B_K with Domain-Wall Valence Quarks and 2+1 Staggered Sea Quarks

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Time Requested: The equivalent of 2,205,350 processor-hours on the Pentium myrinet cluster ("qcd") at Fermilab, but on any of the Fermilab clusters

Abstract

We propose a precise measurement of the neutral kaon mixing parameter, B_K. We will use the existing MILC lattice configurations with 2+1 dynamical flavors of "Asqtad" staggered quarks and generate new external quark propagators with domain-wall valence quarks. This mixed action approach will enable us to reach the chiral regime in the sea sector while minimizing four-fermion operator mixing and allowing the use of nonperturbative renormalization.

Scientific Motivation

Experimental measurements of CP violation can be used to extract information about the CKM matrix. In particular, the size of indirect CP violation in the neutral kaon system, ϵ_K, combined with theoretical input, places an important constraint on the apex of the CKM unitarity triangle [1]. Because ϵ_K is well-known experimentally [2], the dominant source of error in this procedure is the uncertainty in the lattice determination of the nonperturbative constant B_K, which parameterizes the mixing between K^0 and $\overline{K}^0$. It is likely that new physics would give rise to extra CP-violating phases in addition to that of the CKM matrix; such phases would manifest themselves as apparent inconsistencies among different measurements of quantities which should be identical within the standard CKM picture. Thus a precise determination of B_K will help to constrain physics beyond the standard model.

Although the error in B_K currently dominates the error in constraining the unitarity triangle with ϵ_K, the magnitude of the CKM matrix element $|V_{cb}|$ is also a large source of uncertainty. Specifically, $|V_{cb}|$ is known to $\sim 2\%$ accuracy, and enters the expression which relates ϵ_K to the apex of the unitarity triangle as the fourth power. Thus, for B_K (and consequently lattice QCD) to cease being the largest source of uncertainty, one must reduce the error in B_K to less than the error from $|V_{cb}|^4$, which is $\sim 10\%$. Furthermore, to enter the era of precision measurements, one must measure B_K to $\sim 5\%$ accuracy. Our proposed procedure for determining B_K is the best way to reduce the many systematic errors which plague other methods.

Recent measurements of both light meson and heavy-light meson quantities with dynamical staggered quarks have shown excellent numerical agreement with experimental results [3]. Thus using the existing staggered lattices for weak matrix elements is a promising avenue to take, especially because they offer the lightest dynamical quark masses currently available [4]. The calculation of B_K with staggered quarks, however, has several problems that are not present in earlier staggered calculations of other quantities. Taste-breaking effects due to the finite lattice spacing introduce significant mixings between the desired four-fermion operator and a number of other operators with incorrect tastes [5]. This fact makes it necessary to use perturbative renormalization because nonperturbative techniques are too complicated to apply, and therefore increases the systematic error in B_K [6]. The taste-violating corrections to the $\Delta S = 2$ operator also greatly increase the number of undetermined coefficients in the staggered chiral perturbation theory expression for B_K, making both the chiral and continuum extrapolations extremely difficult [7].

Domain-wall quarks, on the other hand, have better chiral properties than staggered fermions, leading to qualitatively simpler lattice discretization errors. Although they do not possess an exact chiral symmetry on the lattice, the degree to which chiral symmetry is broken can be controlled through the length of the fifth dimension. Consequently, while the $\Delta S = 2$ operator still mixes with other operators, there are significantly fewer such operators and non-perturbative renormalization can be used in the determination of B_K. Unfortunately, however, domain-wall quarks are computationally expensive and are therefore presently impractical for realizing light quark masses.

Our proposal to combine staggered and domain-wall fermions takes advantage of the best properties of both discretizations. By using staggered sea quarks and domain-wall valence quarks we can better approach the chiral regime in the sea sector while minimizing operator mixing and allowing the use of nonperturbative renormalization. Mixed action simulations have already been successfully used by the LHPC and NPLQCD collaborations to study quantities of interest to nuclear physics [8, 9]. Thus we expect that a similar method can be used to determine the weak matrix element B_K. We cannot, however, reuse the propagators generated by LHPC because they truncated the MILC lattices in half in the time direction and imposed Dirichlet boundary conditions. This introduces an unknown systematic error that is not acceptable for B_K, which must be known precisely in order to have any phenomenological impact.

We list the available fine ($a = 0.09$ fm), coarse ($a = 0.125$ fm), and medium-coarse ($a = 0.15$ fm) MILC ensembles in Table 1. The mass of the strange (i.e. heavy) quark in these configurations is roughly that of the physical strange quark, while the two degenerate light quark masses range from $m_s/10 \leq m_l \leq m_s$. Because the quantity B_K is known to have weak dependence on the sea quark masses, and depends more heavily on the valence masses [10], we initially plan to use many valence masses with a modest number of dynamical ensembles. We must, of course, have enough sea quark masses to allow a chiral extrapolation to the physical up and down quark masses, as well as two lattice spacings to allow a continuum extrapolation.

In our first study we plan to use a subset of the dynamical staggered configurations with lattice spacings $a = 0.125$ fm and $a = 0.15$ fm. The combinations of valence and sea quark masses with which we will first calculate B_K are shown in Table 3. We have based our choices for domain-wall valence quark masses loosely on the mixed DW-staggered pion masses measured by the LHPC collaboration (see Table 2) such that two of our masses allow interpolation around the physical kaon and all or most are in the chiral regime. We plan to add additional valence masses on the $a = 0.09$ fm ensembles in the following year. As in all simulations with domain-wall quarks, it is essential to measure the residual mass, m_{res}, to ensure that the amount of chiral symmetry breaking is not too large and to study the spectral flow to ensure that one is not simulating too close to the unphysical Aoki phase. This has already been done by the LHPC collaboration for the coarse lattices, which were shown to be acceptable for lattice simulations. The medium-coarse lattices, however, have not yet been studied, and because they have a larger lattice spacing, are more likely to be affected by the Aoki phase. We will perform appropriate studies of the medium-coarse lattice ensembles and determine if they are acceptable for mixed-action simulations. If it turns out that the medium-coarse lattices cannot be used, we will use the additional allocation time to begin

Table 1: Available MILC fine, coarse and medium-coarse gauge configurations. The pion masses on the fine and coarse lattices are given in Ref. [11]. Data from the medium-coarse lattices has not yet been analyzed, however, we were able to measure the pion masses corresponding to the heaviest two valence quark masses with propagators stored at FNAL. The remaining three medium-coarse pion masses are estimates based upon the heavier masses and the behavior on the coarse lattices.

a(fm)	L	m_l	m_s	m_π(MeV)	approx. # configs.
0.09	40	0.0031	0.031	—	600
0.09	28	0.0062	0.031	336	600
0.09	28	0.0124	0.031	467	600
0.125	24	0.005	0.05	254	600
0.125	20	0.007	0.05	300	800
0.125	20	0.01	0.05	357	800
0.125	20	0.02	0.05	494	600
0.125	20	0.03	0.05	600	600
0.125	20	0.04	0.05	–	600
0.125	20	0.05	0.05	–	600
0.15	20	0.00484	0.0484	212	600
0.15	16	0.0097	0.0484	327	600
0.15	16	0.0194	0.0484	453	600
0.15	16	0.0290	0.0484	550	600
0.15	16	0.0484	0.0484	700	600

Table 2: Correspondence between domain-wall valence quark masses and pion masses on the coarse lattices (neglecting sea-quark effects) [14].

a(fm)	L	$m_{val.}^{\mathrm{dwf}}$	m_π(MeV)
0.125	20	0.0138	341
0.125	20	0.0306	474
0.125	20	0.0474	575
0.125	20	0.0642	662
0.125	20	0.0810	736

Table 3: Proposed valence and sea quark combinations for the calculation of B_K if the medium-coarse lattices can be used.

a(fm)	L	m_l	m_s	$m_{val.}^{\mathrm{dwf}}$
0.125	20	0.007	0.05	0.01, 0.02, 0.03, 0.04, 0.05
0.125	20	0.01	0.05	0.01, 0.03, 0.05
0.125	20	0.02	0.05	0.01, 0.03, 0.05
0.15	16	0.0097	0.0484	0.00484, 0.0097, 0.0194, 0.0290, 0.0484
0.15	16	0.0194	0.0484	0.0097, 0.0194, 0.0290, 0.0484
0.15	16	0.0290	0.0484	0.0097, 0.0194, 0.0290, 0.0484

running on the fine lattices.

In order to further reduce systematic errors, we will use the nonperturbative renormalization (NPR) technique of the Rome-Southampton group [12] to match our lattice value for B_K to continuum results. This will add to the amount of processor-hours required, as discussed in the following paragraph, but is essential for a precise determination of B_K. After renormalization, we will extrapolate to the physical up and down quark masses and to the continuum limit using the appropriate mixed-action chiral perturbation theory expression for domain-wall valence and staggered sea quarks [13]. The calculation of B_K in mixed-action chiral perturbation theory is currently in progress by the authors of this proposal.

Codes and Resources

B_K comes from the matrix element of the $\Delta S = 2$ weak four-quark operator between a K^0 and $\overline{K}^0$ state, but is normalized to be a dimensionless quantity of $O(1)$. Thus each calculation of B_K requires four quark propagators. However, all propagators can be reused and combined to form various degenerate and nondegenerate "kaons"; thus one really needs two propagators – one from the K^0 to the operator and the other from the operator to $\overline{K}^0$ – per valence quark per configuration. An additional propagator per valence quark per configuration is also needed for the nonperturbative renormalization.

The most computationally intensive portion of this project by far is the propagator inversions, for which we will be using the Chroma lattice software. We have timed propagator inversions for the various valence masses on the coarse and medium-coarse lattices on the Fermilab P4 myrinet cluster; these times are given in Table 4. Although Chroma cannot yet do nonperturabative renormalization, an NPR routine for Chroma is currently being written and will be available in the next few months

Table 4: Time to calculate a single domain-wall propagator with $L_5 = 16$ using Chroma on the Fermilab cluster.

a(fm)	L	$m_{val.}^{\rm dwf}$	nodes (FNAL)	time (hours)
0.09	28	0.0062	112	6.28
0.09	28	0.0124	112	3.57
0.09	28	0.0186	112	2.58
0.09	28	0.0248	112	2.07
0.09	28	0.031	112	1.73
0.125	20	0.01	64	1.61
0.125	20	0.02	64	0.96
0.125	20	0.03	64	0.71
0.125	20	0.04	64	0.62
0.125	20	0.05	64	0.48
0.15	16	0.00484	64	0.80
0.15	16	0.0097	64	0.57
0.15	16	0.0194	64	0.37
0.15	16	0.0290	64	0.28
0.15	16	0.0484	64	0.19

Table 5: Computer time needed to determine B_K using the valence quark masses and ensembles listed in Table 3. Each calculation of B_K requires three propagator inversions per valence quark mass per configuration. We will write the codes to calculate the B_K matrix element and perform nonperturbative renormalization ourselves using QDP++, as it does not exist in Chroma.

Matrix Elements	1,336,576 processor-hours
Nonperturbative Renormalization	668,288 processor-hours
Code Development/Analysis ($\sim$ 10%)	200,486 processor-hours
Total	2,205,350 processor-hours

[15]. Chroma does not, however, have software to calculate the $\Delta S = 2$ matrix element, so we will write this ourselves using QDP++. Thus we have allocated an additional 10% of the time the propagator inversions will take for code development and analysis. The proposed allocation time is given in Table 5.

Summary

At the end of this project we expect to have a precise determination of the neutral kaon mixing parameter B_K including dynamical quark effects. Use of two lattice spacings and multiple quark masses will give us control over the systematic errors associated with both the chiral and continuum extrapolations. This measurement, when used in a unitarity-triangle analysis, will place an important constraint on physics beyond the standard model.

All propagators generated for this work will be available to the SciDAC community shortly after they have been used for the calculation of B_K. Researchers who wish to use these propagators more promptly should contact us to arrange access.

References

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