

Pion and kaon decay constants, quark masses,
and B_K with close-to-physical light-quark masses
from mixed-action lattice QCD

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Time Requested: 11.8 million Jpsi core-hours on Fermilab clusters and 7.2 million BG/P core-hours (equivalent to 3.9 million Jpsi core-hours) plus 21.9 Tbytes of tape storage (the equivalent of 0.06×10^6 Jpsi core-hours) at Fermilab.

Project Webpage: <http://www.nevis.columbia.edu/~tgadfort/Ruth/KaonPhysics/index.html>

Abstract

We propose to compute domain-wall propagators on the most chiral MILC Asqtad ensemble with $a \approx 0.09$ fm and $am_l/am_s = 1/20$. We also propose to double our existing data sets on the remaining MILC “coarse” and “fine” ensembles with $a \approx 0.12$ fm and $a \approx 0.09$ fm, respectively.

Over the past few years we have obtained computing time through USQCD in order to calculate the kaon bag parameter B_K . Last year we published a two-lattice spacing result with a quoted $\sim 4\%$ precision [1]. This is currently the best published unquenched determination of B_K , and fulfills one of the key goals in flavor physics of the U.S. lattice QCD community stated in the 2007 white paper. Combined with the recent determinations of B_K from RBC/UKQCD and Bae *et al.* (with which we are in good agreement) there is now some tension in the unitarity triangle fits. This tension is driven in large part by the new precision in the constraint from kaon mixing. Given this situation, it is crucial to nail down the combined chiral-continuum extrapolation of B_K by simulating with lighter pion masses and by reducing the statistical errors.

We therefore request the equivalent of 11.8 million Jpsi core-hours on Fermilab clusters and 7.2 million BG/P core-hours (equivalent to 3.9 million Jpsi core-hours) at Argonne and 21.9 Tbytes of tape storage (the equivalent of 0.06×10^6 Jpsi core-hours) at Fermilab for our mixed-action kaon physics project.

Scientific Objectives

Over the past few years we have carried out a successful project to calculate the kaon bag parameter B_K using USQCD resources. B_K parameterizes the hadronic contribution to mixing between K^0 and $\bar{K}^0$ mesons, and is one of the most sensitive probes of new physics beyond the Standard Model. Last year we published our result [1],

$$\hat{B}_K = 0.724(8)(29), \quad (1)$$

where the first error is statistical, and the second is the sum of all systematic errors in quadrature. This is currently the best published unquenched determination of B_K , with all systematic errors under control, and fulfills one of the key goals in flavor physics of the U.S. lattice QCD community stated in the 2007 white paper “Fundamental parameters from future lattice calculations” [2]. Combined with the recent determination of B_K from RBC/UKQCD [3] and Bae *et al.* [4] (with whom we are in good agreement) we confirm the earlier claim of Ref. [5] that there is some tension in the unitarity triangle fits [6]. This tension is driven in large part by the new precision in the constraint from kaon mixing. In particular, using the latest averages of all lattice inputs to the unitarity triangle fit, as well as some previously neglected corrections to ϵ_K [7], the fit prefers the value $\hat{B}_K = 0.889 \pm 0.083$ [8] when the lattice input for $\hat{B}_K$ is excluded from the fit. Given this tension, it is crucial to continue our precision studies of kaon physics using multiple methods including our mixed-action approach. We are requesting computing resources for our current mixed-action kaon physics project that would allow us to reduce the error from the chiral-continuum extrapolation, and, when combined with other improvements, obtain B_K with a total uncertainty of $\sim 2\%$.

Our project combines staggered and domain-wall fermions in the method pioneered by the LHP Collaboration [9], and takes advantage of the best properties of both discretizations. This method uses domain-wall valence quarks on top of an improved staggered sea (the MILC configurations [10]). By using staggered sea quarks we can take advantage of the vast library of MILC ensembles with multiple lattice spacings and “2+1” flavors of light sea quarks including masses as low as 1/20th of the strange quark mass. The use of domain-wall valence quarks allows us to minimize operator mixing and makes it much easier to implement nonperturbative renormalization. In the case of a purely staggered calculation, taste-breaking effects due to the finite lattice spacing introduce significant mixings between the desired four-fermion operator and a number of other operators with incorrect tastes [11]. This makes nonperturbative techniques difficult to apply, and thus far the impressive three-lattice spacing staggered B_K calculation of Bae *et al.* only uses lattice perturbation theory to compute the renormalization factor [4]. The taste-violating corrections to the $\Delta S = 2$ operator also greatly increase the number of undetermined coefficients in the $SU(3)$ staggered chiral perturbation theory expression for B_K , making the chiral and continuum extrapolation quite complicated [12], although this can be avoided through the use of $SU(2)$ χ PT for the chiral extrapolation [4]. Domain-wall quarks, on the other hand, have better chiral properties than staggered fermions, leading to qualitatively simpler lattice discretization errors. Although they do not possess an exact chiral symmetry on the lattice, the degree to which chiral symmetry is broken can be controlled through the length of the fifth dimension. Consequently, while the $\Delta S = 2$ operator still mixes with other operators, there are significantly fewer such operators and non-perturbative renormalization can be used more easily in the determination of B_K . Domain-wall quarks, however, are computationally expensive, and although a new domain-wall result including data at a second lattice spacing was recently obtained by RBC/UKQCD [3], given the expense of dynamical domain-wall configuration generation, results at a third, finer lattice spacing are unlikely to appear soon.

Table 1: Total uncertainty in B_K ; each error is given as a percentage. The second column shows our published error budget from Ref. [1], while the third column shows our project error budget given the requested running in this proposal. The larger renormalization factor error assumes one-loop perturbative matching and an error in Z_{B_K} comparable to that recently obtained by RBC/UKQCD using the same NPR method that we have adopted [3]. The smaller error assumes completion of the two-loop perturbative matching factor calculation by Almeida and Strm, and is a conservative estimate based on power-counting and the size of the known 1-loop terms.

uncertainty	PRD '09	<i>projected</i>	
		1-loop P.T.	2-loop P.T.
statistics	1.2%	0.8%	
chiral & continuum extrapolation	1.9%	1.3%	
scale and quark mass uncertainties	0.8%	0.8%	
finite volume errors	0.6%	0.6%	
renormalization factor	3.4%	2.0% $\longrightarrow$	1.0%
total	4.2%	2.7% $\longrightarrow$	2.1%

Over the past year we have been making several significant improvements with respect to our published work. On Intrepid at Argonne, we computed the unrenormalized $K^0\text{-}\overline{K}^0$ mixing matrix element with several valence quark masses on two $a \approx 0.06$ fm “superfine” sea quark ensembles. We are now generating domain-wall propagators on the $48^3 \times 144$ superfine ensemble with $am_l/am_s = 1/5$ for the nonperturbative renormalization, and will have completed this by the end of the current USQCD allocation cycle. After including data at a third lattice spacing and NPR using momentum-sources, twisted boundary conditions, and non-exceptional kinematics, we expect the largest source of systematic uncertainty to be the chiral extrapolation.¹ We therefore request time to compute B_K on the most chiral MILC Asqtad ensemble with $am_l/am_s = 1/20$ and $a \approx 0.09$ fm. We also request time to double our available data set on the remaining “coarse” and “fine” MILC ensembles with $a \approx 0.12$ fm and $a \approx 0.09$ fm, respectively. Inclusion of data with pseudoscalar masses close to the physical pion as well as smaller statistical errors will nail down the combined chiral-continuum extrapolation and reduce the total uncertainty in B_K . Given the requested resources, we expect to determine B_K to approximately 2% precision. These resources will also enable us to determine the pion and kaon decay constants with a precision competitive with the world’s best calculations by BMW, HPQCD, and MILC [13, 14, 15] and the light-quark masses with an accuracy competitive with RBC/UKQCD [16].

Improvements to Published Work

Table 1 shows the error budget for B_K from Ref. [1]. Since the publication of our two-lattice spacing result in 2009, we have been working to reduce the two largest sources of uncertainty from the nonperturbative renormalization and the chiral-continuum extrapolation.

¹Note that reducing the renormalization factor uncertainty to the same level as the chiral-continuum extrapolation will also require a continuum two-loop matching calculation, which is in progress by Almeida and Strm.

Nonperturbative Renormalization

We are using the nonperturbative renormalization (NPR) method of Rome-Southampton [17] to obtain the renormalization factor Z_{B_K} . This approach requires the computation of Landau gauge-fixed propagators at several valence quark masses and on multiple ensembles in order to obtain Z_{B_K} in the chiral limit. We are implementing some significant improvements over the method used to determine Z_{B_K} in our publication to reduce both the statistical and systematic errors. This is essential for reducing the total error in B_K , since the largest single source of uncertainty is from the NPR.

Recently, both the RBC/UKQCD Collaborations and Lytle (on behalf of Bae *et al.*) have begun using an improved volume source to significantly reduce the statistical errors in Z_{B_K} [18, 19]. Although the propagators require a momentum projection at the source, and thus require a new inversion for each momentum, the averaging over the spatial volume allows one to use many fewer configurations to get small statistical errors. (For example, Lytle showed that as few as 8 configurations was sufficient to obtain the quark mass renormalization factor Z_q on the coarse MILC lattices to sub-percent statistical accuracy [19].) Therefore, the use of gauge-fixed momentum sources is preferable for the larger-volume $a \approx 0.06$ fm lattices, and has allowed us to reduce the percentage of computing time used for the NPR by approximately a factor of 5 while at the same time reducing the size of the statistical errors by an order or magnitude.

Stürm *et al.* have also developed a new “non-exceptional” momentum scheme for determining Z_{B_K} [20]. Use of non-exceptional kinematics significantly reduces the amount of chiral symmetry breaking between Λ_V and Λ_A [21]. Chiral symmetry breaking is currently one of the largest sources of systematic uncertainty in our published determination of Z_{B_K} , but with the use of non-exceptional kinematics it will soon be one of the smallest. Use of non-exceptional kinematics can also reduce the size of higher-order corrections in the conversion from the RI/(S)MOM scheme to the $\overline{MS}$ scheme, as has been demonstrated for the quark mass renormalization factor Z_m [20].

We have written our own code to compute and write out the un-amputated momentum-space Greens Functions using the Chroma and QDP++ libraries [22], and our own python/cython analysis code for off-line amputation and projection. We have checked our code by computing the renormalization factors on one of RBC/UKQCD’s 16^3 domain-wall ensembles and comparing with the results presented in Ref. [21]. Figure 1 shows preliminary results for the quantity $2(\Lambda_A - \Lambda_V)/(\Lambda_A + \Lambda_V)$ on the coarse $am_l/am_s = 0.007/0.05$ ensemble. The amount of chiral symmetry breaking is consistent with zero within the sub-percent statistical errors.

We are also using this new NPR scheme with volume momentum sources to renormalize the light-quark masses. Recently the perturbative expression needed to convert the renormalization factor Z_m from the RI/SMOM scheme to the $\overline{MS}$ scheme was calculated to two-loop order [23, 24]. In this new scheme the size of the two-loop term is only $\sim 2\%$, so the neglected higher-order terms in the perturbative series are expected to be similarly small. Thus we expect to obtain a competitive determination of the light-quark masses with all sources of systematic uncertainty under good control, including the renormalization factor.

Third ($a \approx 0.06$ fm) Lattice Spacing

The next largest source of uncertainty in our published determination of B_K is the chiral-continuum extrapolation [1]. In order to reduce this error we have therefore computed the K^0 - $\bar{K}^0$ mixing matrix element with several valence quark masses on two sea-quark ensembles with $a \approx 0.06$ fm. Table 2 gives a summary of our full data set. Addition of data at a third lattice spacing gives us a much better handle on the a^2 -dependence of B_K and other matrix elements. The staggered

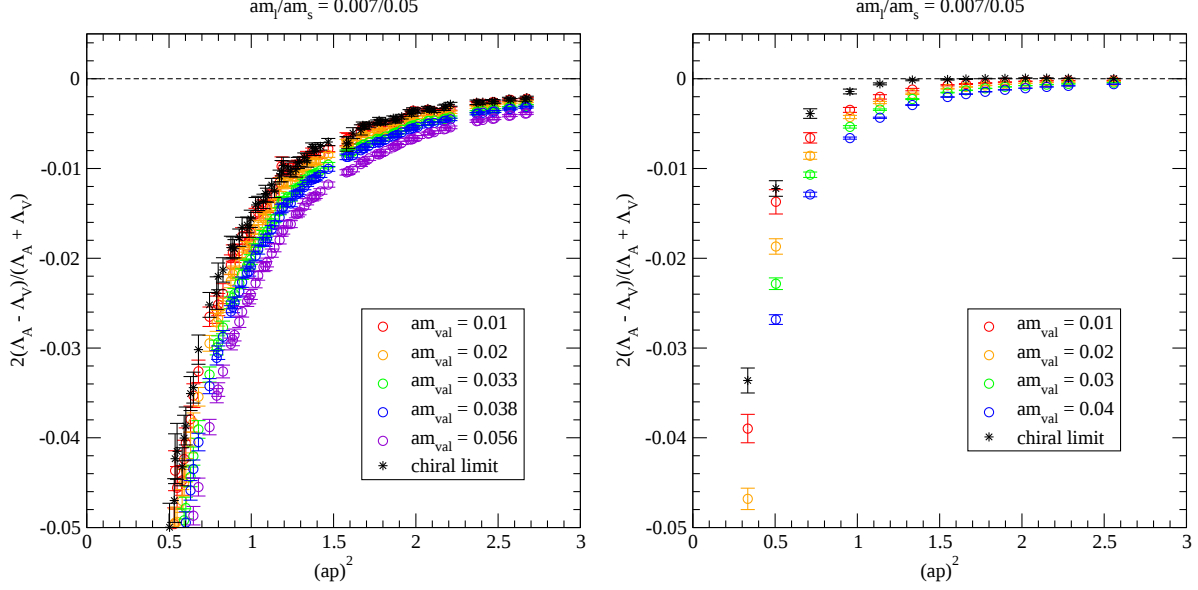


Figure 1: The quantity $2(\Lambda_A - \Lambda_V)/(\Lambda_A + \Lambda_V)$ versus $(ap)^2$ on the $am_l/am_s = 0.007/0.05$ coarse ensemble for several valence quark masses and in the chiral limit $am_{\text{val}} = am_l = 0$. The size of this quantity indicates the amount of chiral symmetry breaking and leads to a systematic uncertainty in Z_{B_K} . The left plot is from Ref. [1] using 836 point-source propagators and exceptional kinematics, while the right plot shows our new data using volume momentum-source propagators with 10 configurations per momentum to reduce the statistical errors and non-exceptional kinematics to reduce the degree of chiral symmetry breaking.

taste-splittings in the sea sector are also smaller on the superfine ensembles by a factor of three (the largest taste-singlet splitting is only $(165 \text{ MeV})^2$). Therefore the superfine data is much closer to the continuum. Figure 2 shows the preliminary chiral-continuum extrapolation of f_π including superfine data; the superfine data points lie extremely close to the continuum full QCD curve and have enabled a significant reduction in the uncertainty due to the chiral-continuum extrapolation.

Run Plan and Resource Allocation

After the inclusion of these improvements to our B_K calculation and the completion of the two-loop perturbative matching factor calculation which is in progress by Almeida and Stürm, we expect the largest source of systematic uncertainty in B_K to be the chiral extrapolation. We therefore propose to compute the pion and kaon decay constants, light-quark masses, and B_K on the most chiral MILC ensemble with a light sea-quark mass 1/20th of the nominal strange-quark mass in the sea sector (corresponding to a Goldstone pion mass of $m_\pi^{\text{sea}} \approx 180 \text{ MeV}$); this will enable much better control of the chiral extrapolation in the sea sector. Because the volume of this ensemble is large enough to accommodate such a light pion mass, we will also compute the axial current and $K^0\text{-}\overline{K}^0$ mixing matrix elements with a light valence quark close to the physical d -quark mass (corresponding to a valence pion mass of $m_\pi^{\text{val}} \approx 180 \text{ MeV}$). Inclusion of such data would approximately halve the distance of the chiral extrapolation in f_π (see Fig. 2) and would significantly reduce the systematic uncertainty due to the chiral extrapolation in the valence sector. Inclusion of this data would almost eliminate the valence chiral extrapolation in B_K (see Fig. 3) because the proposed valence quark mass is close to the physical d -quark, and the kaon is

an $\bar{s}d$ meson.

We also propose to double the statistics on our existing coarse and fine data sets. We will do so by adding a second time source on each configuration separated by $T/2$ (where T is the temporal extent of the lattice) from the original source. As shown in Fig. 3, the statistical errors in the data points closest to the physical value of B_K have $\gtrsim 1.5\%$ statistical errors. Reducing the statistical errors would therefore not only improve the statistical error in B_K , but it would also better constrain the extrapolation to the physical value of B_K and hence reduce the uncertainty due to the chiral extrapolation.

Table 3 shows the proposed valence- and sea-quark mass combinations for the calculation of B_K . The calculation of the $K^0\text{-}\bar{K}^0$ mixing matrix element requires two Coulomb gauge-fixed wall-source propagators per valence-quark mass – one with periodic boundary conditions and one with antiperiodic boundary conditions in order to effectively double the time extent of the lattice. We have timed the propagator inversions for a number of valence quark masses on the $a \approx 0.09$ fm MILC lattices with $am_l/am_s = 1/20$ on the Argonne BG/P; these times are given in Table 4. In order to save computing resources, we plan to use the MIT Moebius domain-wall inverter in Chroma for this ensemble [27]. This will allow us to reduce the extent of the fifth dimension by a factor of two from $L_5 = 16 \rightarrow 8$ while keeping the size of the residual quark mass m_{res} and the amount of chiral symmetry breaking approximately fixed [28]. Based on this estimate, the total computing time needed to compute B_K on this ensemble is 3.9 million J/psi core-hours. Because of the large lattice volume of this ensemble ($64^3 \times 96 \times L_5 = 8$), this running is most efficient on the BG/P Intrepid at Argonne (see Table 4). The computing time needed to double the number of Coulomb gauge-fixed wall-source propagators on the remaining coarse and fine ensembles is 10.3 million J/psi core-hours. The time to gauge-fix the lattices, and later to compute the 2-point and 3-point correlation functions, is much smaller than that needed for the propagator generation, and we estimate that an additional 10% will be sufficient. All of this running can be done efficiently on the clusters at Fermilab, and we request this portion of our allocation there. Our total computing request is summarized in Table 5.

We would like to save the Coulomb gauge-fixed domain-wall propagators to tape at Fermilab for enough time to allow their use in other projects (such as $K \rightarrow \pi\pi$ matrix elements) and by other groups. All of the propagators that we have generated for B_K have already been re-used for the calculation of $K \rightarrow \pi\pi$ in the $\Delta I = 3/2$ channel, and we presented preliminary results for the quantity $\text{Re}(A_2)$ using a subset of our available data at Lattice 2010 [29]. The proposed propagators on the coarse and fine ensembles with the time source at $T/2$ will also be particularly useful for our ongoing computation of $K \rightarrow \pi\pi$ matrix elements in the $\Delta I = 1/2$ channel, for which propagators originating from two different time sources are necessary for some of the contractions. Table 6 shows the file sizes of the domain-wall quark propagators for the four different lattice volumes, in both GB and Jpsi-equivalent node hours assuming that they are stored on tape. A comparison of Table 6 and Table 4 reveals that calculating the domain-wall propagator is $\sim 100 - 200$ times more expensive than storing it. Thus it is more efficient to save and reuse the domain-wall propagators than to recalculate them. The total storage space needed to save all of the propagators listed in Table 3, plus the storage currently in use holding existing propagators is given in Table 7. Our total mass storage request is 21.9 TBytes of new tape at Fermilab. In order to save our correlators and other analysis files, we also require a small amount of additional disk space: ~ 0.22 TBytes in the “/project” area at Fermilab.

Summary

With the addition of data at close to the physical pion mass in the sea sector ($m_\pi^{\text{sea}} \approx 180$ MeV) and close to the physical kaon mass in the valence sector, plus twice the statistics on the $a \approx 0.12$ fm and $a \approx 0.09$ fm ensembles, we expect to have a precise determination of the neutral kaon mixing parameter B_K including dynamical quark effects with a total uncertainty of approximately 2%. This will fulfill one of the key goals in flavor physics of USQCD stated in the 2002 strategic plan and the 2007 white paper “Fundamental parameters from future lattice calculations” [2]. We also expect to obtain results for the pion and kaon decay constants and light-quark masses that are competitive with the world’s best determinations of these quantities. At the level of precision we expect to achieve for B_K , it is also important to consider the effects of $K \rightarrow \pi\pi$ matrix elements in order to use ε_K as a constraint on new physics, and we are currently working on computing the necessary kaon matrix elements. In fact, all of the propagators that we are generating for $K^0\text{-}\bar{K}^0$ can also be re-used for the calculation of $K \rightarrow \pi\pi$ matrix elements. Other improvements needed to maximize the impact of our current proposal are the three-loop corrections to the Inami-Lim functions and an improved determination of $|V_{cb}|$, both of which are underway. The result of our published B_K calculation (along with that of RBC/UKQCD and Bae *et al.*) has already made a significant impact on the global unitarity-triangle fit and revealed a tension with the Standard Model prediction; pursuing this lead by further reducing the uncertainties is essential and may ultimately lead to definitive evidence for new physics.

The domain-wall propagators that we are generating can be used for the calculation of other interesting physics quantities, and we encourage other members of the lattice QCD community to do so. For example, the NPLQCD collaboration plans on using our lightest quark propagators to compute the π - K scattering length. All of the propagators listed in Table 2 are stored at Fermilab and can be made available immediately for non-competing analyses. Researchers who wish to use them should contact us to arrange access. We would therefore like to retain exclusive rights to use our domain-wall propagators for both direct and indirect determinations of $K \rightarrow \pi\pi$ matrix elements. We would also like to retain exclusive rights to calculate the long distance contributions to ΔM_K .

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Table 2: Number of Coulomb gauge-fixed wall source propagators available. Both spectrum quantities (m_π , f_π , *etc.*) and B_K have been calculated with all of these propagators.

$a(\text{fm})$	$L^3 \times T$	m_l	m_s	$m_{val.}^{\text{dwf}}$	# configs.
≈ 0.06	$64^3 \times 144$	0.0018	0.018	0.0026, 0.0469, 0.0108, 0.033	96
≈ 0.06	$48^3 \times 144$	0.0036	0.018	0.0036, 0.0072, 0.0108, 0.033	128
≈ 0.09	$40^3 \times 96$	0.0031	0.0031	0.004, 0.0124, 0.0186, 0.046	102
≈ 0.09	$40^3 \times 96$	0.0031	0.031	0.004, 0.0124, 0.0186, 0.046	150
≈ 0.09	$28^3 \times 96$	0.0062	0.031	0.0062, 0.0124, 0.0186, 0.046	374
≈ 0.09	$28^3 \times 96$	0.0093	0.031	0.0062, 0.0124, 0.0186, 0.046	198
≈ 0.09	$28^3 \times 96$	0.0124	0.031	0.0062, 0.0124, 0.0186, 0.046	198
≈ 0.09	$28^3 \times 96$	0.0062	0.0186	0.0062, 0.0124, 0.0186, 0.046	160
≈ 0.125	$32^3 \times 64$	0.005	0.005	0.007, 0.02, 0.03, 0.05	175
≈ 0.125	$24^3 \times 64$	0.005	0.05	0.007, 0.02, 0.03, 0.05, 0.065	216
≈ 0.125	$20^3 \times 64$	0.007	0.05	0.01, 0.02, 0.03, 0.04, 0.05, 0.065	268
≈ 0.125	$20^3 \times 64$	0.01	0.05	0.01, 0.02, 0.03, 0.05, 0.065	220
≈ 0.125	$20^3 \times 64$	0.02	0.05	0.01, 0.03, 0.05, 0.065	117
≈ 0.125	$20^3 \times 64$	0.01	0.03	0.01, 0.02, 0.03, 0.05, 0.065	160

Table 3: Proposed valence and sea quark mass combinations for the calculation of Coulomb gauge-fixed wall-source propagators. The propagators listed in the top panel will be computed with their origin at $t = 0$ (modulo a random 4D translation of the gauge field), while the propagators listed in the bottom panel will be computed with their origin at the lattice temporal midpoint $t = T/2$ (again modulo a translation of the gauge field) in order to double the existing data set.

$a(\text{fm})$	$L^3 \times T$	m_l	m_s	$m_{val.}^{\text{dwf}}$	# configs.
≈ 0.09	$64^3 \times 96$	0.00155	0.031	0.0024, 0.0124, 0.0186, 0.046	100
≈ 0.09	$40^3 \times 96$	0.0031	0.0031	0.004, 0.0124, 0.0186, 0.046	102
≈ 0.09	$40^3 \times 96$	0.0031	0.031	0.004, 0.0124, 0.0186, 0.046	150
≈ 0.09	$28^3 \times 96$	0.0062	0.031	0.0062, 0.0124, 0.0186, 0.046	374
≈ 0.09	$28^3 \times 96$	0.0093	0.031	0.0062, 0.0124, 0.0186, 0.046	198
≈ 0.09	$28^3 \times 96$	0.0124	0.031	0.0062, 0.0124, 0.0186, 0.046	198
≈ 0.09	$28^3 \times 96$	0.0062	0.0186	0.0062, 0.0124, 0.0186, 0.046	160
≈ 0.125	$32^3 \times 64$	0.005	0.005	0.007, 0.02, 0.03, 0.05	175
≈ 0.125	$24^3 \times 64$	0.005	0.05	0.007, 0.02, 0.03, 0.05, 0.065	216
≈ 0.125	$20^3 \times 64$	0.007	0.05	0.01, 0.02, 0.03, 0.04, 0.05, 0.065	268
≈ 0.125	$20^3 \times 64$	0.01	0.05	0.01, 0.02, 0.03, 0.05, 0.065	220
≈ 0.125	$20^3 \times 64$	0.02	0.05	0.01, 0.03, 0.05, 0.065	117
≈ 0.125	$20^3 \times 64$	0.01	0.03	0.01, 0.02, 0.03, 0.05, 0.065	160

Table 4: Time to calculate a single domain-wall propagator with $L_5 = 8$ using Chroma on the Argonne “intrepid” BG/P. For comparison, the lower panel shows the time for the same computation on the Ds cluster at Fermilab.

$a(\text{fm})$	$L^3 \times T$	m_{val}^{dwf}	nodes (intrepid)	time (hours)	Jpsi core-hours
≈ 0.09	$64^3 \times 96$	0.0024	1024	4.46	9,865
≈ 0.09	$64^3 \times 96$	0.0124	1024	1.88	4,158
≈ 0.09	$64^3 \times 96$	0.0186	1024	1.53	3,384
≈ 0.09	$64^3 \times 96$	0.046	1024	1.02	2,256
≈ 0.09	$64^3 \times 96$	0.046	64 (Ds)	1.78	4,848

Table 5: Computer time needed to generate the Coulomb gauge-fixed wall-source propagators for the sea quark ensembles, valence quark masses and numbers of configurations listed in Table 3.

64^3 $a = 0.09$ fm propagators	3.9×10^6 Jpsi core-hours	Argonne
other $a = 0.09$ fm propagators	9.1×10^6 Jpsi core-hours	Fermilab
all $a = 0.12$ fm propagators	1.2×10^6 Jpsi core-hours	Fermilab
correlation functions and analysis	1.5×10^6 Jpsi core-hours	Fermilab
Total	15.7×10^6 Jpsi core-hours	

Table 6: File sizes of domain-wall propagators for various lattice volumes. The equivalent cost to store the file on tape uses the conversion 1 Tbyte tape = 2,694 Jpsi core-hours.

$a(\text{fm})$	$L^3 \times T$	size (GB)	tape storage cost (Jpsi core-hours)
≈ 0.09	$64^3 \times 96$	28	74
≈ 0.09	$40^3 \times 96$	6.6	18
≈ 0.09	$28^3 \times 96$	2.3	6
≈ 0.125	$32^3 \times 64$	2.3	6
≈ 0.125	$24^3 \times 64$	0.9	2
≈ 0.125	$20^3 \times 64$	0.6	2

Table 7: Tape storage needed to save all of the domain-wall propagators used to compute B_K . The current storage determination reflects actual usage, while the future storage requirement is an estimate including both the proposed new propagators in Table 3 and the superfine NPR propagators currently being generated with the remainder of the 2010–2011 USQCD allocation. The equivalent cost to store the file on tape uses the conversion 1 Tbyte tape = 2,694 Jpsi core-hours.

currently in use	≈ 153.2 TB
additional space for new runs	21.9 TB
Total	175.1 TB
	$= 0.47 \times 10^6$ Jpsi core-hours

Table 8: Disk storage needed to save 2-point and 3-point correlators, logfiles, and analysis files in the “/project” area at Fermilab. The current storage determination reflects actual usage, while the future storage requirement is an estimate that includes both the data that would be generated with the proposed running and the data currently being generated (primarily for the NPR) with the remainder of the 2010–2011 USQCD allocation. The equivalent cost to store the file on disk uses the conversion 1 Tbyte disk = 26,940 Jpsi core-hours.

currently in use	0.38 TB
additional space for new runs	0.22 TB
Total	0.60 TB
$\approx 16,000$ Jpsi core-hours	

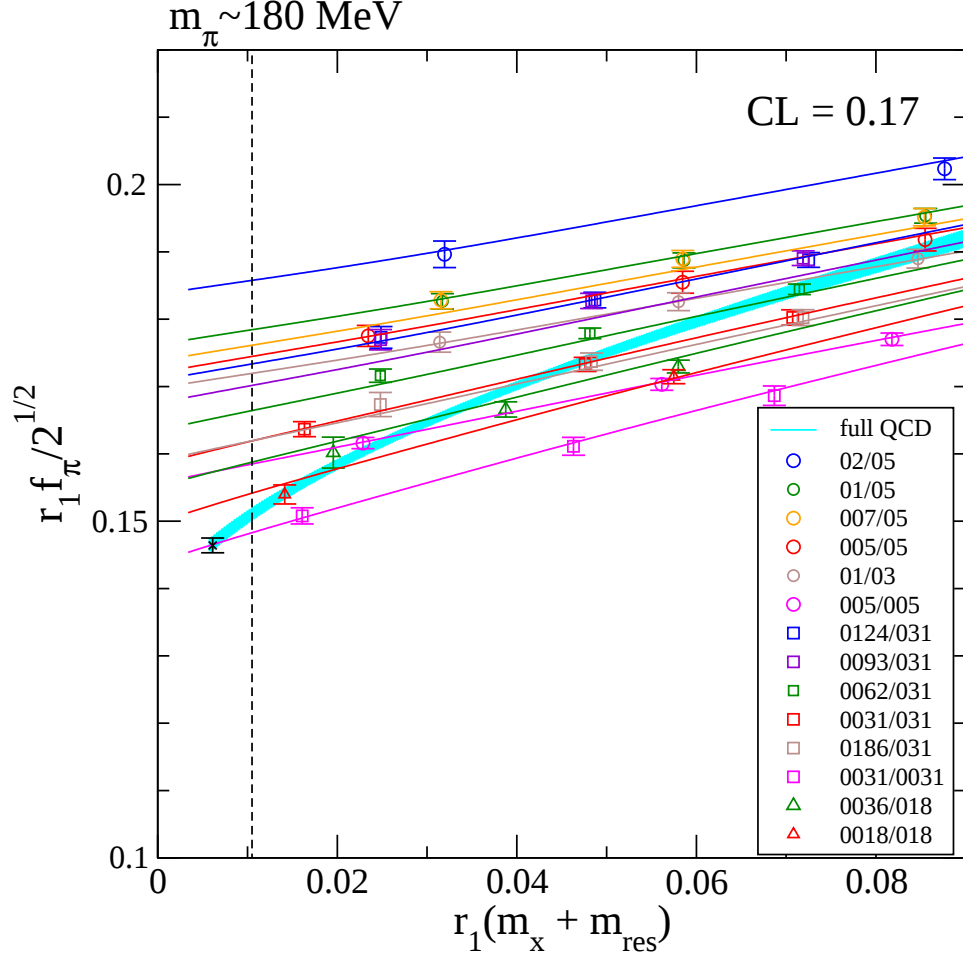


Figure 2: Pseudoscalar decay constant versus sum of valence and residual quark masses. The circles are coarse data points, squares are fine data points, and triangles are superfine data points. The cyan band is the full QCD curve (with statistical errors only) that results from fitting the lattice data to the mixed-action χ PT expression, supplemented by higher order polynomial terms. The “x” is the experimental value using the latest HPQCD lattice spacing determination [25, 26]. Note that this plot only shows data for degenerate pions; the fit, however, also includes nondegenerate pions below $m_\pi \lesssim 600 \text{ MeV}$. The dashed vertical line denotes the location of the proposed data point at a pion mass of approximately 180 MeV.

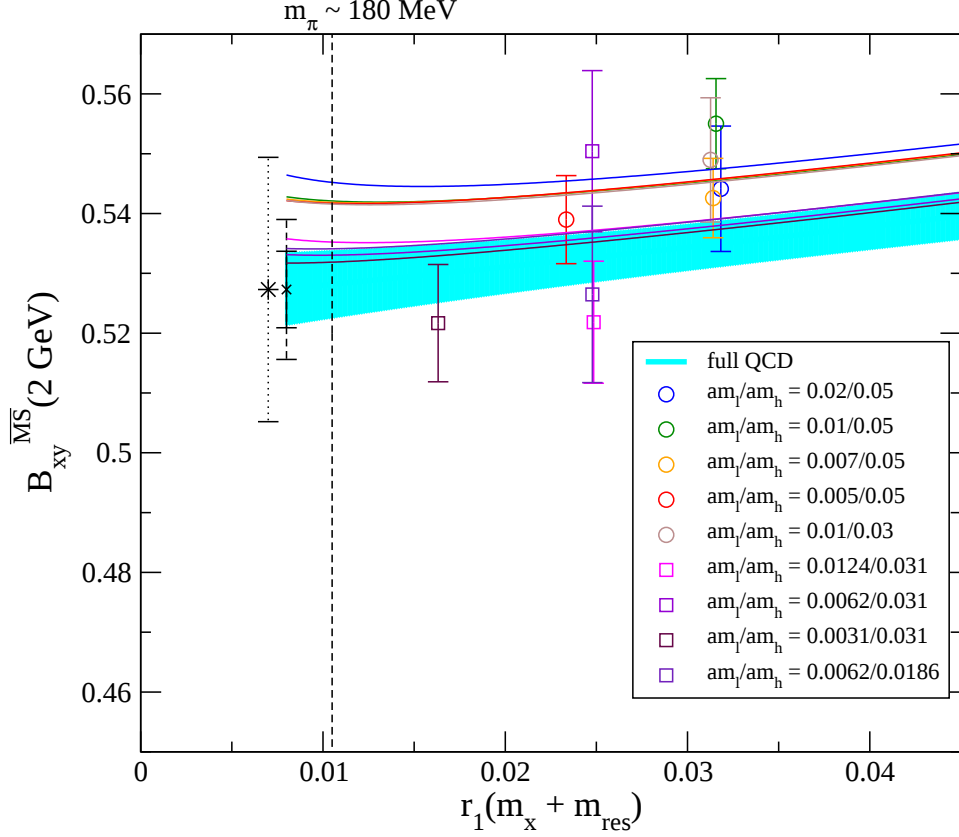


Figure 3: B_K versus light valence quark mass $r_1(m_x + m_{\text{res}})$ [1]. Only a small subset of the data points included in the fit are shown for illustration. Each point corresponds to a nondegenerate kaon in which the heavier mass is fixed close to the strange-quark mass and the lighter mass is the lightest simulated on that sea-quark ensemble. The fit curve is the full QCD continuum extrapolated curve with the strange quark fixed to its physical value. The extrapolated value of B_K is shown, including the statistical error (solid error bar with X) and the systematic error due to the chiral extrapolation, combined with the statistical error in quadrature (dashed error bar). The dotted error bar (star, slightly offset) shows the total error for B_K . The dashed vertical line denotes the location of the proposed data point close to the physical d -quark mass.