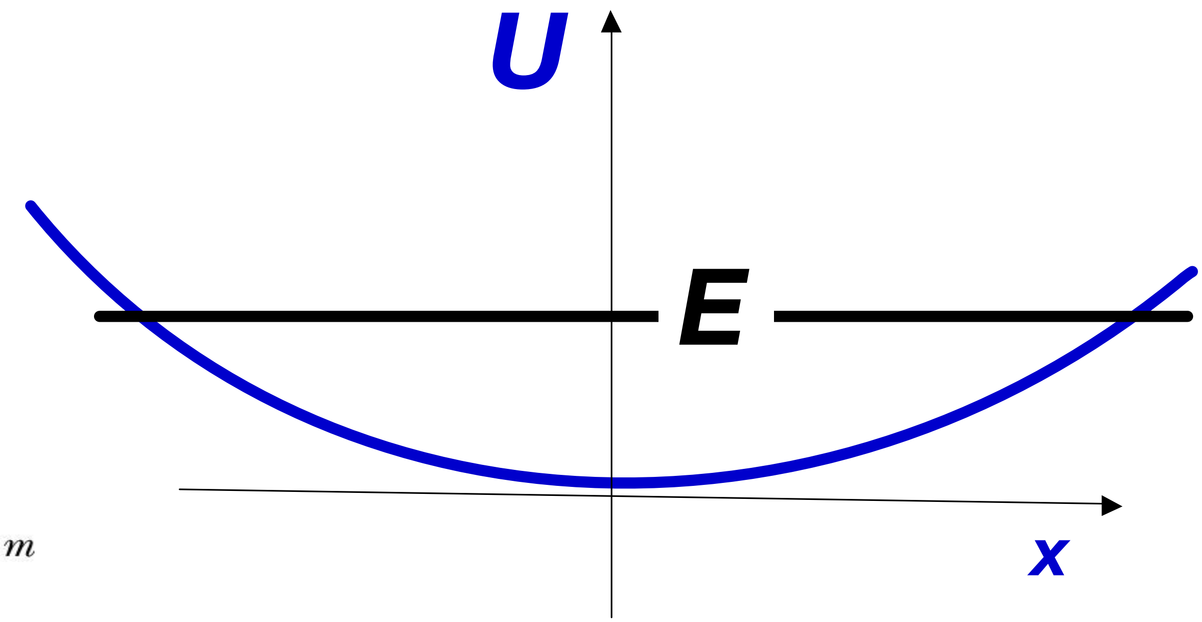
$$x = L \sin q \approx L q$$

$$U = mgy \approx \tfrac{1}{2} mgL \left(\frac{x}{L} \right)^2$$

$$U \approx \tfrac{1}{2} \frac{mg}{L} x^2 \qquad F_x = -\frac{mg}{L} x$$

Pendulum Potential Well





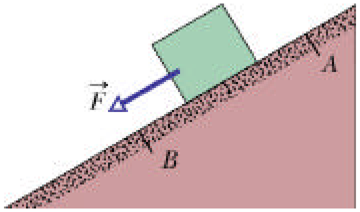
$$U \approx \tfrac{1}{2}\frac{mg}{L}x^2 \qquad F_x = -\frac{mg}{L}x$$

- Pendulum "trapped" between extremes determined by *E* (fixed)
- Kinetic Energy is difference of $K(x)=E-U(x)$

DEMO

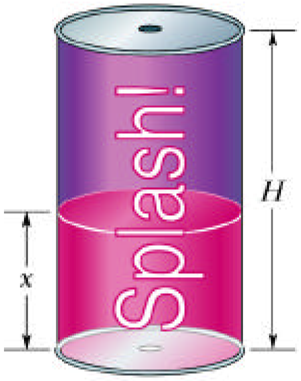
New Stuff - Ch 9-10

- New principle: conservation of momentum
- So far, we have dealt with simple systems
  - **point-like objects**
  - **symmetric objects**

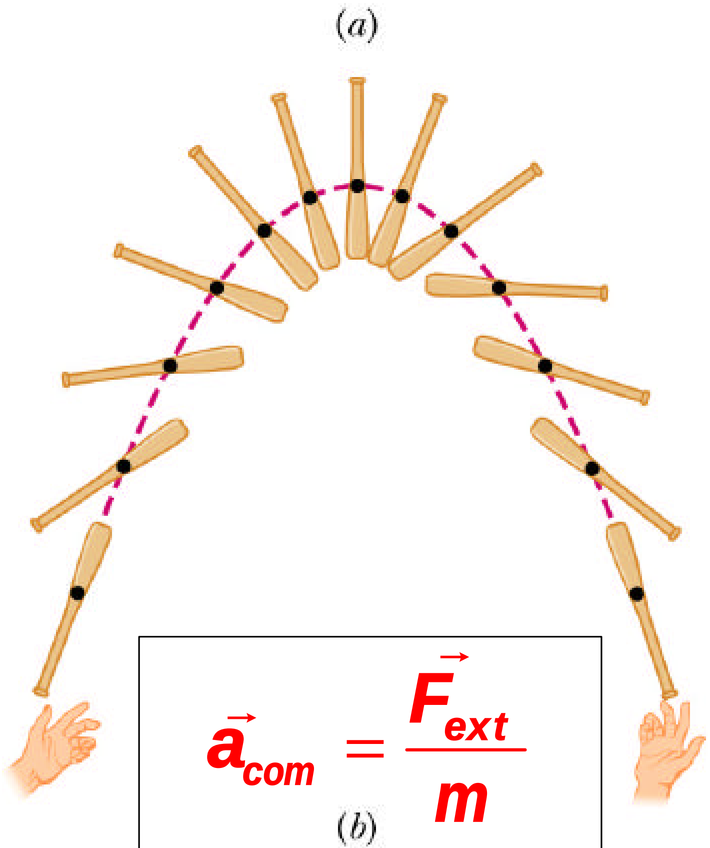


- Now we learn how Newton's Laws apply to complicated systems
  - **Example of issues --- *F=ma* --- but for what point?**
  - **Assert the answer: Center of Mass!**
  - **Prove later and get more!!**

Complications because most masses are not simple



Extended body: Center of mass obeys *F=ma* (like point particle)



$$\vec{a}_{com} = \frac{\vec{F}_{ext}}{m}$$

Above case:

$$\vec{a}_{com} = \frac{m\vec{g}}{m} = \vec{g}$$

Demos

- wrench (no gravity)
- pix of bat in text

cm motion of wrench no force.MOV

RULES

- First specify the <u>system</u> under discussion and know its mass (*m*)
- Determine (calculate) the center of mass (*com*) within the body
- Determine the total <u>external</u> force, F_{ext}, on the body

Conclusion

- Check for your MT1 blue books at entry
- See all relevant exam info at website
- Next lecture, we will continue discussing chapters 9 and 10
  - Systems of particles (macroscopic bodies)
  - Momentum and collisions