$K \to \pi\pi$ matrix elements from mixed action lattice QCD

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Motivation

Lattice calculations of $K \to \pi\pi$ matrix elements are important for understanding the Standard Model and in constraining physics beyond the Standard Model. For example, they are needed to explain the origin of the $\Delta I = 1/2$ rule and to compute the long-distance contributions to neutral kaon mixing [1]. Because the lowest-order Standard Model contributions to ϵ'/ϵ are from 1-loop electroweak penguin diagrams, $K \to \pi\pi$ decay is sensitive to physics at very high scales. Many extensions of the Standard Model lead to new particles that enter the loops, and these contributions to $K \to \pi\pi$ may be sufficiently large that they can be observed once the hadronic uncertainties in the weak matrix elements are small enough.

Two general approaches to evade the Maiani-Testa no-go theorem [2] and determine $K \to \pi\pi$ matrix elements on the lattice have been developed. The "direct" Lellouch-Lüscher finite-volume method [3] is the most straightforward to implement, but it is computationally demanding because it requires a large (~ 6 fm) box and physical light-quark masses. The "indirect" method constructs $K \to \pi\pi$ matrix elements using the low-energy constants (LEC's) of $SU(3)$ χPT obtained from calculating simpler lattice quantities such as $K \to 0$ and $K \to \pi$. Although it was shown in Ref. [4] that all LEC's through next-to-leading order can be obtained from such "simple" lattice quantities, this approach relies on the use of $SU(3)$ χPT at the kaon mass, where the convergence of the chiral expansion is quite slow. We present a new approach for computing $K \to \pi\pi$ matrix elements that is lest costly than the direct method but that does not rely on the use of $SU(3)$ χPT to extrapolate to the physical kaon mass as in the indirect method.

Tension in the CKM unitarity triangle

Recent $2+1$ flavor lattice calculations of B_K with $\sim 4\%$ precision [5] have revealed a 2-3σ tension in the CKM unitarity triangle [1, 6, 7] which may be due to kaon or B_d-meson mixing. Almost all constraints on the CKM unitarity triangle, however, come from the B-meson sector. Thus it is essential to place other constraints on the unitarity triangle from the kaon sector in order to test whether the amount of observed CP-violation in the B-meson sector is the same as in the kaon sector. Once lattice QCD calculations of $K \to \pi\pi$ matrix elements are sufficiently precise, they can be combined with the experimental measurement of ϵ'_K/ϵ_K to impose an additional constraint on the apex of the CKM unitarity triangle.

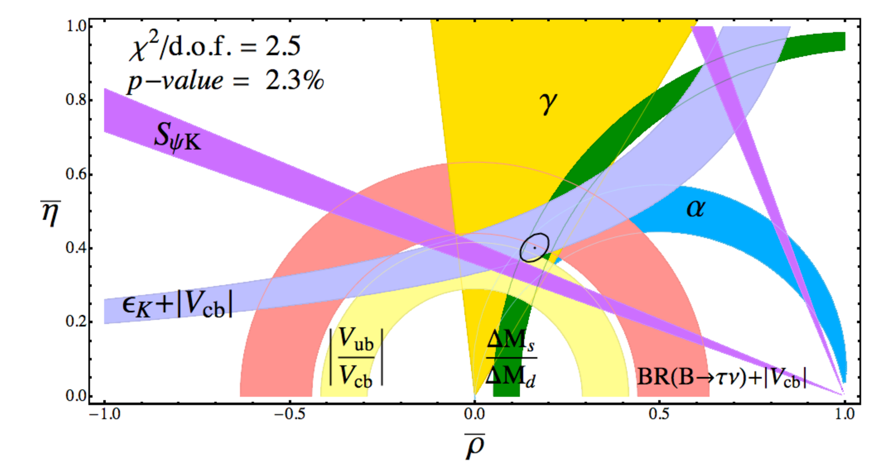

Figure: Global fit of the CKM unitarity triangle [7].

Proof of concept: f_π and f_K

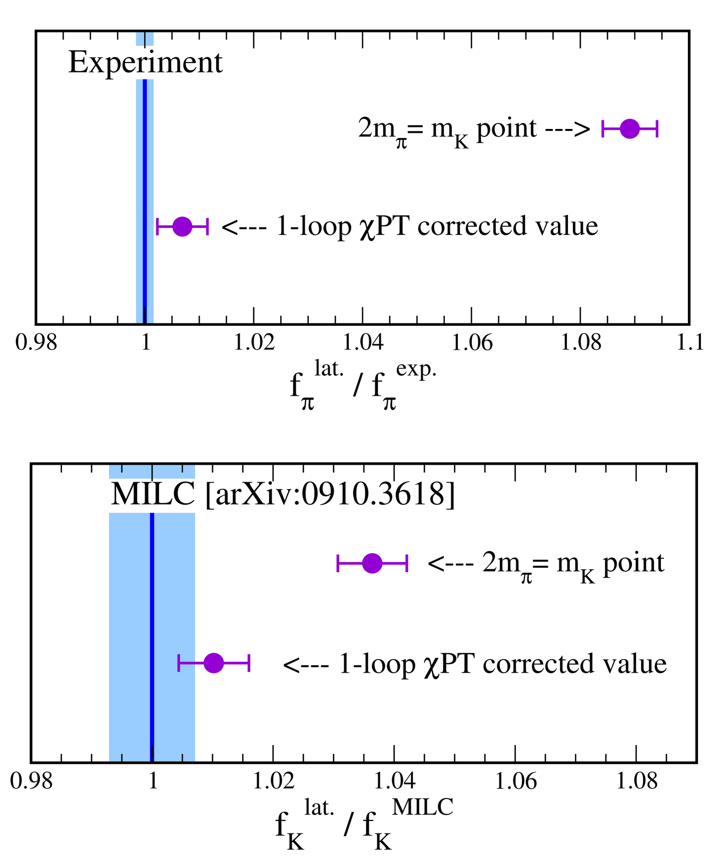

Figure: Demonstration of the method for f_π (left plot) and f_K (right plot). Errors on the circular points only show the statistical errors in the numerator. Vertical error bands denote the total (statistical plus systematic) uncertainties in f_π and f_K. After the interpolation to the unphysical kinematics point $m_K = m_K^{\rm phys}$ and $m_\pi = m_K^{\rm phys}/2$, the size of the 1-loop χPT correction is below 10% for f_π and below 5% for f_K.

Advantages of mixed-action lattice calculations

We compute $K \to \pi\pi$ matrix elements in unquenched lattice QCD using asqtad-improved staggered sea quarks and domain-wall valence quarks. We use the publicly-available 2+1 flavor MILC gauge configurations [8], and simulate with several valence and sea quark masses. Although our preliminary analysis only uses two lattice spacings, we have generated data at three lattice spacings ($a \sim 0.06$, 0.09 and 0.12 fm) and will include all of it in the publication. Our lightest taste-pseudoscalar sea-sea pion has $m_{\pi,5} = 240$ MeV while our lightest valence-valence pion has $m_\pi = 210$ MeV. On the $a \sim 0.06$ fm ensembles, the heaviest (taste-singlet) sea-sea pion is also quite light $m_{\pi,I} = 270$ MeV. This gives us good control over our combined chiral-continuum extrapolation using mixed-action χPT. The approximate chiral symmetry of the valence sector ($m_{\rm res} < 3$ MeV on all three lattice spacings) makes analysis of the mixed-action simulation data simpler than the purely staggered case. Only two additional parameters appear at 1-loop in the mixed action χPT expressions for m_{PS}, f_{PS}, and B_K as compared to the purely domain-wall case [9], and they can both be obtained from spectrum calculations. Furthermore, nonperturbative renormalization using the method of Rome-Southampton [11] can be carried out in a straightforward manner. Finally, the success of our earlier mixed-action lattice calculation of B_K [12] indicates that the mixed-action method is also a good way to determine $K \to \pi\pi$ matrix elements.

Lattice simulation parameters

Table: Ensembles and quark masses used for the preliminary determination of Re(A_2). The columns show the (i) approximate lattice spacings, (ii) lattice volumes, (iii), nominal up/down (m_l) and strange quark (m_h) masses in the sea, (iv) corresponding pseudoscalar taste pion mass, (v) partially quenched valence quark masses (m_x), (vi) lightest available domain-wall pion mass, and (vii) number of configurations analyzed on each ensemble.

		sea sector		valence sector		
a(fm)	$\left(\frac{L}{a}\right)^3 \times \frac{T}{a}$	am_l/am_h	$am_{\pi,5}$	am_x	am_π	$N_{\rm conf.}$
0.06	$64^3 \times 144$	0.0018/0.018	0.06678(3)	0.0026, 0.0108, 0.033	0.06376(96)	96
0.06	$48^3 \times 144$	0.0036/0.018	0.09353(7)	0.0036, 0.0072, 0.0108, 0.033	0.07458(76)	129
0.12	$24^3 \times 64$	0.005/0.05	0.15970(13)	0.007, 0.02, 0.03, 0.05, 0.065	0.1718(11)	218
0.12	$20^3 \times 64$	0.007/0.05	0.18887(8)	0.01, 0.02, 0.03, 0.04, 0.05, 0.065	0.1968(08)	279

Preliminary determination of Re(A_2)

We illustrate the new approach using the $(27,1)$ $\Delta I = 3/2$ $K \to \pi\pi$ matrix element, which, when combined with continuum Wilson coefficients [10], allows us to determine Re(A_2). The figure at right shows the interpolation to the unphysical kinematics point $m_K = m_K^{\rm phys}$ and $m_\pi = m_K^{\rm phys}/2$. We then correct this point using leading-order $SU(3)$ χPT:

$$\langle \pi^+\pi^- | \mathcal{O}^{\Delta I = 3/2}_{(27,1)} | K^0 \rangle_{\rm LO} = -4i\alpha_{27}(m_K^2 - m_\pi^2)/f_K f_\pi^2 \quad (1)$$

$$(\alpha_{27})_{\rm LO} = -B_0 f_0^4/3\,, \quad (2)$$

where f_0 and B_0 are the pion decay constant and B_K in the chiral limits, respectively. Because the renormalization factor for the $(27,1)$ $\Delta I = 3/2$ operator is the same as B_K, we can use the result for Z_{B_K} from Ref. [12]. We obtain

$$\mathrm{Re}(A_2) = 1.496(70) \times 10^{-8}, \quad (3)$$

where the error is statistical only. This agrees with the experimental measurement, $\mathrm{Re}(A_2)_{\rm exp} = 1.50 \times 10^{-8}\mathrm{GeV}$ [15].

As shown in the table at right, we estimate that the total uncertainty in our preliminary result is below 20%, and this will improve further with the use of our full data set. The χPT truncation error and error from the uncertainty in the LEC's will also decrease with the use of the 1-loop χPT correction factor. We restrict our lightest valence quark mass to maintain $m_\pi L \gtrsim 3.5$, and estimate that our finite volume errors are a few percent using 1-loop FVχPT. We will perform an explicit finite-volume study, however, before publication. Our preliminary result is renormalized using lattice perturbation theory, but we will complete the nonperturbative renormalization on the $a \sim 0.06$ fm ensembles in the future.

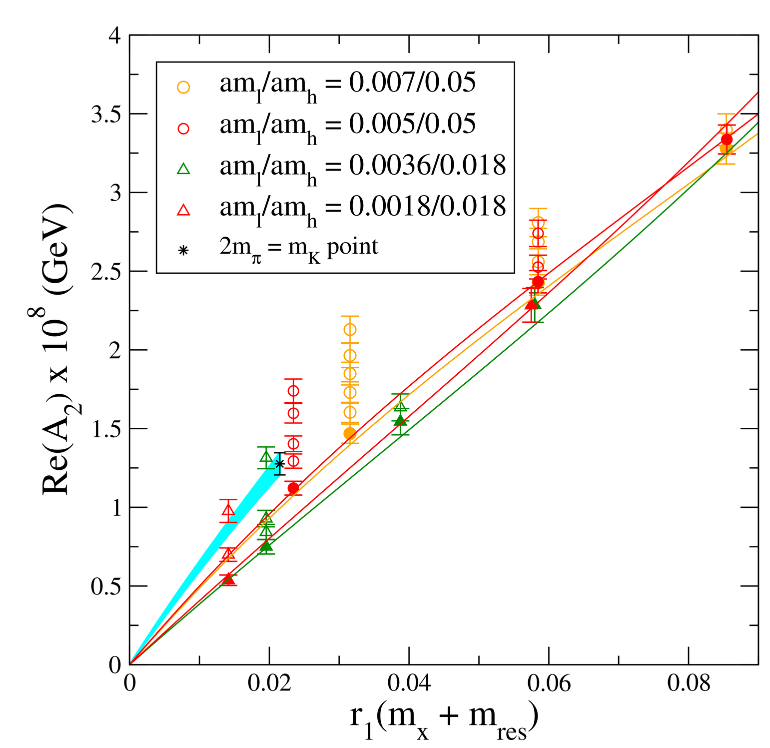

Figure: Determination of Re(A_2) at the unphysical kinematics point $m_K = m_K^{\rm phys}$ and $m_\pi = m_K^{\rm phys}/2$. Circles (squares) denote $a \sim 0.12$ fm ($a \sim 0.06$ fm) data. Fit lines correspond to the degenerate mass case and should pass through the filled symbols.

New approach to $K \to \pi\pi$ matrix elements

Li and Christ concluded in Ref. [13] that large uncertainties in the LO and NLO $SU(3)$ LEC's and the slow convergence of $SU(3)$ χPT at the scale of the kaon mass lead to large errors that make the extraction of $K \to \pi\pi$ matrix elements using the "indirect" method unreliable. Our method therefore addresses these drawbacks of the traditional approach and improves upon it in several ways. In the combined chiral-continuum extrapolation, we use the physical pseudoscalar meson masses and decay constant. This leads to better fits as measured by the correlated χ^2/d.o.f.; nontrivial agreement between the NLO mixed-action χPT prediction for the isovector scalar correlator and lattice simulation data lends support to this approach [14]. When the fixed-order (NLO) fit is bad, we approximate higher order terms in the chiral expansion by polynomials. This leads to larger leading-order terms and hence suggests better convergence than was found in Ref. [13]. For example, the two figures at right show the $SU(3)$ χPT fit of B_K along with the sizes of the various contributions [12].

Despite these findings, NLO χPT corrections can still be as much as 50% for some quantities. Therefore, to achieve the precision needed for $K \to \pi\pi$ we do not rely on the "indirect" method alone. Rather, we combine indirect and direct methods in a cost-effective way. We bypass the Maiani-Testa theorem by simulating with both pions at rest. We fit the numerical data to NLO mixed-action χPT plus higher-order analytic terms, extrapolate to the continuum, and interpolate to the point at which $m_K = m_K^{\rm phys}$ and $m_\pi = m_K^{\rm phys}/2$. Thus we avoid needing to rely on $SU(3)$ χPT to extrapolate to the physical kaon mass, where we expect higher-order corrections to be significant. We then correct this unphysical kinematics point using fixed-order $SU(3)$ χPT. The low energy constants needed for this correction can be obtained from simpler quantities such as f_K, K^0-$\bar{K}^0$, and $K \to \pi$. Since the kaon is tuned to its physical value, terms involving only kaons are correct to all orders in the $SU(3)$ chiral expansion; we therefore expect higher-order corrections to be small.

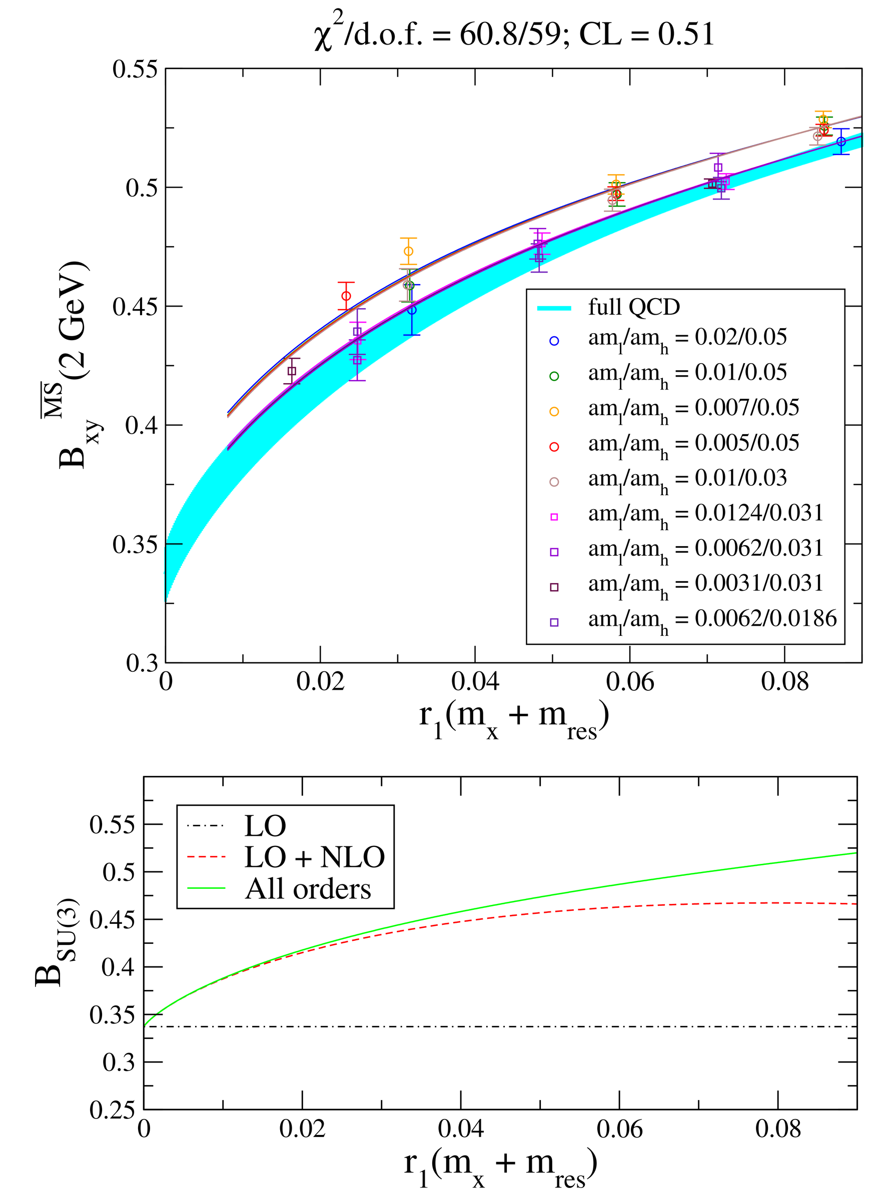

Figure: Chiral and continuum extrapolation of B_K [12] (upper plot) along with convergence of the $SU(3)$ χPT fit (lower plot). Circles (squares) denote $a \sim 0.12$ fm ($a \sim 0.09$ fm) data. Only degenerate points are shown, but the fit also includes non-degenerate data. The cyan band is the degenerate quark mass full QCD curve ($m_x = m_y = m_l = m_h$) in the continuum limit. The y-intercept of the band gives the LEC B_0, the value of B_K in the $SU(3)$ chiral limit. The right-most point on both plots corresponds to $\sim m_s/2$.

Conclusions

This new method for determining $K \to \pi\pi$ matrix elements from lattice simulations is less costly than direct simulations of $K \to \pi\pi$ at physical kinematics. It improves, however, upon the traditional "indirect" approach of constructing the $K \to \pi\pi$ matrix elements using NLO $SU(3)$ χPT, which can lead to large higher-order chiral corrections. Using the explicit example of the $\Delta_I = 3/2$ $(27,1)$ operator to illustrate the method, we obtain a value for Re(A_2) that agrees with experiment and has a total uncertainty of $\lesssim 20\%$. Although our simulations use domain-wall valence quarks on the MILC asqtad-improved gauge configurations, this method is more general and can be applied to data computed with any fermion formulation.

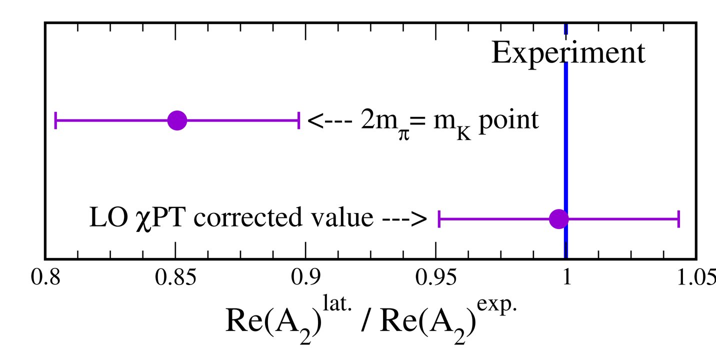

Figure: Correction of Re(A_2) to physical kinematics using LO χPT. Note that the leading-order χPT correction is only 15%.

uncertainty	$\mathrm{Re}(A_2)$
statistics	4.7%
χPT truncation error	9%
uncertainty in leading-order LECs	4%
discretization errors	4%
finite volume errors	few percent%
renormalization factor	3.4%
scale and quark-mass uncertainties	3%
Wilson coefficients	few percent
total	less than 20%

Table: Estimated error budget for $\mathrm{Re}(A_2)$.

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