ACADEMIC TRAINING No. 51

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"THERMODYNAMICS OF STRONG INTERACTIONS"

by

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This series of 6 lectures will be given on

Tuesdays and Thursdays, at 11.00, in the Auditorium

on the following dates:

February 23 and 25, March 2, 4, 9 and 11

PROGRAMME

Separation of kinematics and thermodynamics.
Limiting fragmentation.
Motivation of thermodynamic description.
A short repetition of the principles of statistical thermodynamics.
Universal hadronic bootstrap as a soluble problem in statistical thermodynamics.
The results: exponential mass spectrum and maximal temperature.
Relation to dual models.
Application to hadron collisions. Productive spectra, various mean values. Inclusive and exclusive experiments.
Application to astrophysics.

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1. INTRODUCTION

Thermodynamics of strong interactions is an idealization, consisting in an attempt to describe the properties of highly excited hadronic matter by means of the technical tools of statistical thermodynamics, in fact of simple-minded equilibrium thermodynamics. Nowhere shall I use irreversible thermodynamics, nor will I consider thermic interaction between systems with relative fast motions. Everything, as far as statistical thermodynamics is concerned, is considered in some sort of local rest frame. Here, however, we use fully relativistic statistical thermodynamics with unlimited creation and absorption of particles. In particular, the energy of a particle is -- in contrast to the convention used in textbooks -- given by $E = \sqrt{p^2 + m^2}$ and the total energy of a system is the sum $\sum E_i$ of all its particles; it includes thus all rest masses.

In order to apply equilibrium thermodynamics to highly excited hadronic matter, which we can produce only in high energy hadron collisions and which therefore, as a whole, is not in peaceful equilibrium but in a state of vehement and complicated collective motion, we have to postulate that the strong forward-backward motions can be kinematically separated from the isotropic heat motion. In fact, we even have to postulate,that the collective motions appear only in the direction of the collision axis and do not give rise to turbulence: if there were turbulence, then we would not know how to separate isotropic turbulence from isotropic heat motion, and the predictions of the thermodynamic description of highly excited hadronic matter could not be compared to the experiment. Thus we formulate:

> Postulate 1
>
> In high energy collisions of hadrons, collective motions have only components in the direction of the collision axis. It is possible to find a continuum of co-moving Lorentz frames (local rest frames), such that a co-moving observer will, in his neighbourhood, see only thermic motion. Turbulence is absent.

This postulate is based on experience; in particular, the absence of turbulence is concluded *a posteriori* from the fact that thermic motion is observable (transverse momentum distribution), and in close agreement with what one expects from pure thermodynamics undisturbed by turbulence. As we will see that the heat motion is small and nearly independent of the collision energy, while turbulence should grow with the collision energy, it is really surprising that even at very high energies ($p_0 \sim 10^6$ GeV/c), it should be negligible compared to thermic motion.

After having stated Postulate 1, we are left with a system without collective motions: the one seen by the co-moving observer in a small neighbourhood.

This system, if describable by statistical thermodynamics, must then have two properties:

- many degrees of freedom,
- some sort of internal equilibrium,

and being taken out of the world of hadrons, it has a third property:

- strong internal interactions.

In view of the difficulties encountered in statistical thermodynamical of real (= interacting) gases, one might expect the presence of strong interactions making the task impossible. As a kind of miracle, it turns out that just the extreme strength of strong interactions enables one to incorporate them in a formalism which is almost as simple as that of a non-interacting (ideal) gas. Our gas will consist of infinitly many components; it is this infinity of components that represents the strong interactions. In a way, this is what everybody has done all the time: if one wishes to describe helium gas, then one does not start with protons, neutrons, electrons, and light quanta in a box, and then apply strong interactions to bind protons and neutrons to α-particles and electromagnetic interactions to bind the electrons to them, one just starts with the atoms and need not care any more about strong- and a good part of electromagnetic interactions. Similarly, in our case, the resonant states

of strong interactions will be treated as particles in their own right. While we are formally dealing with free particles, the bulk of strong interactions will be represented by the rich spectrum of hadrons.

It will turn out, that the very fact that we assume the object with the above three properties to exist and to be describable by statistical thermodynamics, fixes up the structure of this theory and forces us into a general hadronic bootstrap resulting in the statement that:

> Each hadron consists of all other hadrons and none is elementary; the number of different kinds of hadrons is infinite and their mass spectrum grows exponentially. The macroscopic sign of this microscopic property is the existence of a universal ultimate temperature T_0 for all matter in equilibrium.

So far for the strong interactions governing our gas. The other two requirements: many degrees of freedom and some sort of internal equilibrium are satisfied: the first, because particle creation and absorption makes the number of degrees of freedom virtually infinite, even if the average number of created real particles is only ten or so. The second, because unlike in ordinary thermodynamics this "some sort of equilibrium" need not be achieved by means of a large number of collisions of already existing particles, namely:

Our system, in high energy collisions, decays in about 10^{-23} s and somehow equilibrium must be achieved in even a shorter time. This cannot go via many collisions among each other of already created secondaries. That would not only take too long, but also it would make it difficult to understand how and why even the masses and numbers of the particles created depend on the very equilibrium which they shall bring about by their mutual collisions. The equilibrium on whose existence we have little doubt, must therefore be something instantaneous, at least it seems to take much less that 10^{-23} s to establish it.

In fact, it will be shown that the particles are created into a pre-established equilibrium.

So, let us assume that the requirements are somehow fulfilled and that a thermodynamic description is worth trying. The plan of these lectures is then to first say something about the separation of collective from thermic motions and then to go through the thermodynamic part in some details. At the end the purely empirical collective motions will be added to the more theoretical thermodynamic description, in order to confront the model with experiments.

Our units are:

$$\hbar = c = k \text{ (Boltzmann's constant)} = 1 \ ,$$

so that the only arbitrary unit is that of the energy, which we measure in MeV and/or GeV. As a convenient inconsequence, we shall sometimes use the words centimeter, second, etc. in the text.

2. SEPARATION OF KINEMATICS AND THERMODYNAMICS

Statistical thermodynamics describes systems with many degrees of freedom by considering the ensemble of all possible states, characterizing classes of them by macroscopic observables and taking the largest classes as representing reality. The largest classes contain states, whose typical property is uniformity. It is for this reason that one obtains isotropic distributions in momentum space and coordinate space, in particular distributions of the Maxwell-Boltzmann-Planck type.

The angular distribution of particles created in high-energy hadron collisions seems to rule out any thermodynamic description of the production process. This need not be true: it is seen by imagining two gas clouds colliding in vacuum with a relative speed exceeding by far the mean thermic velocity inside each one: seen from the collision frame the angular distribution of particles after the collision is anisotropic, but it might well be that at any place a co-moving coordinate frame can be found seen from which all motion in a small neighbourhood is isotropic and thermic.

Seen from the collision frame, there might be a continuum of such co-moving frames with different speeds. Our Postulate 1 supposes that such frames do indeed exist and that they all move in the direction of the collision axis, representing thus a continuum of collective motions. It is by no means *a priori* evident that Postulate 1 is true for colliding hadronic matter, but it is empirically true and will not be discussed further; we simply take it for granted.

Once we know two things:

- the local thermodynamic parameters (temperature, density, pressure etc.),
- the distribution of collective velocities,

then we can describe the highly anisotropic global aspect in any coordinate frame. As this global picture is a superposition of Lorentz-transformed local pictures, it is clear that no coordinate frame for the global description is better than any other one. We shall formulate everything in the collision centre-of-mass frame, but also consider the laboratory frame and the projectile rest frame.

2.1 Collective velocity distribution

Our present consideration is purely classical. At a given instant, say t = 0, a hadron-hadron collision may look like this:

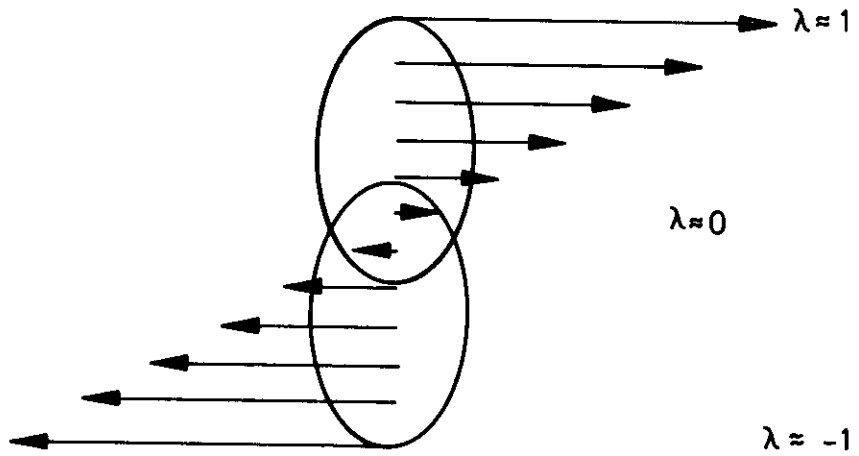

Fig. 1

At each point $\vec{x}$, there is a velocity distribution centred around a mean value $\vec{v}$ which we call the local collective velocity. This velocity distribution is a rapidly varying function of $\vec{x}$, t and $\vec{v}$, and it depends also on the impact parameter b. We measure collective velocities conveniently, not by v itself, but by

$$\lambda = \text{sign}(v)\,\frac{\gamma - 1}{\gamma_0 - 1} \tag{2.1}$$

where $\gamma = (1 - v^2)^{-\frac{1}{2}}$ is the local Lorentz factor and $\gamma_0 = (1 - v_0^2)^{-\frac{1}{2}}$ the centre-of-mass (c.m.s.) Lorentz factor of the incoming particles. For definiteness we assume here a proton-proton collision. One can easily see that $|\lambda|$ is the ratio of local kinetic energy density to incoming kinetic energy density. This variable has the advantage of being restricted to an energy-independent interval. Let then

$u(\lambda, \vec{x}, t, b)$ be the probability density to find the collective velocity λ at the place $\vec{x}$ at time t in a collision with the impact parameter b. (2.2)

2.2 Local excitation energy density

We now go into one of the co-moving Lorentz frames (λ-frame) and find ourselves surrounded by excited hadronic matter without collective motions -- at least in a small neighbourhood. The local energy density is greater than that of the incoming proton, because part of the kinetic energy has been transformed into excitation energy. In order to calculate the local energy density belonging to λ, we formulate:

Postulate 2

All the kinetic energy of the incoming particle ($|\lambda| = 1$) which has disappeared in decelerating hadronic matter to $|\lambda| < 1$ is adiabatically converted into local excitation energy (heat).

This postulate is as little *a priori* obvious as the first, but it is plausible: locally created heat can be transported away only through emission of particles, but before that starts, it must become hot. We will see that production rates are governed by a Boltzmann factor exp (-m/T) and that there is a maximum temperature so that always $T < T_0 \approx m_\pi$; this means that, whatever the temperature actually reached may be, particle production is relatively exponentially negligible at any lower temperature, and it is absolutely negligible for any much lower temperature. In other words: if a piece of hadron matter is decelerated and heated up, then, at any state of deceleration and heating, all previous heat losses are negligible compared to the actual heat content just reached. Thus it is heated up (almost) adiabatically and then decays. The very fact that we observe high mass resonant states may be interpreted in these same words.

If we now take Postulate 2 for granted, we follow a piece of hadron matter through the process of deceleration. We define

$$\begin{aligned} \varepsilon_0 &= m_p/V_0 \quad = \text{average proton rest energy density} \\ V_0 &= \frac{4\pi}{3} m_\pi^{-3} = \text{natural hadron volume} \quad . \end{aligned} \tag{2.3}$$

Our piece of matter (filling a unit volume) has total energy $E_0 = \varepsilon_0 \gamma_0$ before the collision. When it reaches a velocity $\lambda < 1$, then it has been excited to an energy density $\varepsilon > \varepsilon_0$, and it moves with Lorentz factor γ. Thus its total energy is now $E = \varepsilon\gamma$. Postulate 2 says $E = E_0$, hence

$$\varepsilon = \frac{\varepsilon_0 \gamma_0}{\gamma} \tag{2.4}$$

2.3 Local production spectrum

By equation (2.4) we assign to any collective velocity λ a definite energy density. The local particle production will, of course, be a function of the local energy density only, i.e. not depend on when, where, and on what impact parameter this local situation was created. Let then

$f_m(p', \varepsilon)$ be the isotropic momentum spectrum of particles m produced in the λ-frame at local energy density ε . (2.5)

We shall determine this function from thermodynamics later.

2.4 The velocity weight functions

Let

$L^{(R)}(\lambda, \gamma_0)$ be the Lorentz transformation which takes the the spectrum from the λ-frame to the arbitrary frame R . (2.6)

Then the spectrum of particles m, seen in R, will be

$$W^{(R)}(\vec{p})\, d^3p =$$

$$= \int d^3x\, dt\, 2\pi b\, db\, u(\lambda, \vec{x}, t, b)\, L^{(R)}(\lambda, \gamma_0) \left\{ f_m(p', \varepsilon) d^3p' \right\} d\lambda \tag{2.7}$$

We see that neither $L^{(R)}(\lambda, \gamma_0)$ nor the spectrum depend on the coordinates $\vec{x}$, t, b, but only on λ and γ_0. We can thus at once integrate over $\vec{x}$, t, b and define the

velocity weight function

$$F(\lambda, \gamma_0) \equiv \int d^3x\, dt\, 2\pi b\, db\, u(\lambda, \vec{x}, t, b, \gamma_0) \tag{2.8}$$

Thus, $F(\lambda)$ collects all contributions to a certain value of λ coming from the whole history and all places of a given collision and then averages it over impact parameters, i.e. over a large ensemble of "identical" collisions. All these contributions have the same λ and therefore the same local momentum spectrum of particles m and the same Lorentz trans-

formation. $F(\lambda)$ attaches a weight to each λ. Thus

$$W^{(R)}(\vec{p})\,d^3p = \int_{-1}^{1} F(\lambda)\, L^{(R)}(\lambda, \gamma_0) \left\{ f_m(p', \varepsilon)\, d^3p' \right\} d\lambda \qquad (2.9)$$

We consider $F(\lambda)$ as an empirical function, which serves to disentangle thermodynamics from the rest and which cannot be calculated within the model. It can be determined from experiments, once $f_m(p', \varepsilon)$ is known. Without going into much detail, we remark a few properties of $F(\lambda)$:

- One might expect that each sort of particle requires its own function $F_i(\lambda)$. In fact, only two such functions are needed to describe rather well all production spectra: one, $F(\lambda)$, for the newly created particles and another one, $F_0(\lambda)$, for the throughgoing particles (i.e. the incoming protons) and their decay products.

- We normalize the functions $F(\lambda)$ and $F_0(\lambda)$ such that:

$$\int_{-1}^{0} F(\lambda)\, d\lambda = \int_{0}^{1} F(\lambda)\, d\lambda = 1 \qquad (2.10)$$

This is convenient in the case of pp collisions, where, for symmetry reasons, $F(\lambda) = F(-\lambda)$ and $F_0(\lambda) = F_0(-\lambda)$. If target and projectile are different (πp etc.), the functions are no longer symmetric.

- One might expect that the functions $F(\lambda)$ and $F_0(\lambda)$ depend on the collision energy. Our choice of a fixed interval $-1 \leqslant \lambda \leqslant 1$ and of a fixed normalization permits only a change of shape, and we thus hope to minimize the possible energy dependence. From experiments between 10 and 70 GeV/c no energy dependence can be concluded.

In the course of the lecture several further remarks will be made about these functions. More details about them are found in the article, where they were first introduced [Hagedorn and Ranft, 1968, from now on quoted as (II)] .

One may guess, how these functions might look like by considering a multi-peripheral graph drawn in a somewhat inconventional way:

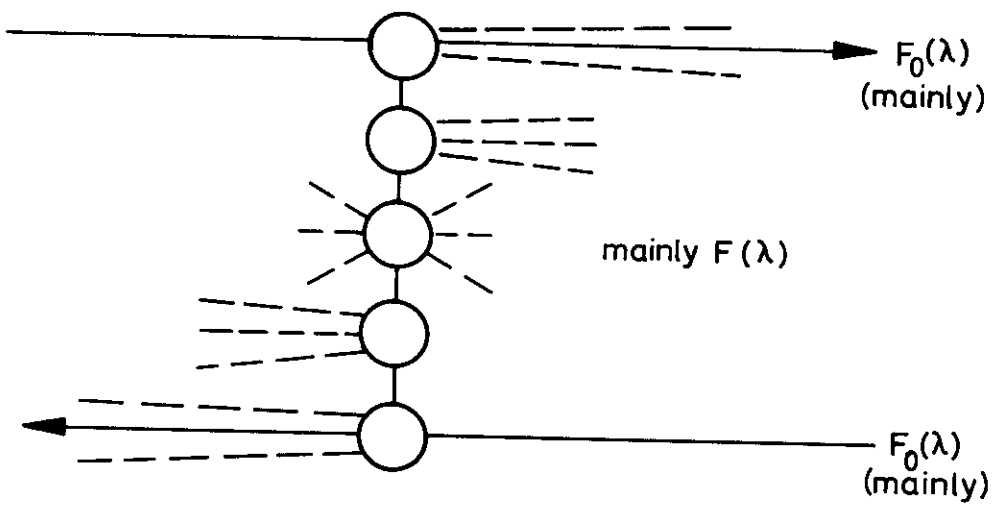

Fig. 2

On this basis one expects that $F(\lambda)$ will be centred around small $|\lambda|$ and $F_0(\lambda)$ around $|\lambda| \approx 1$. This is indeed what one finds experimentally. Fig. 3 shows the actual functions as determined by fitting the model to data. A calculation based on a multi-Regge description has yielded a theoretical function $\phi(\lambda)$ which is rather close to the empirical $F(\lambda)$ (Ranft and Ranft, 1970).

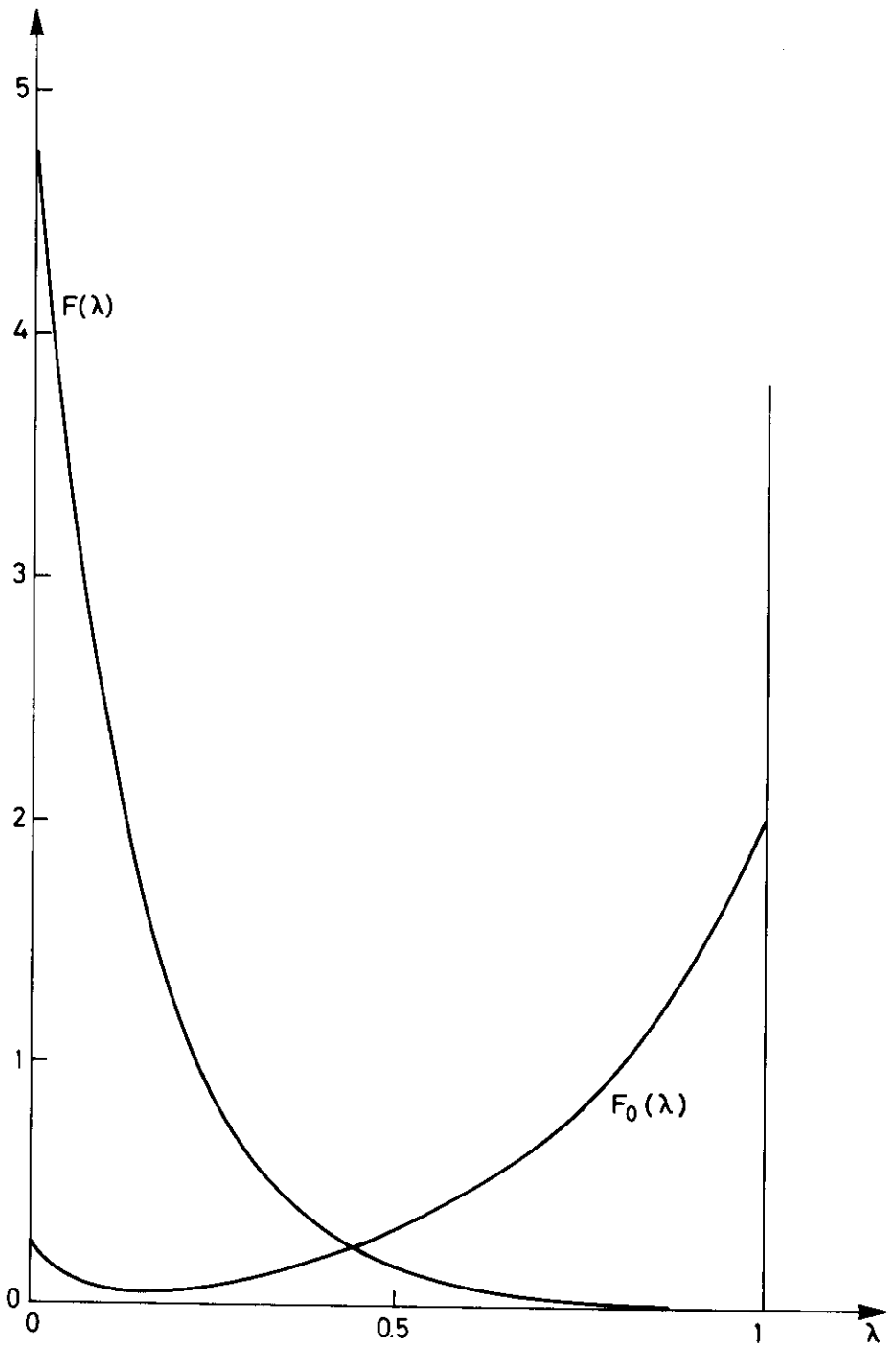

Fig. 3 The velocity weight functions $F(\lambda)$ and $F_0(\lambda)$. The first is for newly created particles and puts more weight on low velocities = = central parts; the second belongs to throughgoing particles which can be very peripheral. In fitting these functions to experiments they had the freedom to choose more complicated shapes including several bumps, even δ-like. They took the simple shapes and there is no evidence for or against δ-like behaviour at $\lambda = 0$ and/or $\lambda = 1$. With two such energy independent functions all spectra from 12 to 70 GeV/c are rather well fitted. So far a systematic change of shape with primary energy of $F(\lambda)$ and $F_0(\lambda)$ could not be detected. Figure taken from (II).

2.5 Limiting fragmentation

This notion, created by Benecke et al., 1969, says that in a collision the target and the projectile break up into fragments, such that for E_0 [collision energy*)] $\to \infty$ the distribution of fragments tends to a limiting function in the respective rest frames of the colliding particles; for the target particle this hypothesis reads

$$\lim_{E_0 \to \infty} W_m^{lab}(\vec{p}, E_0) = \rho_m^{(1)}(\vec{p}) \tag{2.11}$$

Similarly for two-particle distributions etc.

This limit is not uniform in $|\vec{p}|$: the higher we choose the value of $|\vec{p}|$ the higher will be the value E_0 above which the deviations from $\rho_m^{(1)}(\vec{p})$ become arbitrarily small.

I shall now show that this hypothesis, if one accepts Eq. (2.9), is equivalent to the hypothesis made in (II) that $F(\lambda)$ and $F_0(\lambda)$ become energy independent for sufficiently large E_0. Consider a fixed value of $|\lambda| = (\gamma - 1)/(\gamma_0 - 1)$. If E_0 becomes very large, then $\gamma_0 = E_{CM}/2m_p$ becomes very large and for fixed λ also γ becomes very large. In that case $\lambda \approx \gamma/\gamma_0$ and the corresponding energy density is $\varepsilon \approx \varepsilon_0/\lambda$. Hence in that case in Eq. (2.9) the spectrum $f_m(p', \varepsilon)$ in the λ-frame becomes $f_m(p', \varepsilon_0/\lambda)$, i.e. it becomes independent of the primary energy and depends only on λ. The same is true by assumption for $F(\lambda)$. Therefore, if we fix a value λ_0 as small as we wish, then the whole contribution

$$W_m^{(\lambda_c)}(\vec{p}) = \int_{-i}^{-\lambda_c} F(\lambda) L(\lambda, \gamma_0) \{ f_m(p', \varepsilon_c/\lambda) \} d\lambda \tag{2.12}$$

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*) We denote the primary laboratory energy by E_0 and the CMS collision energy by E_{CM}.

depends on γ_0 only through the Lorentz transformation $L(\lambda,\gamma_0)$ if we chose the collision energy E_0 sufficiently large, namely such that $\lambda_0\gamma_0 >> 1$, or, with $\gamma_0 = E_{CM}/2m$: such that $E_{CM} >> 2m/\lambda_0$. But this whole contribution (2.12) is moving backward in CMS with velocities λ corresponding to Lorentz factors $\gamma = \lambda\gamma_0$. The CMS moves forward in lab. with Lorentz factor γ_0 and therefore, seen from the lab., the forward Lorentz transformation with γ_0 and the backward motion in CMS with $\gamma \approx \lambda\gamma_0$ compensate to some extent. This becomes clear if one writes down the laboratory four-velocity of a piece of excited matter (in short "fireball") moving backward with velocity $-|\lambda|$ in CMS: for γ and $\gamma_0 >> 1$ one finds

$$V_{lab}(\lambda) \equiv (\gamma_\lambda, \beta_\lambda\gamma_\lambda) = \left(\frac{1+\lambda^2}{2|\lambda|}, \frac{1-\lambda^2}{2|\lambda|}\right) \qquad (2.13)$$

This is then, for $\lambda > \lambda_0$ and $E_{CM} >> 2m/\lambda_0$, independent of the primary energy; furthermore, particle emission from this fireball is energy independent too ($\varepsilon = \varepsilon_0/\lambda$) and therefore the fireball belonging to λ contributes an energy independent part to the lab. spectrum. This is true for all $-1 \leq \lambda \leq -\lambda_0$. Thus -- under the assumption that $F(\lambda)$ and $F_0(\lambda)$ are energy independent -- the whole contribution (2.12), transformed to the lab. system, becomes energy independent:

$$W_{m,lab}^{(\lambda_c)}(\vec{p}) = \int_{-1}^{-\lambda_c} F(\lambda)\, L_{lab}(\lambda) \left\{ f_m(p', \varepsilon_c/\lambda) \right\} d\lambda \qquad (2.14)$$

with $L_{lab}(\lambda)$ transforming from the λ-frame to the lab. frame using the four velocity (2.13) which depends only on λ and no longer on γ_0. In the words of Benecke et al.: "it stems from breaking up the target particle". The next contribution, from $-\lambda_0$ to λ_0 is "at rest" in CMS and therefore in lab. has energies of the order of $\gamma_0 \sim \sqrt{E_0}$ while the part coming from λ_0 to 1 goes forward with $\gamma \sim \lambda\gamma_0$ in CMS and therefore in the lab. has energies of order $\gamma_0^2 \sim E_0$. This contribution comes from the breakup of the projectile particle. Thus, there is, in the lab. frame, one part from $(-1 \leq \lambda \leq -\lambda_0)$ of the spectrum which for $E_0 \to \infty$ stays unchanged, one part (from $-\lambda_0 < \lambda < \lambda_0$) moving on to higher and higher energies of order $\sqrt{E_0}$ and one part $(\lambda_0 \leq \lambda \leq 1)$ moving still faster towards higher energies of order E_0 . In symmetric collisions like pp there is of course a complete symmetry between these classes of contributions: by transforming from the lab. frame to the projectile frame one interchanges them in an obvious way. For non-symmetric collisions like πp there is only a qualitative or approximate symmetry because then $F(-\lambda) \neq F(\lambda)$.

The value of λ_0, that is, the borderline between the contribution to the "limiting fragmentation" or "breakup of the target" and the other parts of the spectrum depends on choice: the smaller λ_0 is chosen, the larger is the primary energy E_0 above which the contribution (2.12), seen from lab., becomes energy-independent: $\lambda_0\gamma_0 >> 1$ implies $E_{CM} >> 2m/\lambda_0$. All our previous considerations are invalid for $\lambda << 2m/E_{CM}$. Thus limiting fragmentation is equivalent to

$$\lim_{E_0 \to \infty} F(\lambda, E_0) = F(\lambda)$$
$$\lim_{E_0 \to \infty} F_0(\lambda, E_0) = F_0(\lambda) \qquad (2.15)$$