

B_K and $K \rightarrow \pi\pi$
with Domain-Wall Valence Quarks
and Staggered Sea Quarks

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Participants: Jack Laiho, Ruth S. Van de Water, Alexei Bazavov, Claude Bernard, Carleton DeTar, Steven Gottlieb, Urs M. Heller, James Hetrick, Ludmila Levkova, Mehmet Oktay, James Osborn, Robert Sugar, Doug Toussaint (MILC Collaboration)

Time Requested: The equivalent of 10.27 million 6n node-hours (with 85% on the Argonne BG/P and 15% on the Fermilab clusters) plus 36 Tbytes of tape storage (the equivalent of $\sim 48,000$ 6n node-hours) at Fermilab.

Abstract

We propose to compute domain-wall propagators on the 2+1 flavor MILC staggered ensembles to be shared by two mixed-action kaon physics projects: B_K by Laiho and Van de Water and $K \rightarrow \pi\pi$ by the MILC Collaboration.

A precise determination of B_K will place an important constraint on the apex of the CKM unitarity triangle, and thus on possible physics beyond the Standard Model. With data at several valence and sea quark masses and two lattice spacings, we have determined B_K to $\sim 1\%$ accuracy [1]. This is currently the best unquenched determination of B_K , and fulfills one of the key goals in flavor physics of the U.S. lattice QCD community stated in the 2007 white paper. In this upcoming allocation year, we plan to generate data at a third lattice spacing of $a \approx 0.06$ fm. This will solidify the chiral-continuum extrapolation and reduce the total uncertainty in B_K . We will also improve upon our preliminary determinations of the light pseudoscalar meson masses and decay constants [2].

A determination of $K \rightarrow \pi\pi$ will «RV: Jack – can you add a 1-sentence summary on the goal?» ... Although the lattice calculation of $K \rightarrow \pi\pi$ is known to be challenging, the success of the mixed-action method for B_K suggests that the mixed-action method is also a good way to approach $K \rightarrow \pi\pi$. «RV: I don't like the wording of the previous sentence!» In the upcoming allocation year, we plan to compute $K \rightarrow \pi\pi$ in the $\Delta I = 3/2$ channel on the $a \approx 0.012$ fm and $a \approx 0.09$ fm MILC lattices using the propagators already computed for B_K , as well as on the $a \approx 0.06$ fm lattices using the newly-generated propagators.

We therefore request the equivalent of 10.27 million 6n node-hours (with $\sim 85\%$ on the Argonne BG/P and $\sim 15\%$ on the Fermilab clusters) and 36 Tbytes of tape storage (the equivalent of $\sim 48,000$ 6n node-hours) at Fermilab to improve upon the calculation of B_K and to compute $K \rightarrow \pi\pi$.

Scientific Objectives $\langle\langle$ Jack add $K \rightarrow \pi\pi$ content $\rangle\rangle$

Experimental measurements of CP violation can be used to extract information about the CKM matrix. In particular, the size of indirect CP violation in the neutral kaon system, ϵ_K , combined with theoretical input, places an important constraint on the apex of the CKM unitarity triangle [3]. Because ϵ_K is well-known experimentally [4], the dominant source of error in this procedure is the uncertainty in the lattice determination of the nonperturbative constant B_K , which parameterizes the mixing between K^0 and $\overline{K}^0$. It is likely that new physics would give rise to extra CP-violating phases in addition to that of the CKM matrix; such phases would manifest themselves as apparent inconsistencies among different measurements of quantities which should be identical within the standard CKM picture. Thus a precise determination of B_K will help to constrain physics beyond the Standard Model. Because the importance of B_K to new physics searches is well known to the lattice community, this quantity was highlighted in the USQCD Collaboration’s 2007 white paper “Fundamental parameters from future lattice calculations” as being one of the key quantities in flavor physics [5]. We are submitting a Type A proposal that would further this goal by allowing us to improve upon the results that we have obtained so far and reduce the total error in B_K to below $\sim 3\%$.

Our project combines staggered and domain-wall fermions in the method pioneered by the LHP Collaboration [6], and takes advantage of the best properties of both discretizations. This method uses domain-wall valence quarks on top of an improved staggered sea (the MILC configurations [7]). By using staggered sea quarks we can take advantage of the vast library of MILC ensembles. This allows us simulate at multiple lattice spacings using “2+1” flavors of light sea quarks with masses as low as 1/10th of the strange quark mass. The use of domain-wall valence quarks allows us to minimize operator mixing, making it much easier to implement nonperturbative renormalization. In the case of a purely staggered calculation, taste-breaking effects due to the finite lattice spacing introduce significant mixings between the desired four-fermion operator and a number of other operators with incorrect tastes [8]. This makes nonperturbative techniques difficult to apply, and thus far staggered B_K calculations have used only perturbative renormalization. This limitation increases the systematic error in B_K [9]. The taste-violating corrections to the $\Delta S = 2$ operator also greatly increase the number of undetermined coefficients in the staggered chiral perturbation theory expression for B_K , making the chiral and continuum extrapolation quite complicated [10]. Domain-wall quarks, on the other hand, have better chiral properties than staggered fermions, leading to qualitatively simpler lattice discretization errors. Although they do not possess an exact chiral symmetry on the lattice, the degree to which chiral symmetry is broken can be controlled through the length of the fifth dimension. Consequently, while the $\Delta S = 2$ operator still mixes with other operators, there are significantly fewer such operators and non-perturbative renormalization can be used more easily in the determination of B_K . Domain-wall quarks, however, are computationally expensive, and the most recent results with 2+1 flavors [11], though impressive, use only one lattice spacing. Although an updated domain-wall result including data at a second lattice spacing is expected in the upcoming year, given the expense of dynamical domain-wall configuration generation, results at a third lattice spacing are unlikely to appear soon.

$\langle\langle$ RV: Jack – I’m not sure what you’ll write on $K \rightarrow \pi\pi$, but you should mention that it’s listed as one of the “future directions” in the 2007 white paper. $\rangle\rangle$

Using USQCD computing resources we have accumulated numerical data at several valence and sea quark masses and two lattice spacings. We can now compute B_K to $\sim ???\%$ accuracy with all sources of systematic uncertainty under control [1], as shown in the “Preliminary Physics Results” section. This is currently the best unquenched flavor determination of B_K , and will place a valuable

constraint on new physics in the quark flavor sector. «RSV: I'm wary of saying that it will improve the constraint, given the uncertainty in V_{cb} ...» We have also determined the light pseudoscalar meson decay constants with similar uncertainties, and have calculated the ratio f_K/f_π to 2% accuracy [2]. We therefore request time in the forthcoming year in order to add data on the $a \approx 0.06$ fm MILC lattices. Use of a third lattice spacing will nail down the combined chiral-continuum extrapolation and reduce the total uncertainty in B_K . Given the requested resources, we expect to determine B_K including dynamical quarks to better than 3% accuracy. «Should we include a footnote with the caveat that this would require 2-loop P.T. here?»

Our calculations of the light pseudoscalar decay constants and B_K benefit from the simpler renormalization properties and less complicated chiral perturbation theory expressions of the mixed-action method. Our approach also appears promising for the determination of the low energy constants of chiral perturbation theory, in particular for $K \rightarrow \pi\pi$ matrix elements. This set of matrix elements is of considerable interest, given both the long-standing puzzle of the $\Delta I = 1/2$ rule and the valuable constraint on the Standard Model that would then be possible given the experimental measurement of ϵ'/ϵ (which parametrizes direct CP violation in kaon decays). The MILC collaboration is therefore proposing to calculate $K \rightarrow \pi\pi$ using the mixed action method in separate USQCD proposal. Their project will use all of the propagators that we generate for B_K to compute $K \rightarrow \pi\pi$ 3-point correlation functions.

Simulation Method and Code

We are using the same parameters for our domain-wall propagators on the MILC lattices [6] as the LHP Collaboration. This allows us to perform cross-checks of simple quantities such as m_{res} and m_π . In particular, we first perform hypercubic-block (HYP) smearing on the MILC lattices with the standard smearing parameters in order to reduce proximity with the Aoki phase [12]. We also use a domain-wall height of 1.7 and an extent in the extra dimension of $L_s = 16$, which produces an acceptably small residual quark mass, of the size of the physical light quark mass on the coarse MILC ensembles. On the fine MILC lattices we find a value of m_{res} that is a factor of 3 times smaller. As in most calculations of B_K , we Coulomb gauge-fix the lattices (after HYP-smearing) and use wall sources for the quark propagators.

Each calculation of the B_K 3-point correlation function requires four quark propagators, though as described below, we only need two inversions per valence mass. As shown in Figure 1, we fix the timeslices of the K^0 and $\bar{K}^0$ and vary the location of the $\Delta S = 2$ operator over all timeslices in between. We first generate both periodic and antiperiodic boundary condition quark propagators and then take symmetric and antisymmetric linear combinations to produce forward- and backward-moving propagators, respectively. We then tie together two forward-moving quarks and two-backward moving quarks with the B_K four-quark operator in order to effectively double the time extent of the lattice. This allows us to increase the size of our plateau region for fitting to all time slices of the lattice; it also allows us to go to lighter valence quark masses by reducing finite volume effects in the time direction. Because all propagators can be reused and combined to form various degenerate and nondegenerate “kaons,” we need two Coulomb gauge-fixed wall source propagators – one periodic and one antiperiodic – per valence quark mass per configuration. We recycle these propagators in the computation of the pseudoscalar meson masses and decay constants. In order to calculate $K \rightarrow \pi\pi$ in the $\Delta I = 3/2$ channel using the method of Refs. [13, 14], we will compute the additional correlation functions that are shown in Fig. 2. This does not require any propagators beyond those needed for B_K . The B_K and $K \rightarrow \pi\pi$ calculations will also share the Landau gauge-fixed point source propagators needed for the nonperturbative renormalization

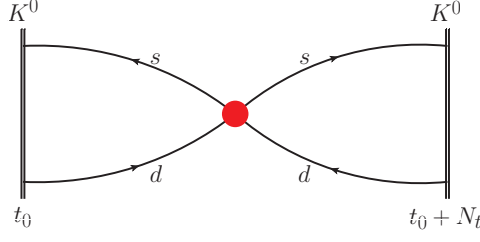


Figure 1: Three-point lattice correlation function for K^0 - $\overline{K}^0$ mixing.

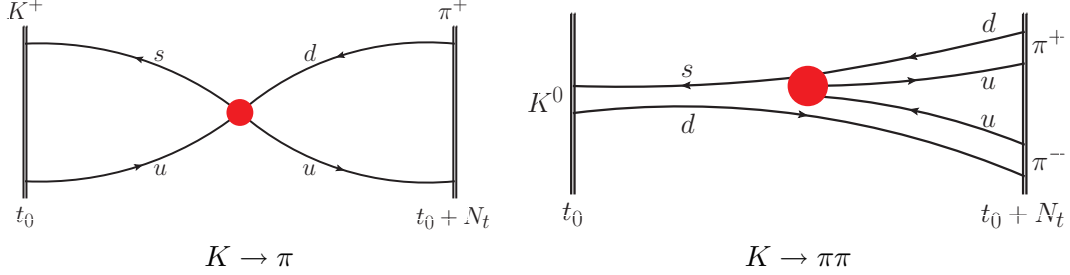


Figure 2: Additional three-point and three-point lattice correlations functions for $K \rightarrow \pi\pi$ in the $\Delta I = 3/2$ channel.

(NPR) of the lattice operators.

The most computationally intensive portion of this project by far is the propagator inversions, for which we are using the optimized domain-wall inverter in the Chroma lattice QCD software package [15]. We are using our own codes, written using the Chroma and QDP++ libraries, to symmetrize and antisymmetrize the quark propagators, to calculate the $\Delta S = 2$ matrix element, and to Fourier transform the Landau gauge-fixed point source propagators for the NPR. All of these codes have been checked against those in the Columbia Physics System [16]. We will write and test Chroma code to calculate the $K \rightarrow \pi\pi$ matrix elements before the beginning of the next allocation cycle.

We have generated a large amount of data to be used in the calculation of B_K . In particular, we have generated Coulomb gauge-fixed wall-source propagators for the masses and ensembles listed in Table 1. We have also generated Landau gauge-fixed point-source propagators for the masses and ensembles given in Table 2; these propagators will be used for the nonperturbative renormalization (NPR). All of these propagators have been stored on tape at Fermilab and will be used by the MILC Collaboration to compute $K \rightarrow \pi\pi$.

Preliminary Mixed-Action Results $\langle\langle\text{Jack}\rangle\rangle$

In this section we review the physics results that we have obtained in the past year. We have calculated the pion and kaon decay constants to $\sim 2\%$ accuracy and their ratio f_K/f_π , to $\sim 2\%$ accuracy [2]. Our results are consistent with experiment and with the more precise lattice QCD determinations of the MILC [17] and HPQCD Collaborations [18]. We have also computed B_K to $\sim 3.5\%$ accuracy [1]. Our result is currently the best 2+1 flavor determination of B_K , and will place a valuable constraint on new physics in the quark flavor sector.

$\langle\langle\text{JWL will update this section ...}\rangle\rangle$

Run Plan and Resource Allocation

We began running promptly at the beginning of the 2008–2009 USQCD allocation cycle, and have already used our allocation at both Fermilab and Argonne. We have calculated B_K on approximately one- to two-hundred configurations on six $a \approx 0.125$ fm ensembles and six $a \approx 0.09$ fm ensembles. In all cases we have used only every fourth configuration in order to minimize the correlations in the lattice data given limited resources. We are currently generating propagators on a larger-volume $40^3 \times 96$ $a \approx 0.09$ fm MILC ensemble with three degenerate light quarks of $am_l = 0.0031$ in the low priority queue at Fermilab. This ensemble will be particularly useful for the determination of low-energy constants, and hence for the MILC Collaboration’s calculation of $K \rightarrow \pi\pi$. We have also begun to compute propagators on an even finer $a \approx 0.06$ fm “superfine” ensemble with $am_l/am_s = 0.0036/0.018$ on the BG/P at Argonne, but were only able to generate propagators on ~ 30 configurations with our allocation for the current year. We expect to obtain propagators on ~ 50 or more configurations, however, by the end of this allocation cycle with the use of the low priority account.

In the 2009–2008 USQCD allocation year we plan to further reduce the uncertainty in B_K due to the extrapolation to the physical light quark masses and the continuum by generating additional data at the $a \approx 0.06$ fm lattice spacing. Table 3 lists the six MILC superfine ensembles, along with the spatial volumes, quark masses, and number of available configurations. We are requesting time to generate propagators on two ensembles with $am_l/am_s = 0.0072/0.018$ and $am_l/am_s = 0.0072/0.018$. The valence and sea quark mass combinations with which we propose to calculate B_K are given Table 4. The use of three lattice spacings will make the continuum extrapolation more reliable and reduce discretization errors in B_K . Because the $am_l/am_s = 0.0072/0.018$ ensemble has a volume of ≈ 4 fm, it will allow us to simulate with an even lighter valence quark mass while keeping finite-volume effects under control. This will reduce the error due to the extrapolation to the physical up and down quark masses, which is currently one of our largest sources of systematic uncertainty in B_K .

In order to use the $a \approx 0.06$ fm data, we also need to compute the renormalization factor Z_{B_K} at this lattice spacing. Table 5 shows the valence and sea quark mass combinations with which we propose to calculate the renormalization factor Z_{B_K} in 2008–2009. We only plan on generating additional NPR propagators on a less expensive, smaller volume 48^3 ensemble. Because we have so much NPR data on the $a \approx 0.0125$ fm and $a \approx 0.09$ fm ensembles, we primarily need the $a \approx 0.06$ fm data to determine the central value of the renormalization factor, Z_{B_K} . We can rely upon the “coarse” and “fine” data to estimate systematic uncertainties in order to save computing time. Recently, the RBC and UKQCD Collaborations have begun using an improved volume source to significantly reduce the statistical errors in Z_{B_K} [23]. Although the propagators require a momentum projection at the source, and thus require a new inversion for each momentum, the averaging over the spatial volume allows one to use many fewer configurations to get small statistical errors. Therefore the use of gauge-fixed momentum sources appears preferable for the larger-volume $a \approx 0.06$ fm lattices, and will hopefully allow us to reduce the percentage of our allocation used for computing the renormalization factor to below 20%. Any reduction in time devoted to the nonperturbative renormalization will go into improving the statistical errors in B_K .

In the 2009–2008 USQCD allocation year the MILC collaboration plans to compute the correlation functions needed for $K \rightarrow \pi\pi$ in the $\Delta I = 3/2$ channel using all of the propagators computed for B_K on the $a \approx 0.012$ fm and $a \approx 0.09$ fm ensembles, which have been saved to tape at Fermilab, as well as with the propagators on the $a \approx 0.06$ fm ensembles generated with the resources requested in this proposal. Use of numerous valence and sea quark masses and three lattice spacings will enable good control over the chiral and continuum extrapolations. $\langle\langle$ RV: Do I need to say

anything else? Is there anything that we can say about why the superfine lattices are especially important for $K \rightarrow \pi\pi$ to make our request more compelling?))

We have timed gauge-fixing for the various sizes of MILC lattices on the Fermilab clusters; these times are given in Table 6. We have also timed the propagator inversions for a number of valence quark masses on the $a \approx 0.06$ fm MILC lattices on the Argonne BG/P; these times are given in Table 7. Using these timings, the total computing resources required to calculate all of the propagators listed in Tables 4 & 5 is given in Table 8. Based on this estimate, our total computing request is 10.27 million 6n node-hours. This includes time to compute correlation functions for both the B_K and $K \rightarrow \pi\pi$ projects. We would like the majority of our allocation, $\sim 85\%$, to be on the Argonne BG/P. The $a \approx 0.06$ fm lattices are too large to compute the propagators at Fermilab, but run easily at Argonne. We would like to retain $\sim 15\%$ of our allocation, however, on the Fermilab clusters to compute correlators, Fourier-transform propagators for the nonperturbative renormalization, and analyze data.

We would like to save all of the domain-wall propagators to tape at Fermilab for enough time to allow their use in other projects (such as $K \rightarrow \pi\pi$ in the $\Delta I = 1/2$ channel in a future year) and by other groups. Table 9 shows the file sizes of the domain-wall quark propagators for the four different lattice volumes, in both GB and 6n-equivalent node hours assuming that they are stored on tape. A comparison of Table 9 and Table 7 reveals that calculating the domain-wall propagator is $\sim 100 - 200$ times more expensive than storing it. Thus it is more efficient to save and reuse the domain-wall propagators than to recalculate them. The total storage space needed to save all of the propagators listed in Tables 4 & 5, plus the storage currently in use holding existing propagators is given in Table 10. Our total mass storage request is 36 TBytes of new tape (plus ≈ 60 TBytes of existing tape) at Fermilab. In order to save our correlators and other analysis files, we also require a small amount of disc space: ~ 0.036 TBytes in the “/project” area at Fermilab for the B_K project and ~ 0.036 TBytes in the “/project” area at Fermilab for the $K \rightarrow \pi\pi$ project.

Summary

With the addition of data at a third $a \approx 0.06$ fm lattice spacing, we expect to have a precise determination of the neutral kaon mixing parameter B_K including dynamical quark effects with a total uncertainty below $\sim 3\%$. (RV: Should we say this given the status of the NPR?) This will fulfill one of the key goals in flavor physics of USQCD stated in the 2002 strategic plan and the 2007 white paper “Fundamental parameters from future lattice calculations” [5]. This measurement, when used in a unitarity-triangle analysis, will place an important constraint on physics beyond the Standard Model. (RV: Jack – can you add a blurb on $K \rightarrow \pi\pi$?)

Although we are requesting time to analyze only 50-100 configurations on two ensembles, we could easily make good use of a substantial increase in our allocation if there was an increase in total USQCD resources. Depending upon the overall error budgets for B_K or $K \rightarrow \pi\pi$ we would use additional computing resources either to increase statistics on one of the existing ensembles or to generate domain-wall propagators on another ensemble. We also plan to use the domain-wall propagators generated by the MILC collaboration for $K \rightarrow \pi\pi$. In addition to B_K , Laiho and Van de Water are currently calculating the pion and kaon decay constants and will calculate the up, down, and strange quark masses in the near future; we would therefore like to retain exclusive rights to use our domain-wall propagators for all of these quantities. We would also like exclusive rights to calculate the $K \rightarrow \pi\ell\nu$ form factor.

The domain-wall propagators that we are generating can be used for the calculation of other interesting physics quantities, and we encourage other members of the lattice QCD community to

do so. For example, the NPLQCD collaboration has been using our lightest quark propagators to compute the π - K scattering length; these results will appear in an upcoming publication. All of the propagators listed in Tables 1 and 2 are stored at Fermilab and can be made available immediately for non-competing analyses. Researchers who wish to use them should contact us to arrange access.

Acknowledgments

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Table 1: Number of Coulomb gauge-fixed wall source propagators available. Both spectrum quantities (m_π , f_π , *etc.*) and B_K have been calculated with all of these propagators. Configurations denoted by a “*” indicate ensembles that are in progress.

$a(\text{fm})$	L	m_l	m_s	$m_{val.}^{\text{dwf}}$	# configs.
≈ 0.06	48	0.0036	0.018	0.0036, 0.0072, 0.0108, 0.033	30*
≈ 0.09	40	0.0031	0.0031	0.004, 0.0124, 0.0186, 0.046	30*
≈ 0.09	40	0.0031	0.031	0.004, 0.0124, 0.0186, 0.046	153
≈ 0.09	28	0.0062	0.031	0.0062, 0.0124, 0.0186, 0.046	213
≈ 0.09	28	0.0093	0.031	0.0062, 0.0124, 0.0186, 0.046	199
≈ 0.09	28	0.0124	0.031	0.0062, 0.0124, 0.0186, 0.046	202
≈ 0.09	28	0.0062	0.0186	0.0062, 0.0124, 0.0186, 0.046	169
≈ 0.125	32	0.005	0.005	0.007, 0.02, 0.03, 0.05	175
≈ 0.125	24	0.005	0.05	0.007, 0.02, 0.03, 0.05, 0.065	218
≈ 0.125	20	0.007	0.05	0.01, 0.02, 0.03, 0.04, 0.05, 0.065	279
≈ 0.125	20	0.01	0.05	0.01, 0.02, 0.03, 0.05, 0.065	228
≈ 0.125	20	0.02	0.05	0.01, 0.03, 0.05, 0.065	117
≈ 0.125	20	0.01	0.03	0.01, 0.02, 0.03, 0.05, 0.065	162

Table 2: Number of Landau gauge-fixed point source propagators available. Configurations denoted by a “*” indicate that the propagators were computed on every 6th trajectory instead of the standard every 24th trajectory.

$a(\text{fm})$	L	m_l	m_s	$m_{val.}^{\text{dwf}}$	# configs.
≈ 0.09	28	0.0062	0.031	0.0119, 0.0171, 0.0287, 0.04	387
≈ 0.09	28	0.0093	0.031	0.0287	251
≈ 0.09	28	0.0124	0.031	0.0119, 0.0171, 0.0287	381
≈ 0.125	24	0.005	0.05	0.01, 0.02, 0.033, 0.038, 0.056	836*
≈ 0.125	20	0.007	0.05	0.01, 0.02, 0.033, 0.038, 0.056	540*
≈ 0.125	20	0.01	0.05	0.01, 0.02, 0.033, 0.038, 0.056	484*
≈ 0.125	20	0.02	0.05	0.01, 0.02, 0.03	81

Table 3: Available MILC $a \approx 0.06$ fm “superfine” lattices. Configurations are recorded every 5 trajectories.

$a(\text{fm})$	L	m_l	m_s	$m_\pi L$	approx. # configs.
≈ 0.06	64	0.0036	0.0108	5.96	500
≈ 0.06	64	0.0018	0.018	4.27	800
≈ 0.06	56	0.0025	0.018	4.39	800
≈ 0.06	48	0.0036	0.018	4.49	800
≈ 0.06	48	0.0054	0.018	5.48	600
≈ 0.06	48	0.0072	0.018	6.33	600

Table 4: Proposed valence and sea quark mass combinations for the calculation of B_K in year four. If we were to receive an increased allocation, we would use the additional time either to add statistics on the listed ensemble or to generate data on another superfine ensemble, depending upon the B_K error budget.

$a(\text{fm})$	L	m_l	m_s	$m_{val.}^{\text{dwf}}$	# configs.
≈ 0.06	64	0.0018	0.018	0.0036, 0.0108, 0.033	50
≈ 0.06	48	0.0072	0.018	0.0036, 0.072, 0.0108, 0.033	100

Table 5: Proposed valence and sea quark mass combinations for the nonperturbative renormalization in year four. The three masses are estimates for timing purposes, and the exact masses will change once we know the value of the taste singlet sea meson on the $a \approx 0.06$ fm lattices.

$a(\text{fm})$	L	m_l	m_s	$m_{val.}^{\text{dwf}}$	# configs.
≈ 0.06	48	0.0036	0.018	0.0036, 0.0072, 0.0108, 0.033	100

Table 6: Time required to Coulomb gauge-fix a MILC lattice using Chroma on the Fermilab cluster. The time for the 64^3 ensemble is an estimate based on scaling the 48^3 time by the ratio of the volumes.

$a(\text{fm})$	L	6n node-hours
≈ 0.06	64	1657
≈ 0.06	48	699

Table 7: Time to calculate a single domain-wall propagator with $L_5 = 16$ using Chroma on the Argonne “intrepid” BG/P. The times for the 64^3 ensemble are estimates based on scaling the 48^3 time by the ratio of the volumes. The time for the lightest mass on the 64^3 ensemble ($am = 0.0026$) is a “guesstimate” based on the scaling behavior on the $a = 0.09$ fm lattices.

$a(\text{fm})$	L	$m_{val.}^{\text{dwf}}$	nodes (intrepid)	time (hours)	6n node-hours
≈ 0.06	64	0.0026	—	—	23755
≈ 0.06	64	0.0072	—	—	9989
≈ 0.06	64	0.0108	—	—	7419
≈ 0.06	64	0.033	—	—	4089
≈ 0.06	48	0.0036	1024	8.08	8936
≈ 0.06	48	0.0072	1024	3.81	4214
≈ 0.06	48	0.0108	1024	2.83	3130
≈ 0.06	48	0.033	1024	1.56	1725

Table 8: Computer time needed to determine the B_K matrix element and renormalization factor using the sea quark ensembles, valence quark masses and numbers of configurations listed in Tables 4 and 5. Each calculation of B_K requires two Coulomb gauge-fixed wall source propagator inversions per valence quark mass per configuration. The nonperturbative renormalization requires one Landau gauge-fixed point source propagator inversion per valence quark mass per configuration.

gauge-fixing	0.10×10^6 6n node-hours
64^3 $a = 0.06$ fm Coulomb propagators	3.53×10^6 6n node-hours
48^3 $a = 0.06$ fm Coulomb propagators	3.60×10^6 6n node-hours
48^3 $a = 0.06$ fm Landau propagators	1.80×10^6 6n node-hours
correlation functions and analysis	1.34×10^6 6n node-hours
Total	10.27×10^6 6n node-hours

Table 9: File sizes of domain-wall propagators for various spatial volumes. The equivalent cost to store the file on tape uses the conversion 1 Tbyte tape = 1,347 6n node-hours.

$a(\text{fm})$	L	size (GB)	tape storage cost (6n node-hours)
≈ 0.06	64	41	54
≈ 0.06	48	17	22
≈ 0.09	40	6.6	9
≈ 0.09	28	2.3	3
≈ 0.125	32	2.3	3
≈ 0.125	24	0.9	1
≈ 0.125	20	0.6	1

Table 10: Tape storage needed to save all of the domain-wall propagators used to compute B_K . The current storage determination reflects actual usage, while the future storage requirement is an estimate based on the proposed new propagators in Tables 4 and 5 and the file sizes in Table 9. The equivalent cost to store the file on tape uses the conversion 1 Tbyte tape = 1,347 6n node-hours.

currently in use	≈ 60 TB
additional space for new runs	36 TB
Total	96 TB
	$= 0.129 \times 10^6$ 6n node-hours

Table 11: Disk storage needed to save 2-point and 3-point correlators, logfiles, and analysis files in the “/project” area at Fermilab. The current storage determination reflects actual usage, while the future storage requirement is an estimate. The equivalent cost to store the file on disk uses the conversion 1 Tbyte tape = 13,470 6n node-hours.

currently in use	0.018 TB
additional space for new runs	0.054 TB
Total	0.072 TB
	$= 970$ 6n node-hours