B_K with Domain-Wall Valence Quarks and 2+1 Staggered Sea Quarks

February 29, 2008

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Time Requested: The equivalent of 4.3 million 6n node-hours on the Fermilab clusters plus 38 Tbytes (the equivalent of 0.15 million 6n node-hours) of tape storage at Fermilab.

Abstract

We propose to continue our mixed-action kaon physics program, with emphasis on the calculation of the neutral kaon mixing parameter, B_K, using domain-wall valence quarks on a staggered sea. A precise determination of B_K will place an important constraint on the apex of the CKM unitarity triangle, and thus on possible physics beyond the Standard Model. We are also calculating the light pseudoscalar meson spectrum, decay constants, and light quark masses. For our calculation, we generate domain-wall quark propagators on top of the existing MILC lattice configurations which include the effects of 2+1 dynamical flavors of "Asqtad"-improved staggered quarks. This mixed-action approach enables us to reach the chiral regime in the sea sector while minimizing four-fermion operator mixing and allowing the use of nonperturbative renormalization, thus reducing two of the most important sources of systematic error in current B_K calculations. This would fulfill one of the key goals in flavor physics of the U.S. lattice QCD community stated in the 2002 strategic plan. In this upcoming allocation year, we plan to increase our statistics and the number of ensembles analyzed on the $a = 0.125$ fm "coarse" and $a = 0.09$ fm "fine" MILC lattices. We therefore request the equivalent of 4.3 million 6n node-hours on the Fermilab clusters plus 38 Tbytes (the equivalent of 0.15 million 6n node-hours) of tape storage at Fermilab to do so.

Scientific Objectives

Experimental measurements of CP violation can be used to extract information about the CKM matrix. In particular, the size of indirect CP violation in the neutral kaon system, ϵ_K, combined with theoretical input, places an important constraint on the apex of the CKM unitarity triangle [1]. Because ϵ_K is well-known experimentally [2], the dominant source of error in this procedure is the uncertainty in the lattice determination of the nonperturbative constant B_K, which parameterizes the mixing between K^0 and $\overline{K^0}$. It is likely that new physics would give rise to extra CP-violating phases in addition to that of the CKM matrix; such phases would manifest themselves as apparent inconsistencies among different measurements of quantities which should be identical within the standard CKM picture. Thus a precise determination of B_K will help to constrain physics beyond the Standard Model. Because the importance of B_K to new physics searches is well known to the lattice community, this quantity was highlighted in the USQCD Collaboration's 2002 strategic plan as being one of its top priorities. We are submitting a Type A proposal which would further this goal and improve upon the results that we have obtained so far. With the resources requested in this proposal, we foresee a B_K determination with errors below 5%.

Our project combines staggered and domain-wall fermions in the method pioneered by the LHP Collaboration [3], and takes advantage of the best properties of both discretizations. This method uses domain-wall valence quarks on top of an improved staggered sea (the MILC configurations [4]). By using staggered sea quarks we can take advantage of the vast library of MILC ensembles. This allows us simulate at multiple lattice spacings using "2+1" flavors of light sea quarks with masses as low as 1/10th of the strange quark mass. The use of domain-wall valence quarks allows us to minimize operator mixing, making it much easier to implement nonperturbative renormalization. In the case of a purely staggered calculation, taste-breaking effects due to the finite lattice spacing introduce significant mixings between the desired four-fermion operator and a number of other operators with incorrect tastes [5]. This makes nonperturbative techniques difficult to apply, and thus far staggered B_K calculations have used only perturbative renormalization. This limitation increases the systematic error in B_K [6]. The taste-violating corrections to the $\Delta S = 2$ operator also greatly increase the number of undetermined coefficients in the staggered chiral perturbation theory expression for B_K, making the chiral and continuum extrapolation quite complicated [7]. Domain-wall quarks, on the other hand, have better chiral properties than staggered fermions, leading to qualitatively simpler lattice discretization errors. Although they do not possess an exact chiral symmetry on the lattice, the degree to which chiral symmetry is broken can be controlled through the length of the fifth dimension. Consequently, while the $\Delta S = 2$ operator still mixes with other operators, there are significantly fewer such operators and non-perturbative renormalization can be used more easily in the determination of B_K. Domain-wall quarks, however, are computationally expensive, and the most recent results with 2+1 flavors [8], though impressive, use only one lattice spacing.

Over the past one and a half years we have accumulated numerical data at two lattice spacings, and we will be able to achieve a precise calculation of B_K with all systematic errors under control with the USQCD allocation of the current year. We are also calculating light pseudoscalar meson decay constants and light quark masses, which benefit from the simpler renormalization properties and less complicated chiral perturbation theory expressions of the mixed-action method. Our approach also appears promising for the determination of the low energy constants of chiral perturbation theory, in particular for $K \to \pi\pi$ matrix elements, an avenue which we are currently exploring. This set of matrix elements is of considerable interest, given both the long-standing puzzle of the $\Delta I = 1/2$ rule and the valuable constraint on the Standard Model that would then

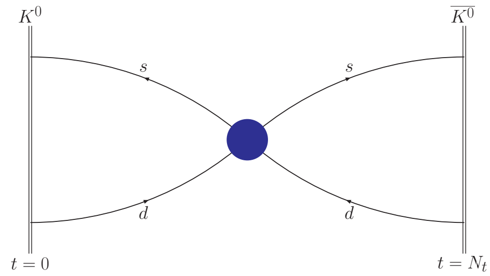

Figure 1: Three-point lattice correlation function for K^0-$\overline{K^0}$ mixing.

be possible given the experimental measurement of ϵ'/ϵ (which parametrizes direct CP violation in kaon decays). We therefore request time in the forthcoming year in order to improve upon our f_π, f_K, light quark masses, and B_K results, and to continue our exploration of $K \to \pi\pi$.

Simulation Method and Code

We are using the same parameters for our domain-wall propagators on the MILC lattices [3] as the LHP Collaboration. This allows us to perform cross-checks of simple quantities such as $m_{\rm res}$ and m_π. In particular, we first perform hypercubic-block (HYP) smearing on the MILC lattices with the standard smearing parameters in order to reduce proximity with the Aoki phase [9]. We also use a domain-wall height of 1.7 and an extent in the extra dimension of $L_s = 16$, which produces an acceptably small residual quark mass, of the size of the physical light quark mass on the coarse MILC ensembles. On the fine MILC lattices we find a value of $m_{\rm res}$ that is a factor of 3 times smaller. As in most calculations of B_K, we Coulomb gauge-fix the lattices (after HYP-smearing) and use wall sources for the quark propagators.

Each calculation of the B_K 3-point correlation function requires four quark propagators, though as described below, we only need two inversions per valence mass. As shown in Figure 1, we fix the timeslices of the K^0 and $\overline{K}^0$ and vary the location of the $\Delta S = 2$ operator over all timeslices in between. We first generate both periodic and antiperiodic boundary condition quark propagators and then take symmetric and antisymmetric linear combinations to produce forward- and backward-moving propagators, respectively. We then tie together two forward-moving quarks and two-backward moving quarks with the B_K four-quark operator in order to effectively double the time extent of the lattice. This allows us to increase the size of our plateau region for fitting to all time slices of the lattice; it also allows us to go to lighter valence quark masses by reducing finite volume effects in the time direction. Currently we start all propagators at timeslice zero. In future running, however, we plan to vary the location of the temporal origin between configurations in order to further decrease correlations and therefore reduce statistical errors. Because all propagators can be reused and combined to form various degenerate and nondegenerate "kaons," we need two Coulomb gauge-fixed wall source propagators – one periodic and one antiperiodic – per valence quark mass per configuration. We recycle these propagators in the computation of the pseudoscalar meson masses and decay constants, and will also use them in the future for $K \to \pi\pi$ matrix elements. In order to determine the nonperturbative renormalization (NPR) factor, Z_{B_K}, we also need one Landau gauge-fixed point source propagator per valence quark mass per configuration.

The most computationally intensive portion of this project by far is the propagator inversions, for which we are using the optimized domain-wall inverter in the Chroma lattice QCD software package [10]. We are using our own codes, written using the Chroma and QDP++ libraries, to

Table 1: Number of Coulomb gauge-fixed wall source propagators available. Note that we have already used approximately 90% of our 2007–2008 USQCD allocation. Both spectrum quantities (m_π, f_π, *etc.*) and B_K have been calculated with all of these propagators.

a(fm)	L	m_l	m_s	$m_{val.}^{\rm dwf}$	# configs.
0.09	40	0.0031	0.031	0.004, 0.0124, 0.0186, 0.046	39
0.09	28	0.0062	0.031	0.0062, 0.0124, 0.0186, 0.046	216
0.09	28	0.0124	0.031	0.0062, 0.0124, 0.0186, 0.046	199
0.125	24	0.005	0.05	0.007, 0.02, 0.03, 0.05, 0.065	128
0.125	20	0.007	0.05	0.01, 0.02, 0.03, 0.04, 0.05, 0.065	210
0.125	20	0.01	0.05	0.01, 0.02, 0.03, 0.05, 0.065	192
0.125	20	0.02	0.05	0.01, 0.03, 0.05, 0.065	117

symmetrize and antisymmetrize the quark propagators, to calculate the $\Delta S = 2$ matrix element, and to Fourier transform the Landau gauge-fixed point source propagators for the NPR. All of these codes have been checked against those in the Columbia Physics System [11].

We have generated a large amount of data to be used in the calculation of B_K and other quantities. In particular, we have generated Coulomb gauge-fixed wall-source propagators for the masses and ensembles listed in Table 1. We have also generated Landau gauge-fixed point-source propagators for the masses and ensembles given in Table 2; these propagators will be used for the nonperturbative renormalization (NPR) of B_K. All propagators have been stored on tape at Fermilab for future use. In order to study both the valence and sea quark mass dependence of B_K, we have used a number of valence quark masses and seven different ensembles. In a mixed-action simulation with domain-wall quarks on a staggered sea there is no unambiguous notion of full QCD at finite lattice spacing, though this ambiguity disappears in the continuum limit. The sea mesons occur in different "tastes", i.e. unphysical flavors, and at finite lattice spacing there are taste violating terms which split the degeneracy of these various staggered mesons. The LHP Collaboration has tuned the bare domain-wall quark mass such that the valence pion mass is equal to the lightest of the staggered pion masses. We do not need to perform such a tuning for our calculation, however, because we can use the correct expression for B_K in partially-quenched, mixed-action chiral perturbation theory as a guide in the extrapolations to the physical light quark mass and to the continuum limit. The calculation of B_K in mixed-action chiral perturbation theory was completed in Ref. [15]. The calculation of pseudoscalar masses and decay constants in mixed-action chiral perturbation theory was done earlier in Ref. [13].

Preliminary Physics Results

In this section we review the physics results that we have obtained so far. We have performed a careful study of discretization effects in the mixed-action theory, and we have demonstrated that these can be controlled using mixed-action chiral perturbation theory [14]. Specifically, we have analyzed the behavior of the isovector scalar (a_0) correlator, which is particularly sensitive to discretization effects and serves as a good consistency check. We have also studied the pion mass and decay constant using mixed-action chiral perturbation theory, and we show preliminary results to demonstrate the promise of the mixed-action method, although this work is still in progress.

At the level of the chiral effective Lagrangian, discretization effects in the mixed-action theory

Table 2: Number of Landau gauge-fixed point source propagators generated thus far using the allocation from year one. Note that we have already used approximately 90% of our 2007–2008 USQCD allocation.

a(fm)	L	m_l	m_s	$m^{\rm dwf}_{val.}$	# configs.
0.09	28	0.0062	0.031	0.0062, 0.0186, 0.046	94
0.09	28	0.0124	0.031	0.0062, 0.0186, 0.046	122
0.125	24	0.005	0.05	0.007, 0.03, 0.05, 0.065	128
0.125	20	0.007	0.05	0.01, 0.02, 0.03, 0.04, 0.05, 0.065	206
0.125	20	0.01	0.05	0.01, 0.03, 0.05, 0.065	161
0.125	20	0.02	0.05	0.01, 0.03, 0.05, 0.065	79

give rise to two new parameters as compared to the lowest order Lagrangian for rooted staggered fermions – the residual quark mass, $m_{\rm res}$, and the mixed valence-sea meson mass-splitting, $\Delta_{\rm mix}$. We have calculated $m_{\rm res}$ at two lattice spacings in order to measure the size of the residual chiral symmetry breaking, and to ensure that it is not too large. We find that our $m_{\rm res}$ is of comparable size to that of simulations by the RBC and UKQCD Collaborations at similar lattice spacings [12]. We also find that the size of $\Delta_{\rm mix}$ is comparable to the size of the smallest of the staggered meson taste-splittings measured by the MILC Collaboration. Because lattice artifacts are different in the valence and sea sectors of the mixed-action theory, they give rise to unitarity-violating effects that disappear in the continuum limit, some of which should be described by mixed-action chiral perturbation theory (MAχPT). Such effects are expected to be mild for many quantities of interest, but are expected to be significant in the case of the isovector scalar (a_0) correlator. Specifically, once the parameters $m_{\rm res}$, $\Delta_{\rm mix}$, and two others that can be determined from the light pseudoscalar meson spectrum are known, the two-particle intermediate state "bubble" contribution to the scalar correlator is completely predicted within MAχPT [17]. We find that the behavior of the scalar meson correlator is quantitatively consistent with the MAχPT prediction; this supports the claim that MAχPT describes the dominant unitarity-violating effects in the mixed-action lattice theory and can therefore be used to remove lattice artifacts and recover physical quantities. We refer to Ref. [14] for more details.

A necessary prerequisite for the calculation of B_K is the determination of the pion decay constant. Because f_π is known experimentally to sub-percent accuracy, it provides a test of the lattice methodology, and allows one to demonstrate an understanding of and control over systematic errors. Figures 2 and 3 show preliminary chiral and continuum extrapolations of the "pion" mass and decay constant data (i.e., $m_x = m_y$) using the approach of the MILC collaboration described in Ref. [16]. They are each simultaneous fits to the coarse and fine data using the next-to-leading order (NLO) partially-quenched, mixed-action χPT expression for m_π or f_π, plus additional NNLO analytic terms. We omit valence points with $m_\pi \gtrsim 500$ MeV and all points on the heaviest ($m_l/m_s = 0.4$) coarse ensemble. We also correct the data by the known 1-loop finite volume effects. Note that, with these judicious cuts, each fit has 36 data points for only 7 or 8 fit parameters, and we do not need to introduce Bayesian constraints.

Although we do not yet have sub-percent statistical errors over the entire range of our data set like the MILC Collaboration, we can still begin to distinguish between fit ansätze that work and those that do not, and hence draw interesting conclusions about the application of χPT to lattice QCD. In order to assess whether or not we have sufficient data in the chiral regime to allow the use of χPT, we must first look for chiral logarithms in the pseudoscalar data. We observe the appropriate curvature in the quantity m_π^2/m_q, which is expected to have large partially-quenched chiral logarithms. We also find that, in order to get a fit of the pion mass data with an acceptable confidence level, we must include the NLO chiral logarithms. Thus we have both qualitative and numerical evidence for the presence of chiral logarithms in our data. Not only must we include chiral logarithms for good fits, but in fact the fit prefers the *correct* mixed-action chiral logarithms. If we set the taste-breaking mass-splittings Δ_I, $\Delta_{\text{mix}} \to 0$ the confidence level decreases from 0.71 to 0.19. Thus we conclude that it is important to use the χPT expression appropriate to the specific lattice action used in simulations, and that this will be even more important as the statistical precision increases. We also implicitly observe finite-volume effects in our data. Despite our reasonably conservative choice to keep the quantity $m_\pi L \gtrsim 4$, the 1-loop finite-volume corrections to our m_π^2 data are as large as $\sim 3\%$ due to enhancements from partial quenching, and we cannot get an acceptable fit to our pion mass data without their inclusion. Thus we are currently performing a finite-volume study on one of the coarse ensembles. Finally, our fits clearly resolve a large $\mathcal{O}(a^2)$ analytic term that causes the coarse data to lie $\sim 10\%$ above the continuum full QCD curve. Because generic discretization effects will likely be of this size for all simulations not using $\mathcal{O}(\alpha_S a^2)$ improved lattice actions, this underscores the importance of having multiple lattice spacings.

Because we are able to correctly determine f_π to $\approx$2.5% statistical accuracy[1] with our existing data, this bolsters confidence in our mixed-action lattice calculations of f_K and B_K. We have accumulated numerical data for B_K on four different coarse MILC ensembles and three fine ensembles. Fig. 4 shows all of our unrenormalized B_K data versus the light quark mass in the "kaon". We have many partially-quenched valence quark masses in order to allow us to extrapolate to the physical down quark mass; our lightest valence quark mass is approximately $m_{\text{strange}}/10$. Because on each ensemble we have a valence quark mass that is slightly heavier than m_{strange}, we can interpolate, rather than extrapolate, to the physical strange quark. In the sea sector, our lightest quark mass is also approximately $m_{\text{strange}}/10$. The statistical errors in our B_K data after use of 90% of our current year's USQCD allocation range from 0.5–2%.

The determination of B_K in the continuum requires renormalization of the $\Delta S = 2$ operator responsible for neutral kaon mixing. We are currently working on NPR code which implements the method of Rome-Southampton [18]. The propagators needed for the NPR have already been computed and archived. Both the chiral extrapolation and nonperturbative renormalization of B_K will be completed soon, and we plan to present a preliminary result for B_K at Lattice 2008. Results for decay constants will appear in a preceding publication.

Run Plan and Resource Allocation

We began running promptly at the beginning of the 2007–2008 USQCD allocation cycle, and have already used approximately 90% of our current allocation. We have calculated B_K on approximately one- to two-hundred configurations on four $a = 0.125$ fm ensembles and two $a = 0.09$ fm ensembles. We have also calculated B_K on approximately forty of the larger-volume $40^3 \times 96$ $a = 0.09$ fm MILC lattices. In all cases we have used only every fourth configuration in order to minimize the correlations in the lattice data given limited resources.

Over the past year, due to requests from consumers, the MILC Collaboration (with computing time from the Fermilab Lattice Collaboration) has extended the size all of the ensembles that we are using, each of which will soon reach a total of 2000 configurations. The MILC collaboration is also generating two new $28^3 \times 96$ $a = 0.09$ ensembles of approximately 1000 configurations.

[1] We have not yet completed a full systematic error analysis for f_π.

Table 3: Proposed valence and sea quark mass combinations for the calculation of B_K in year three. We plan to use all of the available configurations from the coarse and fine ensembles that the Fermilab Lattice and MILC Collaborations have extended for high statistics running. For these ensembles the number of configurations in the last column is therefore 500 (the planned total) minus the number that we have already generated, which can be found in Table 1. We also plan to increase our data on the large volume 40^3 fine ensemble to 100 configurations, and to generate data for 250 configurations on two new 28^3 fine ensembles that we have not used previously.

a(fm)	L	m_l	m_s	$m_{val.}^{\rm dwf}$	# configs.
0.09	40	0.0031	0.031	0.004, 0.0124, 0.0186, 0.046	61
0.09	28	0.0062	0.031	0.0062, 0.0124, 0.0186, 0.046	284
0.09	28	0.0062	0.0186	0.0062, 0.0124, 0.0186, 0.046	250
0.09	28	0.0093	0.031	0.0062, 0.0124, 0.0186, 0.046	250
0.09	28	0.0124	0.031	0.0062, 0.0124, 0.0186, 0.046	301
0.125	24	0.005	0.05	0.007, 0.02, 0.03, 0.05, 0.065	372
0.125	20	0.007	0.05	0.01, 0.02, 0.03, 0.05, 0.065	290
0.125	20	0.01	0.05	0.01, 0.02, 0.03, 0.05, 0.065	308
0.125	20	0.02	0.05	0.01, 0.02, 0.03, 0.05, 0.065	383

In the 2008–2009 USQCD allocation year we plan to further reduce the following sources of uncertainty: statistics, discretization errors, chiral extrapolation errors, and finite volume errors. Table 3 summarizes our running plan: it shows the valence and sea quark mass combinations with which we propose to calculate B_K. In order to reduce statistical errors we plan to calculate B_K on all of the new coarse MILC lattices. We also plan to calculate B_K on all of the new configurations in the two $28^3 \times 96$ fine ensembles for which we already have data, and to calculate B_K on 250 configurations in two new $28^3 \times 96$ fine ensembles. This will allow us to better resolve both lattice-spacing and sea-quark mass dependence, thereby reducing chiral and continuum extrapolation errors. Finally, in order to understand the size of and control finite volume effects, we plan to calculate B_K on 60 additional configurations in the $40^3 \times 96$ fine ensemble. This run plan is flexible, and may be modified somewhat when we have a more complete error budget for B_K. If there were to be a substantial increase in our allocation due to other projects moving to leadership class machines, we would use the time to analyze more of the $40^3 \times 96$ ensemble, which MILC is also extending.

Table 4 shows the valence and sea quark mass combinations with which we propose to calculate the renormalization factor Z_{B_K} in 2008–2009. In order to conserve computing resources, we only plan on using half as many configurations for the nonperturbative renormalization factor as for the B_K 3-point correlation function. This would still give us ∼250 configurations per ensemble, which should be sufficient for a precise, ∼1–2%, determination of Z_{B_K}. We only plan on generating additional NPR propagators on the less expensive, smaller volume (20^3 for the coarse, 28^3 for the fine) MILC ensembles in order to save computing time.

We have timed gauge-fixing for the various sizes of MILC lattices and propagator inversions for a number of valence quark masses on the coarse and fine MILC lattices on the Fermilab clusters; these times are given in Tables 5– 7.[2] Using these timings, the total computing resources required to calculate all of the propagators listed in Tables 3 & 4 is given in Table 8. Based on this estimate,

[2]Note that we used the P4 cluster "qcd" for the small 20^3 lattices, while we used the opteron cluster "kaon" for the larger 24^3, 28^3, and 40^3 lattices.

Table 4: Proposed valence and sea quark mass combinations for the nonperturbative renormalization in year three. The number of configurations in the last column is the planned total (250) minus the number that we have already generated, which can be found in Table 2.

a(fm)	L	m_l	m_s	$m_{val.}^{\rm dwf}$	# configs.
0.09	28	0.0062	0.031	0.0062, 0.0124, 0.0186, 0.046	156
0.09	28	0.0093	0.031	0.0062, 0.0124, 0.0186, 0.046	250
0.09	28	0.0124	0.031	0.0062, 0.0124, 0.0186, 0.046	128
0.125	20	0.007	0.05	0.01, 0.02, 0.03, 0.05, 0.065	44
0.125	20	0.01	0.05	0.01, 0.03, 0.05, 0.065	89
0.125	20	0.02	0.05	0.01, 0.03, 0.05, 0.065	171

Table 5: Time required to Coulomb gauge-fix a MILC lattice using Chroma on the Fermilab cluster.

a(fm)	L	6n node-hours
0.09	40	221
0.09	28	77
0.125	24	20
0.125	20	12

our total computing request is 4.3 million 6n node-hours.

We would like to continue to run on the Fermilab clusters during the next allocation year, with a 12%-88% split between time on pion/qcd and time on kaon. As suggested in Tables 5–7, although we are able to calculate domain-wall propagators efficiently on the 20^3 lattices using pion/qcd, we require the larger number of nodes, increased memory, and increased scratch disk space available on kaon for the 24^3, 28^3, and 40^3 lattices. All of our code has been compiled and checked on the Fermilab machines, and our runscripts are written assuming that our propagators and data live in locations specific to the Fermilab head nodes.

We would also like to save all of the domain-wall propagators to tape at Fermilab for enough time to allow their use in other projects and by other groups. Table 9 shows the file sizes of the domain-wall quark propagators for the four different lattice volumes, in both GB and 6n-equivalent node hours assuming that they are stored on tape. A comparison of Table 9 and Tables 6 & 7 reveals that calculating the domain-wall propagator is 6 times more expensive than storing it for the cheapest propagator, and over 100 times more expensive for the costliest propagator. Thus it

Table 6: Time to calculate a single domain-wall propagator with $L_5 = 16$ using Chroma on the Fermilab "qcd" cluster.

a(fm)	L	$m_{val.}^{\rm dwf}$	nodes (qcd)	time (hours)	6n node-hours
0.125	20	0.01	32	4.63	74
0.125	20	0.02	32	2.98	47
0.125	20	0.03	32	2.11	34
0.125	20	0.05	32	1.52	24
0.125	20	0.065	32	1.30	21

Table 7: Time to calculate a single domain-wall propagator with $L_5 = 16$ using Chroma on the Fermilab "kaon" cluster.

a(fm)	L	$m_{val.}^{\mathrm{dwf}}$	nodes (kaon)	time (hours)	6n node-hours
0.09	40	0.004	160	9.44	2654
0.09	40	0.0124	160	3.97	1116
0.09	40	0.0186	160	2.99	841
0.09	40	0.046	160	1.69	475
0.09	28	0.0062	64	4.29	482
0.09	28	0.0124	64	2.39	269
0.09	28	0.0186	64	1.69	190
0.09	28	0.046	64	1.00	112
0.125	24	0.007	16	4.99	140
0.125	24	0.02	16	2.03	57
0.125	24	0.03	16	1.47	41
0.125	24	0.05	16	0.98	28
0.125	24	0.065	16	0.81	23

Table 8: Computer time needed to determine the B_K matrix element and renormalization factor using the sea quark ensembles, valence quark masses and numbers of configurations listed in Tables 3 and 4. Each calculation of B_K requires two Coulomb gauge-fixed wall source propagator inversions per valence quark mass per configuration. The nonperturbative renormalization requires one Landau gauge-fixed point source propagator inversion per valence quark mass per configuration.

40^3 $a = 0.09$ fm propagators	0.633 $\times 10^6$ 6n node-hours
28^3 $a = 0.09$ fm propagators	2.972 $\times 10^6$ 6n node-hours
24^3 $a = 0.12$ fm propagators	0.222 $\times 10^6$ 6n node-hours
20^3 $a = 0.12$ fm propagators	0.456 $\times 10^6$ 6n node-hours
Total	4.283 $\times 10^6$ 6n node-hours

Table 9: File sizes of domain-wall propagators for various spatial volumes. The equivalent cost to store the file on tape uses the conversion 1 Tbyte tape = 4,000 6n node-hours.

a(fm)	L	size (GB)	tape storage cost (6n node-hours)
0.09	40	6.592	26
0.09	28	2.261	9
0.125	24	0.949	4
0.125	20	0.550	2

Table 10: Mass storage needed to save all of the domain-wall propagators listed in Tables 3 and 4.

40^3 $a = 0.09$ fm propagators	3.14 TB
28^3 $a = 0.09$ fm propagators	24.64TB
24^3 $a = 0.12$ fm propagators	3.45 TB
20^3 $a = 0.12$ fm propagators	5.95 TB
Total	37.18 TB
	$= 0.15 \times 10^6$ 6n node-hours

is more efficient to save and reuse the domain-wall propagators than to recalculate them. The total storage space needed to save all of the propagators listed in Tables 3 & 4 is given in Table 10. Our total mass storage request is 38 TBytes of tape at Fermilab.

Summary

At the end of this project we expect to have a precise, few percent, determination of the neutral kaon mixing parameter B_K including dynamical quark effects. This would fulfill one of the key goals in flavor physics of USQCD stated in the 2002 strategic plan and the 2007 white paper "Fundamental parameters from future lattice calculations" [19]. Use of two lattice spacings, multiple quark masses, multiple spatial volumes, and mixed-action lattice chiral perturbation theory will give us control over the systematic errors associated with both the chiral and continuum extrapolations. This measurement, when used in a unitarity-triangle analysis, will place an important constraint on physics beyond the Standard Model.

The domain-wall propagators that we are generating can be used for the calculation of other interesting physics quantities, and we encourage other members of the lattice QCD community to do so. In addition to B_K, we are currently calculating the pion and kaon decay constants and will calculate the up, down, and strange quark masses in the near future; we would therefore like to retain exclusive rights to use our domain-wall propagators for all of these quantities. We would also like exclusive rights to calculate $K \to \pi\pi$ and the $K \to \pi\ell\nu$ form factor. We have begun writing code to calculate $K \to \pi\pi$ in the $\Delta I = 3/2$ channel, which does not require the generation of new domain-wall propagators. Both ϵ'/ϵ and the $K \to \pi\ell\nu$ form factor, however, require propagators in addition to those for B_K. We would therefore consider the possibility of a larger collaboration on them, especially if we were able to share computing resources. All of the propagators listed in Tables 1 and 2 are stored at Fermilab and can be made available immediately for non-competing analyses. Researchers who wish to use them should contact us to arrange access.

Acknowledgments

We thank Robert Edwards, Bálint Joó, and Kostas Orginos for invaluable assistance in using Chroma. We also thank Saul Cohen, Huey-Wen Lin and Meifeng Lin for help in compiling and running CPS. We thank Martin Savage for sharing an example of how to write and compile new code that uses the Chroma libraries, and we thank MILC for the use of their configurations. We thank Norman Christ and Chris Dawson for useful physics discussions and suggestions. We thank Jim Simone for helping us integrate Chroma with FermiQCD. Finally, we thank Sam Li for help with the nonperturbative renormalization.

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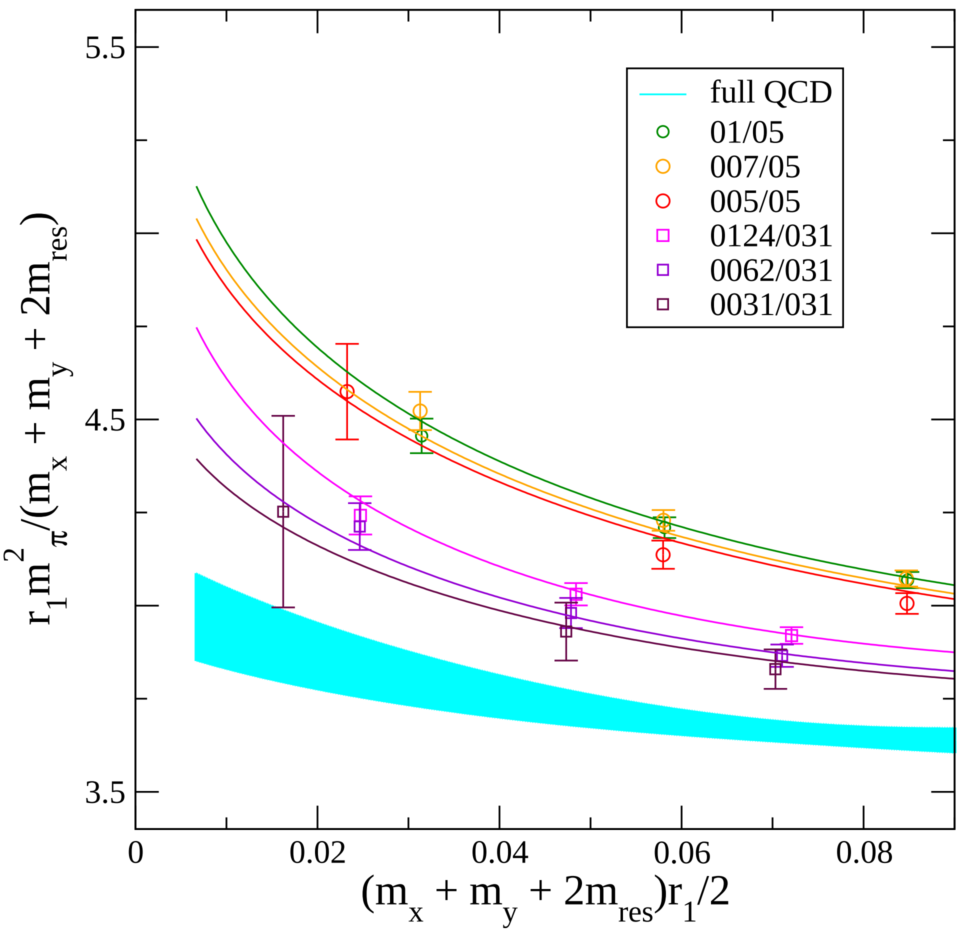

Figure 2: "Pion" mass-squared divided by quark mass versus sum of valence and residual quark masses. The circles are coarse data points and the squares are fine data points. The cyan band is the full QCD curve (with statistical errors only) that results from fitting the lattice data to the appropriate mixed-action χPT expression. The CL of the correlated fit is 0.71. Note that this plot only shows data for degenerate pions; the fit, however, also includes nondegenerate pions below $m_\pi \lesssim 500$ MeV.

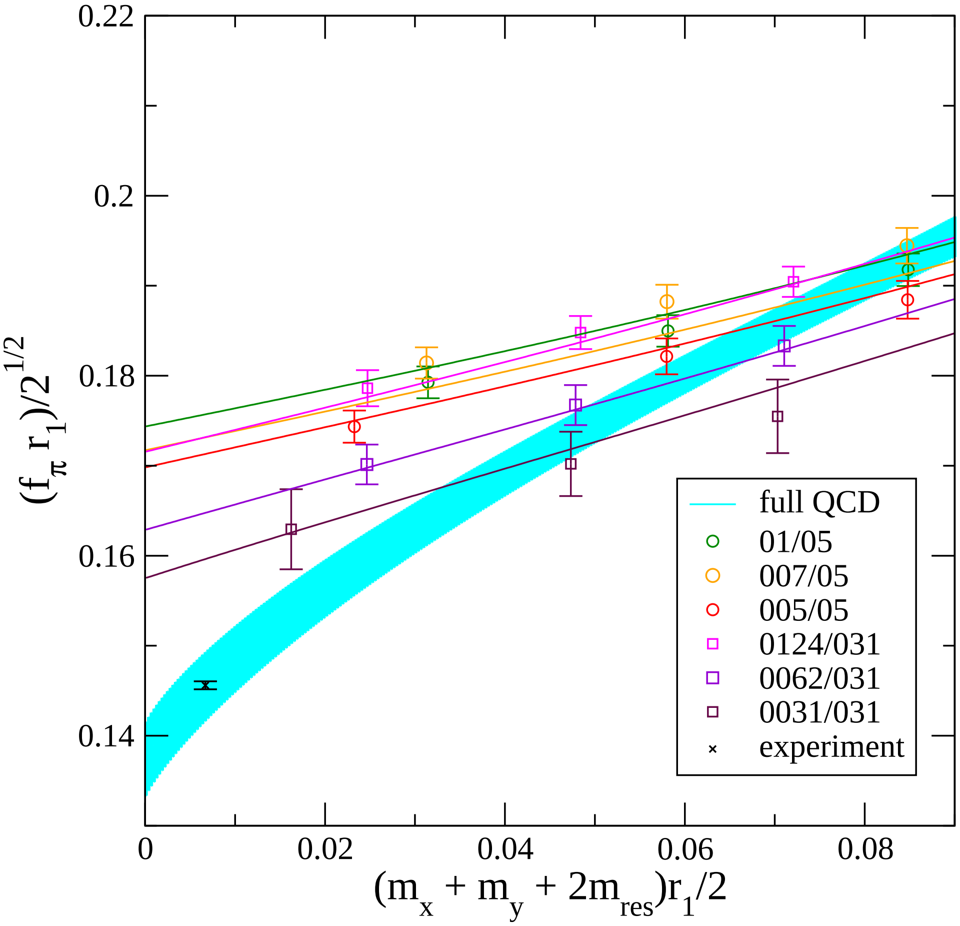

Figure 3: Pseudoscalar decay constant versus sum of valence and residual quark masses. The circles are coarse data points and the squares are fine data points. The cyan band is the full QCD curve (with statistical errors only) that results from fitting the lattice data to the mixed-action χPT expression. The cross is the experimental value, using the latest MILC determination of the lattice spacing. The CL of the correlated fit is 0.95. Note that this plot only shows data for degenerate pions; the fit, however, also includes nondegenerate pions below $m_\pi \lesssim 500$ MeV.

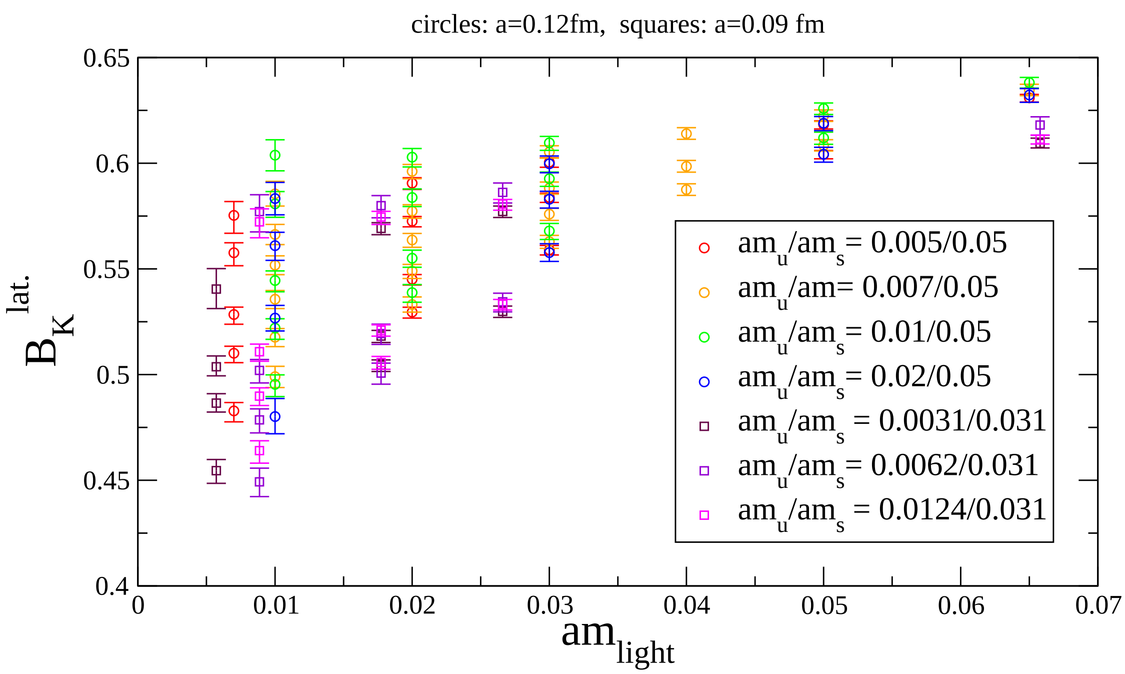

Figure 4: All partially-quenched data for $B_K^{\text{lat.}}$ shown versus the light quark mass in the "kaon". The colors indicate different sea quark ensembles. Circular symbols correspond to coarse lattice data and square symbols correspond to fine lattice data.