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4th lecture

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"THERMODYNAMICS OF STRONG INTERACTIONS"

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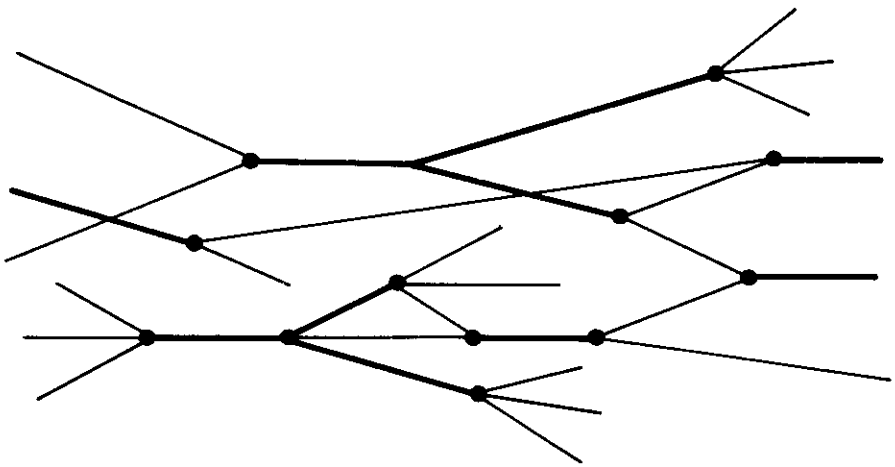
4.5 Duality and bootstrap

Let us now look at a fireball graphically: its particles interact strongly, two colliding ones may be just deflected or they may go into a resonant state, in either case any number of other particles of any sort may be created, which interact in the same way, -- on the whole, it looks like this:



Now, if in this picture we draw *all* possible lines, allow *any* number of lines emerging from a vertex and allow the lines to carry *any* of the masses of strong interactions: we claim, that then this *is* strong interactions. One might object, that all this are S-channel resonances, and that the typical high-energy behaviour of the virtual subcollisions are not taken into account. However, if our $\rho(m)$ is going to contain the contributions of all phaseshifts, then nothing has been forgotten.

Also duality (Dolen, Horn, Schmid, 1967, 1968; Schmid 1968) tells us that even the high-energy Regge-behaviour of the subcollisions is

properly taken into account by a sum over s-channel resonances plus a background [which we included in $\rho(m)$].

Thus, supposing the *complete* $\rho(m)$ to be known, the thermodynamics of our gas will amount to a complete (and dual) picture of strong interactions, as far as a thermodynamic description is possible at all.

Now we shall see that the simple assumption that this thermodynamical picture corresponds to something real, forces us to accept a bootstrap mechanism which fixes the mass spectrum (almost uniquely) and thereby the whole theory. Thus, either this theory works or it does not -- but it is essentially fixed up by the assumption that it exists at all. That goes as follows (here comes the promised circular syllogism):

1) Suppose that a thermodynamic description of strong interactions is possible; in other words: that its objects, the fireballs, exist.

2) If fireballs exist, they may have any total energy $E = m$; their level density $\sigma(m, V)$ will be a function [Laplace transform of $Z(V, T)$] which is calculable inside the theory, once the mass spectrum $\rho(m)$ is given.

3) Consider a continuum of fireballs with different masses: somewhere when the mass becomes smaller and smaller, selection rules begin to make a thermodynamic description less and less applicable, and fireballs gradually obtain, according to their quantum numbers, individual personalities: they become what we call resonances. On the other hand, we know that with increasing mass, resonances begin to decay statistically into more and more decay channels, some of which contain resonances themselves. In other words: resonances with greater and greater mass gradually become what we call fireballs. Thus, fireballs *are* resonances and vice versa. We call all of them fireballs.

4) A thermodynamic description of fireballs which aims to include all internal strong interactions as completely as possible, ought to do so by using the most *complete* $\rho(m)$, *which counts all particles and all resonances and, since resonances are fireballs, also all fireballs.*

5) As $\sigma(E, V_0)$ is the number of different fireballs of hadronic size V_0 in $\{E, dE\}$, the mass spectrum $\rho(m)$ must be continued beyond the experimentally known region to include $\sigma(m, V_0)$. In other words: $\rho(m)$ and $\sigma(m, V_0)$ are thermodynamically equivalent:

$$\lim_{m\to\infty} \frac{\ln \rho(m)}{\ln \sigma(m, V_0)} = 1 \qquad (4.10)$$

We shall show that this condition can be fulfilled.

6) A thermodynamical system is *completely* described by its partition function or, equally well, by its density of states $\sigma(E, V)$, which can be calculated once $\rho(m)$ is given. The condition of the equivalence of $\rho(m)$ and $\sigma(m, V_0)$, determines essentially their asymptotic behaviour (as we shall prove) and, since $\rho(m)$ is known experimentally for small m, it is known completely; hence, our thermodynamics is essentially (we shall see to what extent) determined; it *is* a thermodynamics of strong interactions.

Thus our supposition No. 1 of above is shown to be fulfilled by construction, the circular reasoning is consistent. In order to show that it corresponds to physical reality, we have to find out its consequences and check them with experiments.

Before we work out the whole formalism, a few comments:

- Our above circular reasoning is not only a logical bootstrap, it indeed corresponds to the idea of hadronic bootstrap:

fireballs ≡ resonances are *at the same time*

i) object of the description,

ii) elementary constituents of this object,

iii) generating the forces creating this object.

Thus a complete and universal hadronic bootstrap is solved, if the condition (4.10) is fulfilled. This bootstrap, because of its generality and its thermodynamic formulation, cannot be expected to give any details

on the bootstrapped particles (as limited bootstraps can do), because, apart from a few low-lying resonances, these particles are fireballs, and for the birds-eye of thermodynamics all fireballs are equal (apart from their mass).

- We shall not try to achieve $\rho(m) \to \sigma(m, V_0)$, because $\sigma(m, V_0)$ contains more states than $\rho(m)$: the hadronic mass spectrum counts each resonance only once [apart from the factor $(2J + 1)(2I + 1)$] disregarding the states due to its possible momenta -- they appear in $\sigma(E, V)$ --, while $\sigma(E, V)$ is calculated under neglect of momentum conservation and therefore contains the states corresponding to the possible motions of the fireball as a whole. Comparing eqs. (3.35a) and (3.36a), we see that for non-relativistic particles $\sigma(E, V)$ is by a factor $E_{kin}^{3/2}(mN)^{3/2}$ greater than $\sigma(E, V, \Delta P)$; our fireballs will turn out to contain mostly heavy, non-relativistic particles, whose total rest mass is $\sum m \approx E$. Thus, for a fireball of energy E, the level densities, with and without momentum conservation, will differ roughly by $E^{2/3}$. Consequently, we expect $\sigma(m, V_0) \approx m^{3/2} \rho(m)$, when m becomes large. The power need not be exactly 3/2: in the first paper on hadron thermodynamics (Hagedorn, 1965), a similar argument, yielding a further factor m^2, was based on the assumption that the resonances in $\rho(m)$ could not have arbitrarily high spin; if Regge trajectories rise linearly to infinity, then that argument fails; if not, the power may become > 3/2.

Our weak condition (4.10), which is sufficient for thermodynamics, makes questions about the precise differences between ρ and σ meaningless.

If, however, we would try to describe everything in the language of phase space (microcanonical ensemble with full energy momentum conservation), then we should require that $\rho(m)$ and $\sigma(m, V_0, \vec{P} = 0)$ themselves (and not only their logarithms) become equal for $m \to \infty$. That this is indeed possible, and leads to a more detailed picture (though to the same thermodynamics), has been shown by Frautschi (1970) in a paper "A statistical bootstrap model of hadrons".

- Once our bootstrap condition is fulfilled, (by an exponential spectrum, as we shall see), we know that $\rho(m)$ *counts everything it should*. However, *what* exactly it counts: how many stable particles, how many sharp or diffuse resonances, how much background?-- these are at present academic questions; their answers may take long and may come from an unexpected direction. In the low-mass region, where particles and resonances are individually known, $\rho(m)$ should be identified with the observed hadronic mass spectrum.

- Our bootstrap reasoning has led to a precise prescription for building up thermodynamics of strong interactions. If one puts the condition (4.10) in words, it reads:

A fire-ball is

> a statistical equilibrium (hadronic blackbbody radiation) of undetermined numbers of all kinds of fireballs, each of which, in turn, is considered to be

- We have gone a long way from a formula, containing a squared matrix element times momentum space [eq. (4.1)], over shifting the interaction from the matrix element via distorted waves into momentum space [eq. (4.6)], to making this procedure as complete as possible by requiring it to be self-consistent [bootstrap condition (4.10)] and thus arriving at an (almost) completely fixed thermodynamics of strong interactions:

> this long way should not obscure the fact that it is only a formal manipulation and generalization (according to known rules of quantum theory) of the starting formula (4.1), namely
>
> Probability = $\int |\text{matrix element}|^2 \times \text{Phase space}$.

It so happens that the result of this generalization fulfils all formal requirements for its description by the formulae of statistical thermodynamics. But the "thermodynamical equilibrium", which is thus described, is not brought about by millions of mutual collisions of given particles;

no: the particles whose equilibrium is described, are born into this *"pre-established" equilibrium which is nothing else than the quantum mechanical probability distribution of all possible final states.* Indeed: Frautschi's (1970) paper, mentioned above, carries through the program resulting from our bootstrap argument, without ever leaving phase space and without ever speaking of temperature or equilibrium. He says: "the reason we choose to work directly with the density of states, at the cost of some awkwardness with the Pauli exclusion principle, is mainly pedagogical: phase space is more familiar to particle physicists, and the independence of the derivation from any assumptions concerning thermal equilibrium is most convincingly demonstrated by not introducing temperature at all".

We shall proceed here with thermodynamics for two reasons: firstly, because by using the notions of thermodynamics, everything becomes surprisingly simple (e.g. the Pauli principle is fulfilled with ease) and physically transparent, secondly, because the applications follow so easily without much formal (though numerical) calculations -- one almost can copy them from elementary textbooks on statistical thermodynamics.

Thus we shall use the words temperature and equilibrium; and these words have their usual physical meaning, but the mechanism, which creates equilibrium and temperature, is entirely different from that of classical statistical thermodynamics. The question, how long does it take to establish equilibrium?, is answered by: no time at all; but what the final state (the one to be observed in a single event) will be, that only turns out when the strong interactions cease to act between the outgoing particles, namely after $\geq 10^{-22}$ sec. Before that, *any* final state has a chance to emerge from the equilibrium. Thus, our "equilibrium" is equivalent to what in field theory are the interpolating fields at $t = 0$, when the particle number operator is not diagonal.

Therefore, the question, whether a resonance lives long enough to participate in this equilibrium, makes no sense; it helps to constitute this equilibrium by its *virtual*, not by its *actual* presence: it is contained in $\rho(m)$ via $(1/\pi)d\delta_1/dp$ and the width of this contribution may be great or small (the life-time may be small or great): it does not matter.