

$\Delta I = 1/2$, $K \rightarrow \pi\pi$ matrix elements
with Domain-Wall Valence Quarks
and Staggered Sea Quarks

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Time Requested: 12.4 million Jpsi core-hours on Fermilab clusters and 12.0 million BG/P core-hours (equivalent to 6.5 million Jpsi core-hours) plus 144 Tbytes of tape storage (the equivalent of 0.39×10^6 Jpsi core-hours) at Fermilab.

Abstract

We propose to compute domain-wall propagators on the 2+1 flavor MILC staggered (asqtad) ensembles for the mixed-action physics project: $K \rightarrow \pi\pi$ in the $\Delta I = 1/2$ channel.

A determination of $K \rightarrow \pi\pi$ matrix elements is of great importance in constraining physics beyond the Standard Model. Although the lattice calculation of $K \rightarrow \pi\pi$ is known to be challenging, the success of our earlier mixed-action calculation of B_K indicates that this method is also a good way to determine $K \rightarrow \pi\pi$ matrix elements. In the upcoming allocation year, we request resources to compute $K \rightarrow \pi\pi$ in the $\Delta I = 1/2$ channel at all three of the MILC lattice spacings ($a = 0.12, 0.09, 0.06$ fm) on which we have already generated many of the necessary propagators.

We therefore request the equivalent of 12.4 million Jpsi core-hours on Fermilab clusters and 12.0 million BG/P core-hours (equivalent to 6.5 million Jpsi core-hours) at Argonne and 144 Tbytes of tape storage (the equivalent of 0.39×10^6 Jpsi core-hours) at Fermilab to compute $K \rightarrow \pi\pi$ matrix elements.

Scientific Objectives

Over the past few years we have carried out a successful project to calculate the kaon bag parameter B_K using USQCD resources. B_K parameterizes the hadronic contribution to mixing between K^0 and $\bar{K}^0$ mesons, and is one of the most sensitive probes of new physics beyond the Standard Model. We have recently published our result [1],

$$\hat{B}_K = 0.724(8)(29), \quad (1)$$

where the first error is statistical, and the second is the sum of all systematic errors in quadrature. This is currently the best published unquenched determination of B_K , with all systematic errors under control, and fulfills one of the key goals in flavor physics of the U.S. lattice QCD community stated in the 2007 white paper “Fundamental parameters from future lattice calculations” [2]. Combined with the recent determination of B_K from RBC/UKQCD [3] (with whom we are in good agreement) we confirm the earlier claim of Ref. [4] that there is some tension in the unitarity triangle fits [5]. This tension is driven mostly by the new precision in the constraint from kaon mixing. In particular, using the latest averages of all lattice inputs to the unitarity triangle fit, as well as some previously neglected corrections to ϵ_K [6], the fit prefers the value $\hat{B}_K = 0.98 \pm 0.10$ [4] when the lattice input for $\hat{B}_K$ is excluded from the fit. Given this tension, it is crucial to continue our precision studies of kaon physics using multiple methods including our mixed-action approach. Under the present year’s allocation, we are continuing our work to update B_K (aiming for an $\approx 3\%$ total error), as well as other supporting calculations such as f_K/f_π , and the light quark masses. A calculation of $K \rightarrow \pi\pi$ in the $\Delta I = 3/2$ channel in collaboration with MILC is also underway. For this allocation year, we are requesting resources to continue our program with calculations of $K \rightarrow \pi\pi$ decays in the $\Delta I = 1/2$ channel, allowing us to address the $\Delta I = 1/2$ rule and ϵ'_K/ϵ_K .

Matrix elements of $K \rightarrow \pi\pi$ in both the $\Delta I = 1/2$ and $3/2$ channels are needed in order to use the experimental determination of ϵ'_K/ϵ_K to constrain new physics, and their calculation was identified as an important future direction in the USQCD Collaboration’s 2007 white paper. The $\Delta I = 1/2$ channel is particularly challenging for the lattice, given the contribution of disconnected diagrams and power divergences arising from mixing with lower-dimensional operators. Nonetheless, given the successful calculation of B_K , the time to begin seriously attacking this problem is now. It has been demonstrated by RBC that the power divergences can be controlled using domain-wall fermions in both the quenched [7] and 2+1 flavor [8] cases. The disconnected diagrams can only be controlled by high statistics, making the long runs on some of the MILC asqtad ensembles particularly attractive to us. Though the purely domain-wall method will also be applied to $K \rightarrow \pi\pi$ matrix elements, more than one independent calculation of the same quantity will be needed in order to confirm results and lend credibility within the broader physics community. For example, the good consistency between the precise determinations of B_K by us and by RBC/UKQCD is one success story of the USQCD program.

Our mixed action approach combines staggered and domain-wall fermions in the method pioneered by the LHP Collaboration in their mixed-action work [9], and takes advantage of the best properties of both discretizations. This method uses domain-wall valence quarks on top of an improved staggered sea (the MILC configurations [10]). By using staggered sea quarks we can take advantage of the vast library of MILC asqtad ensembles. The use of domain-wall valence quarks allows us to minimize operator mixing and makes it much easier to implement nonperturbative renormalization. In the case of a purely staggered calculation, taste-breaking effects due to the finite lattice spacing introduce significant mixings between the desired four-fermion operators and a number of other operators with incorrect tastes [11]. This makes nonperturbative techniques difficult to apply with staggered valence quarks, both in the case of B_K , as well as in the case of

the $\Delta S = 1$ operators needed for $K \rightarrow \pi\pi$ decays. Domain-wall quarks, on the other hand, have better chiral properties than staggered fermions, leading to qualitatively simpler lattice discretization errors. Although they do not possess an exact chiral symmetry on the lattice, the degree to which chiral symmetry is broken can be controlled through the length of the fifth dimension. Consequently, the mixing with wrong chirality operators can be reduced to a negligible level for most quantities and to a manageable level for those involving power divergent mixing with lower dimensional operators, as is the case for matrix elements of $\Delta I = 1/2$ operators.

Our studies of f_K , B_K , and the isovector scalar in mixed-action χ PT suggest that $SU(3)$ χ PT is a valid framework at the 10-20% level for the quantities we are considering, even up to the heavier masses and splittings used in our simulations, including masses close to the physical strange quark. This nontrivial agreement was demonstrated in Ref. [12] for the isovector scalar, and our studies of light pseudoscalar decay constants and B_K corroborate this [1, 13]. Although higher-order analytic terms are necessary to obtain reasonable confidence levels in our quantities where the raw data has sub-percent statistical errors, the contributions through next-to-leading order in $SU(3)$ χ PT are accurate at the 10-20% level even at the physical strange quark mass. Note that these conclusions are consistent with those of MILC [14, 15].

The indirect approach to obtaining $K \rightarrow \pi\pi$ matrix elements at the $\approx 30\%$ level by constructing them from the low energy constants (LEC's) of χ PT thus appears promising. Even this level of precision is interesting to phenomenology, and would allow us to use the experimental value of $\varepsilon'_K/\varepsilon_K$ as a constraint on the unitarity triangle for the first time. It was shown that all of the LEC's needed to construct $K \rightarrow \pi\pi$ matrix elements through next-to-leading order (NLO) in χ PT can be obtained from relatively simple quantities in Refs. [16, 17]. These simpler quantities are $K \rightarrow 0$, $K \rightarrow \pi$, and $K \rightarrow \pi\pi$ matrix elements at unphysical kinematics that evade the Maiani-Testa no-go-theorem [18] for multi-hadron processes in Euclidean space. A calculation of the LEC's through NLO in χ PT is complementary to a direct calculation of $K \rightarrow \pi\pi$ matrix elements using the direct finite-volume method of Lellouch-Lüscher [19]. Additional propagators beyond this year's request would be needed for a calculation of $K \rightarrow \pi\pi$ via the direct method, and the propagators that we propose to generate in this request could be recycled and used in such a future work. Because of the numerical demands of the $\Delta I = 1/2$ channel, and because we cannot expect to be awarded most of the available USQCD resources in a single year, we have chosen the indirect method relying on χ PT as a first step. Fortunately, the indirect approach is a natural stepping-stone to the more expensive direct approach. During subsequent years we can recycle the propagators from all of the previous years, while at the same time maximizing the physics impact at the end of each stage.

The disconnected diagram appearing in Figure 2(f) presents a special numerical challenge, and must be evaluated at unphysical kinematics even in the indirect method where the NLO LEC's are extracted and then used to construct $K \rightarrow \pi\pi$ through NLO in χ PT. Nonetheless, there is some advantage to the indirect method, because the pions are strictly at rest (without noisy three-momentum insertion), and a set of double ratios can be constructed to minimize statistical errors and extract the needed LEC's. Even so, it is possible that we will need a future dedicated run at high statistics for a restricted set of parameters to address the disconnected diagram. Such a run may be part of a future proposal.

Because we have saved all of the domain-wall propagators that we have generated since the inception of the mixed-action program, we can re-use them in the computation of $K \rightarrow \pi\pi$ in the $\Delta I = 1/2$ channel. This leads to significant savings for this request compared to starting from scratch. In order to compute the $\Delta I = 1/2$ matrix elements, we still need to compute so-called "eye" diagrams, where a quark propagator is contracted with itself at a point. This requires the use of random sources, and involves an additional propagator inversion per configuration. Our request

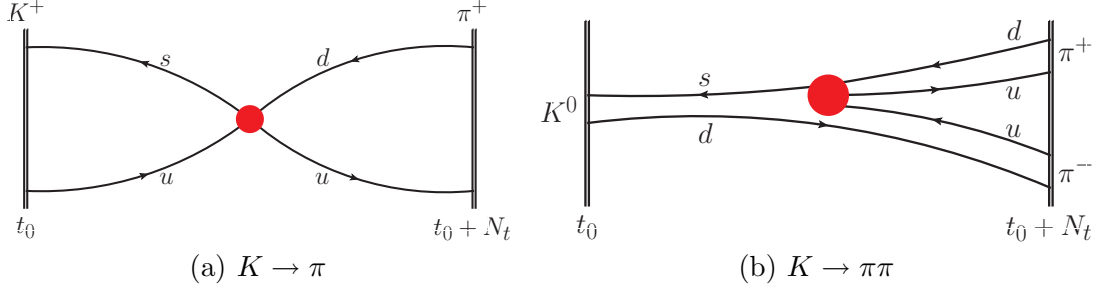


Figure 1: Three-point and four-point lattice correlation functions for $K \rightarrow \pi\pi$ in the $\Delta I = 3/2$ channel.

to USQCD is for time to compute these random source propagators on all of the lattices we have analyzed so far. Note that we do not have computing resources elsewhere.

Simulation Method and Code

We are using the same parameters for our domain-wall propagators on the MILC lattices [9] as those used by the LHP Collaboration in their mixed-action work. This has allowed us to perform cross-checks of simple quantities such as m_{res} and m_π . In particular, we first perform hypercubic-block (HYP) smearing on the MILC lattices with the standard smearing parameters in order to reduce proximity with the Aoki phase [20]. We also use a domain-wall height of 1.7 and an extent in the extra dimension of $L_s = 16$, which produces an acceptably small residual quark mass, of the size of the physical light quark mass on the coarse MILC ensembles. On the fine MILC lattices we find a value of m_{res} that is a factor of 3 times smaller in physical units. We Coulomb gauge-fix the lattices (after HYP-smearing) and use wall sources for the quark propagators.

Figure 1 shows the contractions needed to extract the LEC's of χPT in order to construct $K \rightarrow \pi\pi$ in the $\Delta I = 3/2$ channel through NLO. Figure 2 shows the additional diagrams needed to construct $K \rightarrow \pi\pi$ in the $\Delta I = 1/2$ channel through NLO. For the purpose of extracting LEC's, it is sufficient to have the pions at rest. We fix the time sources and sinks of the initial and final meson states and vary the location of the $\Delta S = 1$ operator over all timeslices in between. We first generate both periodic and antiperiodic boundary condition quark propagators and then take symmetric and antisymmetric linear combinations to produce forward- and backward-moving propagators, respectively. We then tie together the forward-moving quarks and backward moving quarks with the four-quark operator in order to effectively double the time extent of the lattice. This allows us to go to lighter valence quark masses by reducing effects due to the finite extent of our lattices in the time direction. For $K \rightarrow \pi\pi$ in the $\Delta I = 1/2$ channel, we also need a self-contracted propagator to evaluate the “eye” diagrams of Figure 2. Because all propagators can be reused and combined to form various degenerate and nondegenerate “kaons,” we need two Coulomb gauge-fixed wall source propagators — one periodic and one antiperiodic — and one self-contracted “eye” propagator, per valence quark mass per configuration. Since we already have the wall source propagators, we have only to compute the “eye” propagator on all configurations that we have analyzed in previous years. The self-contracted propagator is inverted on a random color source spanning multiple timeslices. The $K \rightarrow \pi\pi$ calculations will also share the Landau gauge-fixed propagators computed in previous years that are needed for the nonperturbative renormalization (NPR) of the lattice operators.

The most computationally intensive portion of this project by far is the propagator inversions,

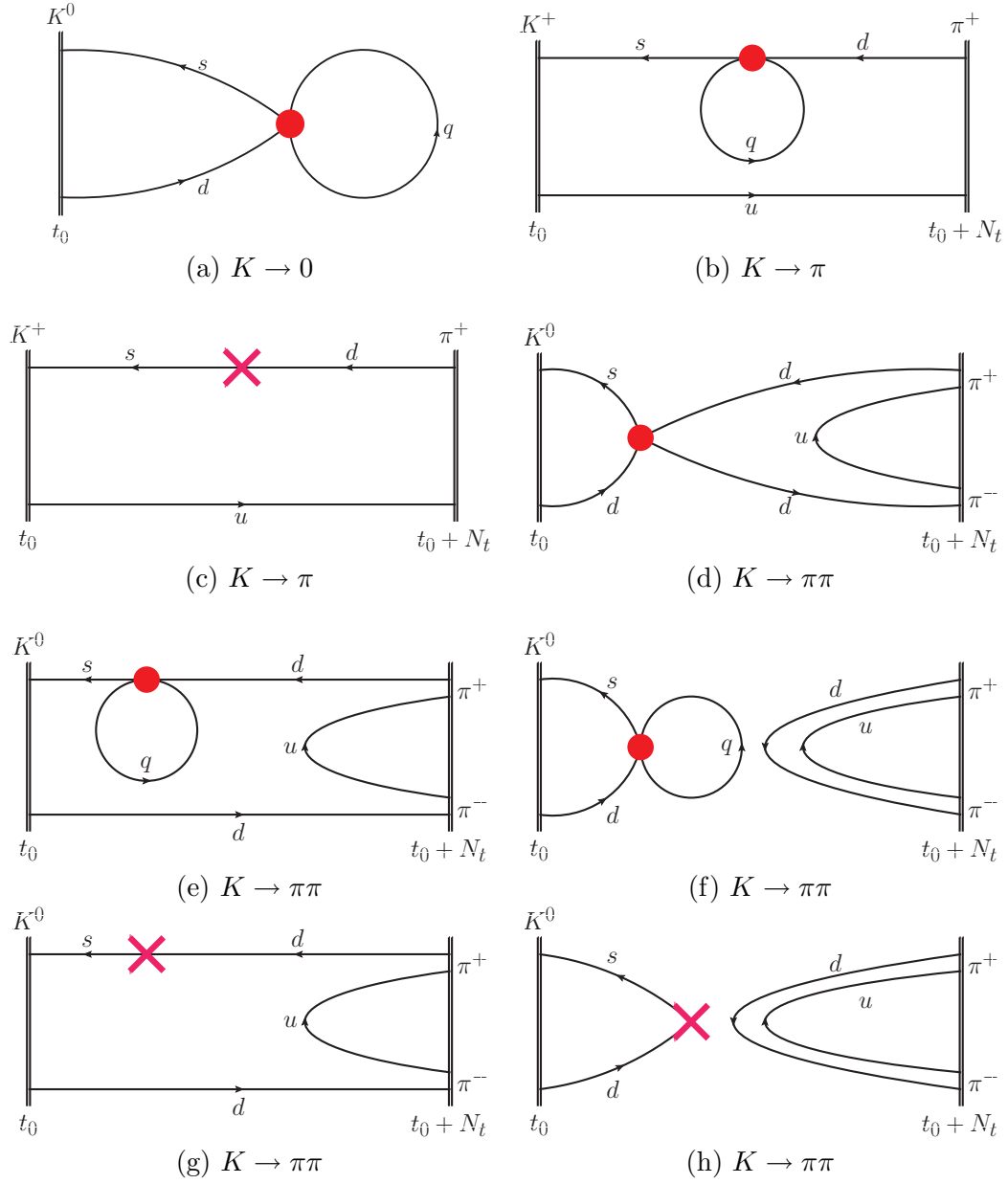


Figure 2: Additional lattice correlation functions for $K \rightarrow \pi\pi$ in the $\Delta I = 1/2$ channel using the indirect method.

for which we are using the optimized domain-wall inverter in the Chroma lattice QCD software package [21]. We have developed our own codes, written using the Chroma and QDP++ libraries, to symmetrize and antisymmetrize the quark propagators, to calculate the required three and four-point correlation functions not involving “eye” diagrams, and to Fourier transform the Landau gauge-fixed propagators for the NPR. All of these codes have been checked against those in the Columbia Physics System [22]. We will write and test Chroma code to calculate the $K \rightarrow \pi$ and $K \rightarrow \pi\pi$ contractions involving an “eye” diagram over the course of the next allocation cycle. We plan to continue to store propagators on tape at Fermilab for use in future projects.

Run Plan and Resource Allocation

We have been generating domain-wall propagators on MILC lattices for the last few years using USQCD resources. We have calculated wall-source propagators on approximately one- to two-hundred configurations on six $a \approx 0.125$ fm “coarse” ensembles, six $a \approx 0.09$ fm “fine” ensembles and two $a \approx 0.06$ fm “superfine” ensembles. In all cases, we have used only every fourth recorded configuration in order to minimize the correlations in the lattice data given limited resources. We are currently finishing a run of propagator generation on the large-volume $64^3 \times 144$ $a \approx 0.06$ fm MILC ensemble at the Argonne BG/P. This ensemble will be particularly useful for anchoring chiral extrapolations and determining low energy constants, given its large physical volume and small sea-pion mass (~ 240 MeV for the taste-Goldstone pion, ~ 270 MeV for the taste-singlet pion). The impact of the superfine running can be seen in our preliminary result for the chiral extrapolation of f_π in Figure 3, where the lightest mass point on the 64^3 superfine ensemble differs by only $\approx 5\%$ from the extrapolated value in the continuum.

In the 2010–2011 USQCD allocation year we propose to calculate self-contracted propagators with random-sources in order to compute “eye” diagrams. This requires an extra domain-wall inversion on top of the two Coulomb gauge-fixed wall source propagators we have already made on each configuration listed in Table 1. We are requesting time to generate these new propagators on all of the ensembles we have analyzed so far, with the mass values given in Table 2. The use of three lattice spacings will make the continuum extrapolation more reliable and reduce discretization errors in $K \rightarrow \pi\pi$ matrix elements. The superfine lattices are particularly important, both because they have smaller taste-symmetry breaking, leading to smaller discretization errors, and because their very fine time resolution will be helpful in calculating disconnected diagrams.

We have timed the propagator inversions for a number of valence quark masses on the $a \approx 0.06$ fm MILC lattices on the Argonne BG/P; these times are given in Table 3. We have also timed the propagator inversions for the coarser ($a \approx 0.09$ and 0.12 fm) ensembles on the kaon cluster at Fermilab; see Table 4. Using these timings, the total computing resources required to calculate all of the propagators listed in Table 2 is given in Table 5. Based on this estimate, our total computing request is 12.4 million Jpsi core-hours on Fermilab clusters and 12.0 million BG/P core-hours at Argonne. This includes time to compute all of the correlation functions for the $K \rightarrow \pi\pi$ project. Inversions on the largest of the $a \approx 0.06$ fm lattices run most efficiently at Argonne, while inversions on the smaller lattices run well on Fermilab clusters.

We would like to save all of the domain-wall propagators to tape at Fermilab for enough time to allow their use in other projects and by other groups. Table 6 shows the file sizes of the domain-wall quark propagators for the four different lattice volumes, in both GB and 6n-equivalent node hours assuming that they are stored on tape. A comparison of Table 6 and Table 3 reveals that calculating the domain-wall propagator is $\sim 100 - 200$ times more expensive than storing it. Thus it is more efficient to save and reuse the domain-wall propagators than to recalculate them. The

total storage space needed to save all of the propagators listed in Table 2, plus the storage currently in use holding existing propagators is given in Table 7. Our total mass storage request is 48 TBytes of new tape (plus ≈ 96 TBytes of existing tape) at Fermilab. In order to save our correlators and other analysis files (both old and new), we also require a small amount of disk space: ~ 0.126 TBytes in the “/project” area at Fermilab.

Summary

With the addition of “eye” diagrams, we will be able to compute $\Delta I = 1/2$ matrix elements, enabling us to probe the $\Delta I = 1/2$ rule and ε'/ε through next-to-leading order in χ PT. This will also lay the groundwork for future direct calculations using the Lellouch-Lüscher finite-volume method [19]. Given our experience with other light pseudoscalar meson quantities, the next-to-leading order low energy constants of χ PT will likely lead to $\sim 30\%$ errors in $\Delta I = 1/2$, $K \rightarrow \pi\pi$ matrix elements, and significantly better precision when combined with direct calculations at close to physical kinematics. Even with 30% errors, ε'/ε would provide a new and powerful constraint on physics beyond the Standard Model; future extensions of this project would do even better. This would fulfill one of the future goals in flavor physics of USQCD stated in the 2002 strategic plan and the 2007 white paper “Fundamental parameters from future lattice calculations” [2].

We could easily make good use of a substantial increase in our allocation if there was an increase in total USQCD resources. Depending upon the overall error budget for $K \rightarrow \pi\pi$ matrix elements we would use additional computing resources either to increase statistics on the proposed ensembles or to generate momentum source domain-wall propagators needed for the direct evaluation of $K \rightarrow \pi\pi$ matrix elements. We would therefore like to retain exclusive rights to use our domain-wall propagators for both direct and indirect determinations of $K \rightarrow \pi\pi$ matrix elements. We would also like exclusive rights to calculate the long distance contributions to ΔM_K .

The domain-wall propagators that we are generating can be used for the calculation of other interesting physics quantities, and we encourage other members of the lattice QCD community to do so. For example, the NPLQCD collaboration is using our lightest quark propagators to compute the π - K scattering length. All of the propagators listed in Table 1 are stored at Fermilab and can be made available immediately for non-competing analyses. Researchers who wish to use them should contact us to arrange access.

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Table 1: Number of Coulomb gauge-fixed wall source propagators available. Both spectrum quantities (m_π , f_π , *etc.*) and B_K have been calculated with all of these propagators.

$a(\text{fm})$	L	m_l	m_s	$m_{val.}^{\text{dwf}}$	# configs.
≈ 0.06	64	0.0018	0.018	0.0026, 0.0108, 0.033	96
≈ 0.06	48	0.0036	0.018	0.0036, 0.0072, 0.0108, 0.033	128
≈ 0.09	40	0.0031	0.0031	0.004, 0.0124, 0.0186, 0.046	102
≈ 0.09	40	0.0031	0.031	0.004, 0.0124, 0.0186, 0.046	150
≈ 0.09	28	0.0062	0.031	0.0062, 0.0124, 0.0186, 0.046	374
≈ 0.09	28	0.0093	0.031	0.0062, 0.0124, 0.0186, 0.046	198
≈ 0.09	28	0.0124	0.031	0.0062, 0.0124, 0.0186, 0.046	198
≈ 0.09	28	0.0062	0.0186	0.0062, 0.0124, 0.0186, 0.046	160
≈ 0.125	32	0.005	0.005	0.007, 0.02, 0.03, 0.05	175
≈ 0.125	24	0.005	0.05	0.007, 0.02, 0.03, 0.05, 0.065	216
≈ 0.125	20	0.007	0.05	0.01, 0.02, 0.03, 0.04, 0.05, 0.065	268
≈ 0.125	20	0.01	0.05	0.01, 0.02, 0.03, 0.05, 0.065	220
≈ 0.125	20	0.02	0.05	0.01, 0.03, 0.05, 0.065	117
≈ 0.125	20	0.01	0.03	0.01, 0.02, 0.03, 0.05, 0.065	160

Table 2: Proposed valence and sea quark mass combinations for the calculation of “eye” propagators.

$a(\text{fm})$	L	m_l	m_s	$m_{val.}^{\text{dwf}}$	# configs.
≈ 0.06	64	0.0018	0.018	0.00469, 0.0108, 0.033	96
≈ 0.06	48	0.0036	0.018	0.0036, 0.0072, 0.0108, 0.033	128
≈ 0.09	40	0.0031	0.0031	0.004, 0.0124, 0.0186, 0.046	102
≈ 0.09	40	0.0031	0.031	0.004, 0.0124, 0.0186, 0.046	150
≈ 0.09	28	0.0062	0.031	0.0062, 0.0124, 0.0186, 0.046	374
≈ 0.09	28	0.0093	0.031	0.0062, 0.0124, 0.0186, 0.046	198
≈ 0.09	28	0.0124	0.031	0.0062, 0.0124, 0.0186, 0.046	198
≈ 0.09	28	0.0062	0.0186	0.0062, 0.0124, 0.0186, 0.046	160
≈ 0.125	32	0.005	0.005	0.007, 0.02, 0.03, 0.05	175
≈ 0.125	24	0.005	0.05	0.007, 0.02, 0.03, 0.05, 0.065	216
≈ 0.125	20	0.007	0.05	0.01, 0.02, 0.03, 0.04, 0.05, 0.065	268
≈ 0.125	20	0.01	0.05	0.01, 0.02, 0.03, 0.05, 0.065	220
≈ 0.125	20	0.02	0.05	0.01, 0.03, 0.05, 0.065	117
≈ 0.125	20	0.01	0.03	0.01, 0.02, 0.03, 0.05, 0.065	160

Table 3: Time to calculate a single domain-wall propagator with $L_5 = 16$ using Chroma on the Argonne “intrepid” BG/P.

$a(\text{fm})$	L	$m_{val.}^{\text{dwf}}$	nodes (intrepid)	time (hours)	Jpsi core-hours
≈ 0.06	64	0.0026	4096	6.7	59476
≈ 0.06	64	0.00469	4096	2.6	38127
≈ 0.06	64	0.0108	—	—	19097
≈ 0.06	64	0.033	—	—	10526
≈ 0.06	48	0.0036	1024	8.08	17693
≈ 0.06	48	0.0072	1024	3.81	8344
≈ 0.06	48	0.0108	1024	2.83	6197
≈ 0.06	48	0.033	1024	1.56	3416

Table 4: Time to calculate a single domain-wall propagator with $L_5 = 16$ using Chroma on the Fermilab “kaon” cluster. Some timings are estimates based on old runs on the Fermilab “qcd” cluster.

$a(\text{fm})$	L	$m_{val.}^{\text{dwf}}$	nodes (kaon)	time (hours)	Jpsi core-hours
0.09	40	0.004	160	9.44	5317
0.09	40	0.0124	160	3.97	2235
0.09	40	0.0186	160	2.99	1684
0.09	40	0.046	160	1.69	952
0.09	28	0.0062	64	4.29	966
0.09	28	0.0124	64	2.39	539
0.09	28	0.0186	64	1.69	381
0.09	28	0.046	64	1.00	225
0.125	24	0.007	16	4.99	281
0.125	24	0.02	16	2.03	114
0.125	24	0.03	16	1.47	83
0.125	24	0.05	16	0.98	55
0.125	24	0.065	16	0.81	46
0.125	20	0.01	—	—	129
0.125	20	0.02	—	—	82
0.125	20	0.03	—	—	59
0.125	20	0.05	—	—	42
0.125	20	0.065	—	—	37

Table 5: Computer time needed to generate the “eye” propagators for the sea quark ensembles, valence quark masses and numbers of configurations listed in Table 2.

64^3 $a = 0.06$ fm propagators	6.5×10^6 Jpsi core-hours
48^3 $a = 0.06$ fm propagators	4.6×10^6 Jpsi core-hours
all $a = 0.09$ fm propagators	4.6×10^6 Jpsi core-hours
all $a = 0.12$ fm propagators	0.6×10^6 Jpsi core-hours
correlation functions and analysis	2.6×10^6 Jpsi core-hours
Total	18.9×10^6 Jpsi core-hours

Table 6: File sizes of domain-wall propagators for various spatial volumes. The equivalent cost to store the file on tape uses the conversion 1 Tbyte tape = 2,694 Jpsi core-hours.

$a(\text{fm})$	L	size (GB)	tape storage cost (Jpsi core-hours)
≈ 0.06	64	41	108
≈ 0.06	48	17	44
≈ 0.09	40	6.6	18
≈ 0.09	28	2.3	6
≈ 0.125	32	2.3	6
≈ 0.125	24	0.9	2
≈ 0.125	20	0.6	2

Table 7: Tape storage needed to save all of the domain-wall propagators used to compute B_K . The current storage determination reflects actual usage, while the future storage requirement is an estimate based on the proposed new propagators in Table 2 and the file sizes in Table 6. The equivalent cost to store the file on tape uses the conversion 1 Tbyte tape = 2,694 Jpsi core-hours.

currently in use	≈ 96 TB
additional space for new runs	48 TB
Total	144 TB
$= 0.39 \times 10^6$ Jpsi core-hours	

Table 8: Disk storage needed to save 2-point and 3-point correlators, logfiles, and analysis files in the “/project” area at Fermilab. The current storage determination reflects actual usage, while the future storage requirement is an estimate. The equivalent cost to store the file on disk uses the conversion 1 Tbyte disk = 26,940 Jpsi core-hours.

currently in use	0.072 TB
additional space for new runs	0.054 TB
Total	0.126 TB
$= 3390$ Jpsi core-hours	

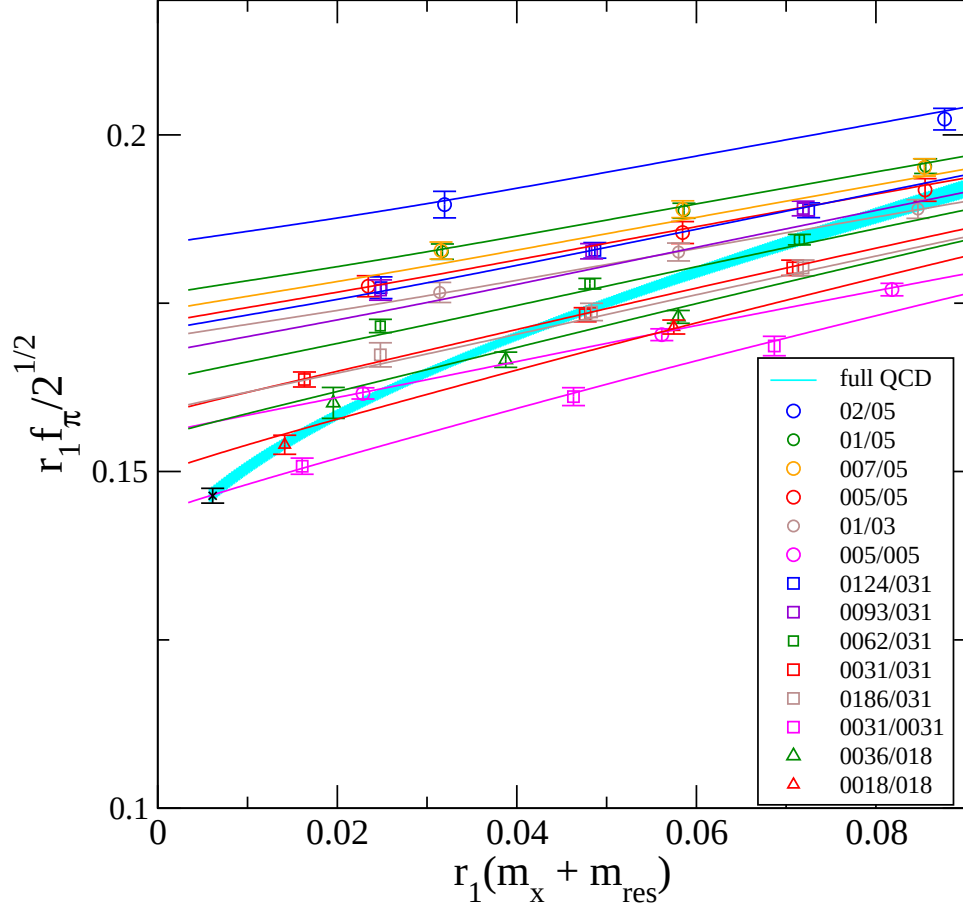


Figure 3: Pseudoscalar decay constant versus sum of valence and residual quark masses. The circles are coarse data points, the squares are fine data points, and the triangles are superfine data points. The cyan band is the full QCD curve (with statistical errors only) that results from fitting the lattice data to the mixed-action χ PT expression, supplemented by higher order polynomial terms. The cross is the experimental value, using the latest HPQCD determination of the lattice spacing. The correlated confidence level of this fit is 17%. Note that this plot only shows data for degenerate pions; the fit, however, also includes nondegenerate pions below $m_\pi \lesssim 600$ MeV.